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Dominator Chromatic Number, Bondage Number and Domatic Number of Some Named Graphs

Research Article*

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Abstract: Let $G = (V, E)$ be an undirected graph. A dominator coloring of a graph G is a proper coloring in which every vertex of G dominates each vertex of at least one color class. The Dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G. In this paper, we found the Dominator chromatic number, Bondage number and Domatic number of some named graphs.

Keywords: Proper coloring, Dominator coloring, Dominator Chromatic number, Bondage number, Domatic number, Fruncht graph, Herschel graph, Wagner graph, Moser Spindle graph.

1. Introduction

In this paper, we have taken the graphs to be finite and undirected graphs. A dominator coloring of a graph G is a proper coloring in which every vertex of G dominates each vertex of at least one color class. In this paper we discuss about the Dominator chromatic number, Bondage number and Domatic number of Fruncht graph, Herschel graph, Wagner graph, and Moser spindle graph.

1.1. Definitions

Definition 1.1 (Proper Coloring). *A graph G having no two adjacent vertices receive the same color is said to be proper coloring. The chromatic number $\chi(G)$, is the minimum number of colors required for a proper coloring of G. A color class is the set of all vertices, having the same color. The color class corresponding to the color i is denoted by V_i.*

Definition 1.2 (Dominator Coloring). *A dominator coloring of a graph G is a proper coloring in which every vertex of G dominates every vertex of at least one color class. The convention is that if $\{v\}$ is a color class, then v dominates the color class $\{v\}$. The dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G.*

Definition 1.3 (Bondage Number). *The Bondage number $b(G)$ of a nonempty graph G is the cardinality of a smallest set of edges whose removal from G results in a graph with domination number greater than $\gamma(G)$.*

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Definition 1.4 (Domatic Number). *In a dominating set one can partition the vertex set of G into atleast two disjoint dominating sets. The maximum number of dominating sets into which the vertex set of a graph G, can be partitioned is called the Domatic number of G and it is denoted by d(G).*

Definition 1.5 (Fruncht Graph). *The Fruncht graph is a 3-regular graph with 12 vertices and 18 edges and no non trivial symmetries.*

Definition 1.6 (Herschel Graph). *The Herschel graph is a bipartite undirected graph with 11 vertices and 18 edges which is the smallest non-Hamiltonian polyhedral graph.*

Definition 1.7 (Wagner Graph). *The Wagner graph is a 3-regular graph with 18 vertices and 12 edges. It is the 8-vertex Mobius ladder graph. Mobius ladder is a cubic circulant graph with an even number n vertices, formed from an n-cycle by adding edges connecting opposite pairs of vertices in the cycle.*

Definition 1.8 (Moser Spindle Graph). *The Moser spindle graph or moser graph is an undirected graph and planar graph. It is a embedded as a unit distance graph in the plane. It has 7 vertices and 11 edges.*

2. Dominator Chromatic Number, Bondage Number and Domatic Number of Fruncht Graph

Theorem 2.1. *Let G be the Fruncht graph, then*

(i) The dominator chromatic number of G is 4. (i.e.,) $\chi_d(G) = 4$.

(ii) The bondage number of G is 3. (i.e.,) $\gamma_b(G) = 3$.

(iii) The domatic number of G is 2 . (i.e.,) $d(G) = 2$.

Proof. Let G be Fruncht graph with 12 vertices and 18 edges. The Fruncht graph is a 3-regular graph with 12 vertices and 18 edges and no non trivial symmetries. Let $V = \{v_1, v_2, ..., v_n\}$ be the vertices of G. Let us now determine the dominator chromatic number, domination number, bondage number and domatic number of G.

Case (i): Let us determine the dominator chromatic number of G.

By the definition of dominator coloring, in which each vertex of the graph dominates every vertex of some color class. Let us assign color 1 to $\{v_1, v_9, v_{10}, v_{12}\}$ and assign color 2 to $\{v_2, v_7, v_5, v_{11}\}$. Then assign color 3 to $\{v_3, v_6, v_8\}$ and color 4 to $\{v_4\}$. So this a valid proper coloring and here each vertex of the graph dominates every vertex of some color class. Hence this is a valid Dominator coloring. Therefore $\chi_d(G) = 4$.

Case (ii): Let us determine the Bondage number of G.

From the definition of Bondage number of G, it is the cardinality of the smallest set of edges E such that the domination number of the graph with edge e removed is greater that the domination number of the original graph. We know that the domination number of G is 4. For a Fruncht graph, we remove 3 edges to make the domination number of G to become greater than the domination number of $G - 3e$. Therefore the bondage number of G is 3.

Case (iii): Let us now determine the Domatic number of G.

From the definition of domatic number is the maximum number of disjoint dominating set. Here $\{v_1, v_9, v_{10}, v_{12}\}$, $\{v_2, v_7, v_5, v_{11}\}$ are the dominating set of G. Therefore the domatic number of G is 2. (i.e.,) $d(G) = 2$. □

Example 2.2.

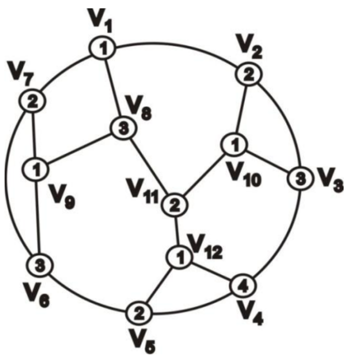

Figure 1. **Fruncht graph**

The color classes of Fruncht graph G is $\{v_1, v_9, v_{10}, v_{12}\} = 1$, $\{v_2, v_7, v_5, v_{11}\} = 2$, $\{v_3, v_6, v_8\} = 3$ *and* $\{v_4\} = 4$. *Therefore* $\chi_d(G) = 4$. *Therefore the Domatic number of Fruncht graph (G) is 2.*

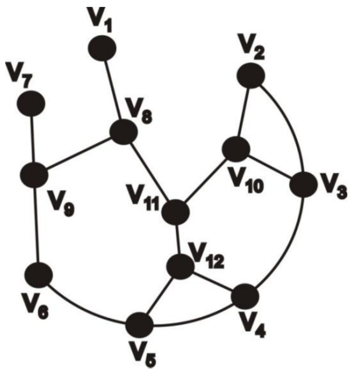

Figure 2. **Bondage number of Fruncht graph (G)**

Here (v_1, v_7), (v_1, v_2), (v_6, v_7) *are removed edges. Therefore the Bondage number of G is 3.*

3. The Dominator Chromatic Number, Bondage Number and Domatic Number of Moser Spindle Graph

Theorem 3.1. *Let G be the Moser spindle graph, then*

(i) The dominator chromatic number of G is 4. (i.e.,) $\chi_d(G) = 4$.

(ii) The bondage number of G is 4. (i.e.,) $\gamma_b(G) = 4$.

(iii) The domatic number of G is 3. (i.e.,) $d(G) = 3$.

Proof. Let G be Moser spindle graph with 7 vertices and 11 edges. Moser spindle graph or moser graph is an undirected graph, with 7 vertices and 11 edges. Let $V = \{v_1, v_2, ..., v_n\}$ be the vertices of G. Let us now determine the dominator chromatic number, domination number, bondage number and domatic number of G.

Case (i): Let us determine the dominator chromatic number of G.

By the definition of dominator coloring, in which each vertex of the graph dominates every vertex of some color class. Let us assign color 1 to $\{v_2, v_5\}$ and assign color 2 to $\{v_6, v_7\}$. Then assign color 3 to $\{v_1, v_4\}$ and color 4 to $\{v_3\}$. So this a valid proper coloring and here each vertex of the graph dominates every vertex of some color class. Hence this is a valid Dominator coloring. Therefore $\chi_d(G) = 4$.

Case (ii): Let us determine the Bondage number of G.

From the definition of Bondage number of G, it is the cardinality of the smallest set of edges E such that the domination number of the graph with edge e removed is greater that the domination number of the original graph. We know that the domination number of G is 2. For a Moser spindle graph, we remove 4 edges to make the domination number of G to become greater than the domination number of $G - 4e$. Therefore the bondage number of G is 4.

Case (iii): Let us now determine the Domatic number of G.

From the definition of domatic number is the maximum number of disjoint dominating set. Here $\{v_2, v_5\}$, $\{v_6, v_7\}$, $\{v_1, v_4\}$ are the dominating set of G. Therefore the domatic number of G is 3. (i.e.,) $d(G) = 3$. □

Example 3.2.

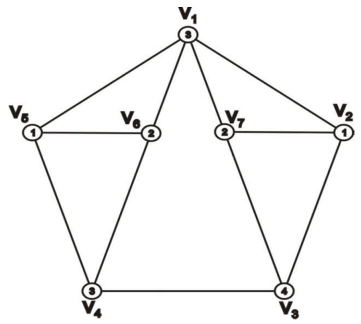

Figure 3. **Moser spindle graph**

The color classes of Moser spindle graph G is $\{v_2, v_5\} = 1$, $\{v_6, v_7\} = 2$, $\{v_1, v_4\} = 3$ *and* $\{v_3\} = 4$. *Therefore* $\chi_d(G) = 4$. *Therefore the Domatic number of Moser spindle graph (G) is 3.*

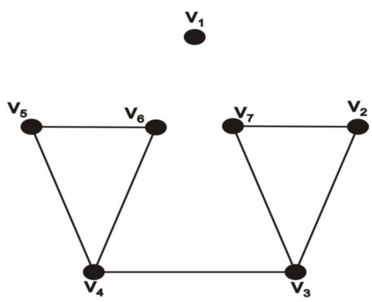

Figure 4. **Bondage number of Moser spindle graph (G)**

Here (v_1, v_6), (v_1, v_5), (v_1, v_7), (v_1, v_2) *are removed edges. Therefore the Bondage number of G is 4.*

4. The Dominator Chromatic Number, Bondage Number and Domatic Number of Wagner Graph

Theorem 4.1. *Let G be the Wagner graph, then*

(i) The dominator chromatic number of G is 4. (i.e.,) $\chi_d(G) = 4$.

(ii) The bondage number of G is 2. (i.e.,) $\gamma_b(G) = 2$.

(iii) The domatic number of G is 2. (i.e.,) $d(G) = 2$.

Proof. Let G be Wagner graph with 8 vertices and 12 edges. The Wagner graph is a 3-regular graph with 8 vertices and 12 edges. It is the 8-vertex Mobius ladder graph. Mobius ladder is a cubic circulant graph with an even number n vertices, formed from an n-cycle by adding edges connecting opposite pairs of vertices in the cycle. Let $V = \{v_1, v_2, ..., v_n\}$ be the vertices of G. Let us now determine the dominator chromatic number, domination number, bondage number and domatic number of G.

Case (i): Let us determine the dominator chromatic number of G
By the definition of dominator coloring, in which each vertex of the graph dominates every vertex of some color class. Let us assign color 1 to $\{v_1, v_3, v_6\}$ and assign color 2 to $\{v_2, v_5, v_7\}$. Then assign color 3 to $\{v_8\}$ and color 4 to $\{v_4\}$. So this a valid proper coloring and here each vertex of the graph dominates every vertex of some color class. Hence this is a valid Dominator coloring. Therefore $\chi_d(G) = 4$.

Case (ii): Let us determine the Bondage number of G.
From the definition of Bondage number of G, it is the cardinality of the smallest set of edges E such that the domination number of the graph with edge e removed is greater that the domination number of the original graph. We know that the domination number of G is 3. For a Wagner graph, we remove 2 edges to make the domination number of G to become greater than the domination number of $G - 2e$. Therefore the bondage number of G is 2.

Case (iii): Let us now determine the Domatic number of G.
From the definition of domatic number is the maximum number of disjoint dominating set. Here $\{v_1, v_3, v_6\}$, $\{v_2, v_5, v_7\}$ are the dominating set of G. Therefore the domatic number of G is 2. (i.e.,) $d(G) = 2$. □

Example 4.2.

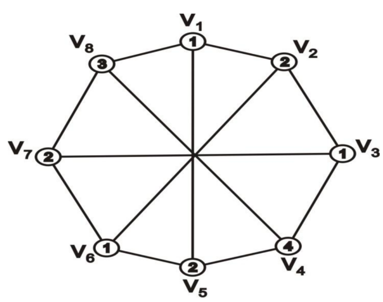

Figure 5. **Wagner graph**

The color classes of Wagner graph G is $\{v_1, v_3, v_6\} = 1$, $\{v_2, v_5, v_7\} = 2$, $\{v_8\} = 3$ *and* $\{v_4\} = 4$. *Therefore* $\chi_d(G) = 4$. *Therefore the Domatic number of Wagner graph (G) is 2.*

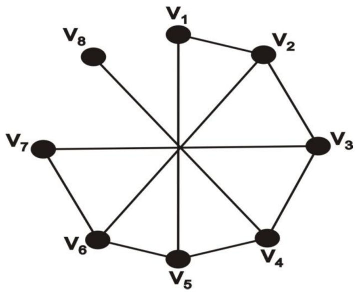

Figure 6. **Bondage number of Wagner graph (G)**

Here (v_1, v_8), (v_8, v_7) *are removed edges. Therefore the Bondage number of G is 2.*

5. The Dominator Chromatic Number, Bondage Number and Domatic Number of Herschel Graph

Theorem 5.1. *Let G be the Herschel graph, then*

(i) The dominator chromatic number of G is 4. (i.e.,) $\chi_d(G) = 4$.

(ii) The bondage number of G is 4. (i.e.,) $\gamma_b(G) = 4$.

(iii) The domatic number of G is 3. (i.e.,) $d(G) = 3$.

Proof. Let G be Herschel graph with 11 vertices and 18 edges. The Herschel graph is a bipartite undirected graph with 11 vertices and 18 edges which is the smallest non-Hamiltonian polyhedral graph. Let $V = \{v_1, v_2, ..., v_n\}$ be the vertices of G. Let us now determine the dominator chromatic number, domination number, bondage number and domatic number of G.

Case (i): Let us determine the dominator chromatic number of G

By the definition of dominator coloring, in which each vertex of the graph dominates every vertex of some color class. Let us assign color 1 to $\{v_9, v_{10}, v_{11}\}$ and assign color 2 to $\{v_1, v_4, v_6\}$. Then assign color 3 to $\{v_2, v_5, v_7\}$ and color 4 to $\{v_3, v_8\}$. So this a valid proper coloring and here each vertex of the graph dominates every vertex of some color class. Hence this is a valid Dominator coloring. Therefore $\chi_d(G) = 4$.

Case (ii): Let us determine the Bondage number of G.

From the definition of Bondage number of G, it is the cardinality of the smallest set of edges E such that the domination number of the graph with edge e removed is greater that the domination number of the original graph. We know that the domination number of G is 3. For a Herschel graph, we remove 4 edges to make the domination number of G to become greater than the domination number of $G - 4e$. Therefore the bondage number of G is 4.

Case (iii): Let us now determine the Domatic number of G.

From the definition of domatic number is the maximum number of disjoint dominating set. Here $\{v_9, v_{10}, v_{11}\}$, $\{v_1, v_4, v_6\}$, $\{v_2, v_5, v_7\}$. are the dominating set of G. Therefore the domatic number of G is 3. (i.e.,) $d(G) = 3$. □

Example 5.2.

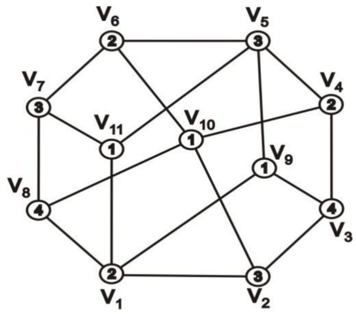

Figure 7. **Herschel graph**

The color classes of Herschel graph G is $\{v_9, v_{10}, v_{11}\} = 1$, $\{v_1, v_4, v_6\} = 2$, $\{v_2, v_5, v_7\} = 3$ *and* $\{v_3, v_8\} = 4$. *Therefore* $\chi_d(G) = 4$. *Therefore the Domatic number of Herschel graph (G) is 3.*

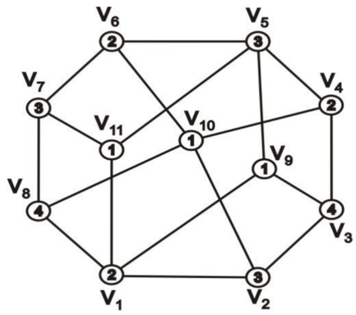

Figure 8. **Bondage number of Herschel graph (G)**

Here (v_6, v_7), (v_6, v_5), (v_5, v_{11}), (v_5, v_9) *are removed edges. Therefore the Bondage number of G is 4.*

6. Conclusion

In this paper, we have found the dominator chromatic number, Bondage number and Domatic number of Fruncht graph, Herschel graph, Wagner graph, and Moser spindle graph.

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