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On 3-Rainbow Domination Number of Silicate Network

Research Article*

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Abstract: Given an undirected graph $G = (V, E)$ and a set of k-colors numbered 1,2. . . ,k. The 3-rainbow domination defined as $f : V(G) \to \{1, 2, 3\}$ such that for each vertex $v \in V(G)$ with $f(v) = \emptyset$. We have,

$$\bigcup_{u \in N(v)} f(u) = \{1, 2, 3\}$$

Such a function f is called a 3-rainbow dominating function (3RDF) and minimum weight of such function is called the 3-rainbow domination number of G and is denoted by $\gamma_{r3}(G)$. In this paper we find the 3-rainbow domination number of interconnection network.

Keywords: Domination number, 3-rainbow domination number, Interconnection network, silicate network.

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1. Introduction

In graphs variety of domination problem solved by using k-rainbow domination. In graph theory k-rainbow domination was introduced by Bresar, Henning & Rall [2] at first. A subset T of the vertex set V(G) of a graph G is said to be dominating set if every vertex in $(V - T)$ is adjacent to a vertex in T. The Minimum Cardinality of dominating set is said to the domination numbers and is denoted by $\gamma(G)$. Consider a graph G with $v \in V(G)$ and the open neighborhood of v is the set $N(v) = \{u \in V(G)/uv \in E(G)\}$ and it closed neighborhood of v is the set $N[V] = \{v\} \cup N(v)$. Let $f : V(G) \to \mathcal{P}\{1, 2, \ldots, k\}$ be a function that assigns to each vertex of G a set of colors chosen from the power set $\{1, 2, \ldots, k\}$. If $v \in V(G)$ and $f(v) = \emptyset$ then $\bigcup_{u \in N(v)} f(u) = \{1, 2, \ldots, k)$. Therefore the function f is called k-rainbow domination function (k-RDF) of G.The Weight of the function is defined by $W(f) = \sum_{v \in V(G)} |f(v)|$. The minimum weight of a k-RDF is called the k-rainbow domination number of G and it is denoted by $\gamma_{rk}(G)$. When $k = 3$ we define a mapping $f : V(G) \to \mathcal{P}\{1, 2, 3\}$ such that for each vertex $v \in V(G)$ with $f(v) = \emptyset$ we have

$$\bigcup_{u \in N(v)} f(u) = \{1, 2, 3\}$$

Such a function f is said to be a 3-rainbow dominating function (3RDF) and minimum weight of such function is said to be 3-rainbow domination number of G and it is denoted by $\gamma_{r3}(G)$.

Example 1.1.

In below figure 3-rainbow domination number is 6.

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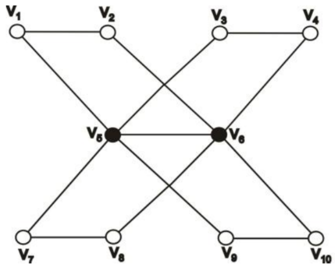
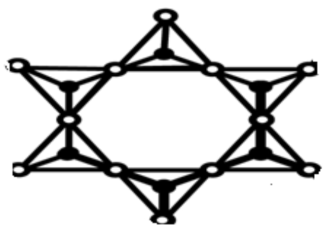


2. Silicate Network

Interconnection network is used for changing data between two processors in a multistage network. Is placed between various devices in the multiprocessor network. It is central role in determining the overall performance in the system. Interconnection network like customary network system consisting of nodes and edges. Interconnection plays major role in multimedia, mass communication etc. There are many types of interconnection networks among these we have chosen the silicate network to determine the 3-rainbow domination number. Origin of silicate is from rock forming and synthetic minerals. The basic unit is S_iO_4. It is Tetrahedron shape, we consider the silicate sheet as a fixed interconnection parallel architecture and is said to be the silicate network. In chemistry S_iO_4-tetrahedron represents oxygen ions in outer points and the center points represents the silicon ion. In graph theory outer vertices are represented as oxygen nodes and center vertices are represented as silicon nodes. This structures is used in various places mainly by determining X-ray diffraction. The ability to conduct electricity, produce a high frequency vibration and provide thermal insulation are some of the unique properties. Hence silicon is the perfect material to make microchips which runs every computers and cell phones and gaming devices. Silicate network ($SL(n)$) of dimension n has $15n^2 + 3n$ vertices and $36n^2$ edges. The diameter of $SL(n)$ is 4n. The 3-degree oxygen nodes of silicate network is said to be boundary nodes [4].

Example 2.1.



3. The 3-Rainbow Domination Number of Silicate Network

Proposition 3.1. *For $n > 0$, the domination number of silicate network of dimension n is $3n^2$. (i.e) $\gamma\,(SL(n) = 3n^2$.*

Proof. Let $SL(n)$ be the silicate network with vertices $15n^2 + 3n$ and edges $36n^2$. Here S be the minimum dominating set of $SL(n)$. Where S is the $3n^2$ vertices which is adjacent to the remaining vertices in silicate network. Let us assume the contrary that S is not a minimum dominating set of $SL(n)$. Let $S\prime$ be the minimum dominating set where $S\prime = V - S$. Let u be any vertex in S (ie) $u \in S$. By the minimality condition of dominating set, we know that for all $u \in S$, $N(u) \cap S = \{v\}$. But $N(u) \cap S\prime = \phi$ for all $u \in S$. Therefore $S\prime$ is not minimum dominating set. And we conclude that S is only minimum dominating set of $SL(n)$ then the domination number is $3n^2$. (i.e) $\gamma(SL(n)) = 3n^2$. □

Proposition 3.2 ([3])**.** *If G be a graph, then for any $k \geq 2$, $\min\{|\text{v(G)}|\,,\ \gamma(G) + K - 2\} \leq \gamma_{rk}(G) \leq k\gamma(G)$.*

The following theorem gives an upper bound for the 3-rainbow domination number of $SL(n)$.

Theorem 3.3. *Let $SL(n)$ be the silicate network of dimension n. The 3-rainbow domination number of silicate network is $9n^2$ (i.e) $\gamma_{r3}(SL(n)) \leq 9n^2 \;\; \forall \;\; n \geq 1$.*

Proof. Let $SL(n)$ be a silicate network. Let us consider the silicate network for dimension one, the domination number for $SL(1)$ is 3 with the vertices 18 and edges 36, we define a mapping $f : v\,(SL\,(1)) \to \mathcal{P}\{1,2,3\}$ such that we shall assign color class $\{1,2,3\}$ to the vertices $\{v_4, v_7, v_9, v_{12}, v_{15}, v_{18}\}$ and the remaining vertices are assigned to the empty color. The minimum sum of numbers of assigned colors overall vertices of $SL(1)$ is 9. Thus the 3-rainbow domination number of $SL(1)$ is 6.

For the silicate network of dimension two, the domination number for $SL(2)$ is 12 with the vertices 66 and edges 144, we define a mapping $f : v\,(SL\,(2)) \to \mathcal{P}\{1,2,3\}$ such that we shall assign color class $\{1,2,3\}$ to the vertices $\{v_3, v_4, v_{11}, v_{12}, v_{13}, v_{17}, v_{18}, v_{19}, v_{28}, v_{29}, v_{30}, v_{31}, v_{36}, v_{37}, v_{38}, v_{39}, v_{48}, v_{49}, v_{50}, v_{54}, v_{55}, v_{56}, v_{63}, v_{64}\}$ and the remaining vertices are assigned to the empty color .Thus the 3-rainbow domination number of $SL(2)$ is 24. Similarly the silicate network of dimension three, the domination number for $SL(3)$ is 27 with vertices 144 and edges 324 thus the 3-rainbow domination number of $SL(3)$ is 54. Repeating this process for dimension n, the silicate network of dimension n, we define a mapping $f : v\,(SL\,(n)) \to \mathcal{P}\{1,2,3\}$ such that for each vertex $v \in SL(n)$ with $f(v) = \emptyset$. We have,

$$\bigcup_{u \in N(v)} f\,(u) = \{1,2,3\}$$

The domination number for $SL(n)$ is $3n^2$ with vertices $15n^2 + 3n$ and edges $36n^2$ and therefore the 3-rainbow domination number of $SL(n)$ is $\gamma_{r3}(SL(n)) \leq 9n^2 \; \forall \; n \geq 1$. □

Example 3.4.

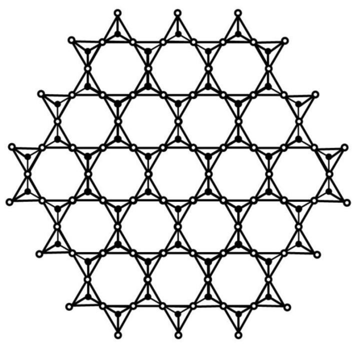


Figure 1. $\gamma_{r3}(SL(3)) = 54$

4. Conclusion

In this paper we established the 3-rainbow domination number of silicate network $SL(n)$. And this work is further extended to find the 3-rainbow domination number for other networks.

References

[1] M.Ali, M.T.Rahim, M.Zib and G.Ali, *On 2-Rainbow Domination of some families of graphs*, International Journal of Mathematics of Soft Computing, 1(1)(2011), 47-53.

[2] B.Bresar, M.A.Henning and D.F.Rall, *Rainbow domination in graphs*, Taiwanese J. Math., 12(2008), 213-225.

[3] B.Hart Neell and D.F.Rall, *On Dominating the Cartesian product of a graph and* k_2, Discussiones Mathematicae Graph Theory, 24(2004), 389-402.

[4] Paul Manuel and Indra Rajasingh, *Topological Properties of Silicate Network*, 5th IEEE GCC Conference, Kuwait, (2009).

[5] P.Sivagami, Indra Rajasingh and Sharmila Mary Arul, *On 3-Rainbow Domination in Hexagonal Network & Honeycomb network*, International Journal of Pure & Applied Mathematics, 101(5)(2015), 839-847.

[6] C.Tong, X.Lin, Y.Yang and M.Luo, *2 Rainbow Domination of generalized Petersen graphs p(n, 2)*, Discrete Applied Mathematics, 157(2009), 1932-1937.