International Journal of Mathematics And its Applications
Volume 4, Issue 4 (2016), 107–114. (Special Issue)
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

The B-Chromaticnumber, Bondage Number and Connected Domination Number of Some Special Graphs

Research Article*

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Abstract: Let $G = (V, E)$ be an undirected and loopless graph. In this paper we determinedtherelationship for b-chromatic number, bondage number and connected domination numberfor Barbell graph, Harary graph, Book graph,Grotzsch graph, Wagner graph respectively.

Keywords: Proper coloring, Chromatic number, b-Coloring, b-Chromatic number, bondage number, connected domination number, Barbell graph, Harary graph, Book graph, Grotzsch graph and Wagner graph.

1. Introduction

Let G = (V, E) be an undirected graph with loopless and multiple edges. A coloring of vertices of graph G is a mapping c: V (G) → {1, 2. . . . . . k} for every vertex. A coloring is said to be proper if any two adjacent vertices of a graph have different colors. The chromatic number χ(G) of a graph G is the smallest integer k which admits a proper coloring. A particular color which is assigned to a certain set of vertices are called color classes. A proper k-coloring c of a graph G is a b-coloring if for every color class c_i, there is a vertex with color i which has atleast one neighbor in every other adjacent color classes. The b-chromatic number χ_b(G) of a graph G is the largest integer k such that G admits a proper k-coloring in which every color class contains at least one vertex adjacent to some vertex in all the other color classes[6]**.**The cardinality of the smallest set of edges E such that the domination of the graph with edge removed is greater than the domination number of the original graph [2]. The connected domination set whose vertices re connected by edges [1]. Irving and Manlove introduce the concept of coloring in 1999, and determined the b-chromatic number of NP-hard problem. The concept of bondage number was introduced by Fink. Hedetniemi and Laskar(1990) note, the domination problem was studied from 1950's onwards, but the research on domination significantly increased in the mid-1970's.In this paper we found the b-chromatic number, bondage number and connected domination number of some special graphs.

Definition 1.1 (Barbell graph B(K_n, K_n))**.** *The n- barbell graph is a simple graph consisting of two non-overlapping n-vertex cliques together with a single edge that has an endpoint in each clique.*

Definition 1.2 (Grotzsch Graph)**.** *The Grotzsch graph is smallest triangle- free graph. It is similar to the Mycielski graph of order four, and is implemented as Graph data. It has 11 vertices,20 edges and it's chromatic number is four. It is non-planar.*

* *Proceedings : National Conference on* `Recent Trends in Applied Mathematics` *held on 22 & 23.07.2016, organized by Department of Mathematics, St. Joseph's College of Arts & Science, Manjakuppam, Cuddalore (Tamil Nadu), India.*

Definition 1.3 (Wagner Graph)**.** *The Wagner graph is a name sometimes given to the 4-Mobius ladder. The Wagner graph is a cubic Hamiltonian graph. The Wagner graph has the most spanning trees among the 8-vertex.*

Definition 1.4 (Book Graph)**.** *The m-book graph is defined as the graph Cartesian product S_{m+1}&P_2, where S_m is a star graph and P_2 is the path graph on two nodes.*

Definition 1.5 (Harary Graph)**.**

Case 1*: If k is even, m is even (i.e) k= 2r, then $H_{2r,n}$can be constructed as follows. It has vertices 0, 1,. . . .n-1 and two vertices i & j are joined if, i-r≤j< i+r (where addition is taken modulo n).*

Case 2: *If k is odd and m is even (i.e) k= 2r+1, then $H_{2r+1,n}$ is constructed by first drawing $H_{2r,n}$ and then adding edges joining vertex i to vertex i+(n/2) for 1≤ i ≤n/2.*

Case 3: *If k is odd and m is odd (i.e) k=2r+1, then $H_{2r+1,n}$ is constructed by first drawing $H_{2r,n}$ and then adding edges joining vertex 0 to vertices (n-1)/2 & (n+1)/2.*

2. Important Results

1. For any complete graph k_n, $\chi_b(k_n)$=n for all n.
2. For any path graph $P_n$and cycle C_n, $\chi_b(P_n)=\chi_b(C_n)= \begin{cases} 2 \text{ for } n<5 \\ 3 \text{ for } n\geq 5 \end{cases}$
3. The bondage number of complete graph $K_n(n \geq 2)$ is $\gamma_b(k_n) = \lceil n/2 \rceil$
4. If G is a connected graph, then, $\gamma(G) \leq \gamma_c(G) \leq \gamma(G)$-2

3. The b-chromatic Number, Bondage Number and Connected Domination of Some Special Graph

In this paper we determined the b-chromatic number, bondage number and connected domination number of some special graphs.

Theorem 3.1. *Let B (K_n, K_n) be the barbell graph. Then the b-chromatic number, bondage number and connected domination number of B (K_n, K_n) are given below:*

1. $\chi_b(B(K_n, K_n)) = n \ \forall\ n \geq 3.$

2. $\gamma_b(B(K_n,K_n)) = \begin{cases} 2 \ \ if\ n\ \leq 4 \\ 3\ otherwise \end{cases}$

3. $\gamma_c(B(K_n,K_n)) = 2, \forall\ n \geq 2.$

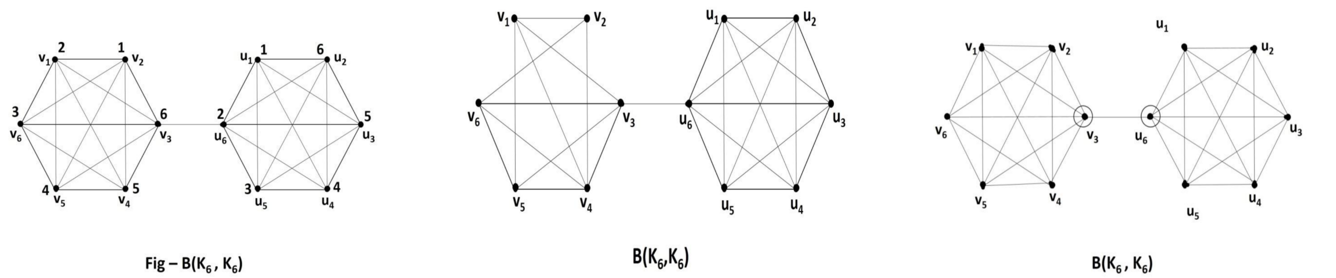
Proof. Let G = B(K_n,K_n). WhereB(K_n, K_n) is the barbell graph with 2n vertices ∀ n > 3. By the definition of barbell graph the B(K_n,K_n) is constructed by two copies of complete graph K_n is connected by an edge. Let A be the first copy of B(K_n, K_n) and B be the 2^{nd} copy of B(K_n,K_n). Here V = {v_1, v_2... v_n} and U = {u_1,u_2,... u_n} are the vertex set of two copies. Let us now determine the b-chromatic number, bondage number and connected domination number of B (K_n, K_n).

Case 1: The b-chromatic number of B(K_n, K_n). Since both the copies are of complete graphs. We know that all the vertices of A are adjacent to each other and similarly all the vertices of B are adjacent to each other so it is possible to color all the vertices of A with distinct colors. i.e Assign colors 1,2,... n to the vertices v_1,v_2,. . . .v_n. Therefore, χ_b (A) = n (since $\chi_b(k_n)$ = n). Similarly χ_b (B) = n. Note that for the particular vertexv_i which is adjacent to the vertex u_i will be assigned with a different color other than the color of v_i. So this is a valid proper coloring and here all the vertices of each color class are adjacent to each other. Hence this is a valid b-coloring. Therefore, $\chi_b(B(K_n, K_n)) = n, \forall\ n \geq 3$.

Case 2: Let us determine the bondage number of B(K_n, K_n). From the definition of bondage number of B(K_n,K_n), we know that the cardinality of the smallest set of edges Esuch that the domination of the graph with edge removed is greater than the domination number of the original graph. We know that the domination number of B(K_n,K_n) is $\gamma_d(B(K_n,K_n)) = 2\ \forall\ n \geq 3$ and for any barbell graph B(K_n,K_n) for $n \leq 4$. We need to remove two edges to make the domination number of B(K_n,K_n) and to become greater that than the domination number of B(K_n,K_n) -2e. Therefore, the bondage number of B (K_n, K_n) is given by $\gamma_b(B(K_n,K_n)) = 2\ \forall\ n\ \leq 4$. When $n > 5$, we need to remove 3 edges from to make the domination number of B (K_n,K_n) to become greater than the domination number B(K_n,K_n)-3e. Therefore, the bondage number of B(K_n,K_n) is given by $= 3\ \forall\ n > 5$.

Case 3: Let us now determine the connected domination number of B(K_n, K_n). We know that from the definition of connected domination set whose vertices are connected by edges. i.e) all vertices of the dominating set lies in the same component. Here we take the vertices $v_i$and u_i which are connected by an edge. Because the vertex v_i is adjacent to all the other vertices of B. Also v_i and $u_i$lie in the same component. Thus v_i and $u_i$are connected. Hence the connected domination set S = $\{v_3, u_6\}$ for $v_i \epsilon$ A and $u_i \epsilon$ B. Therefore, $\gamma_c(B(K_n, K_n)) = 2$. Hence, in general we conclude that $\gamma_c(B(K_n, K_n)) \leq \gamma_b(B(K_n,K_n)) \leq \chi_b(B(K_n, K_n))$. □

Example 3.2.



(a) $\chi_b(B(K_6, K_6)) = 6$ (b) $\gamma_b(B(K_6, K_6)) = 3$ (c) $\gamma_c(B(K_6, K_6)) = 2$

Theorem 3.3. *If $H_{(k,m)}$is a Harary graph when k= 2n and m= 2n+2 for all positive values of n. Then the b-chromatic number, bondage number, connected domination number of Harary graph are given by*

1. $\chi_b(H_{(k,m)})$= n+1 for all n.

2. $\gamma_b(H_{(k,m)})$ = 2n+1 for all n. 3. $\gamma_c(H_{(k,m)})$ = 2for all n.

Proof. This theorem can be proved by induction hypothesis. Let $H_{(k,m)}$ be a Harary graph. Let V ($H_{(k,m)}$) = $\{v_0,v_1,v_2,\ldots\ldots v_{m-1}\}$ and E ($H_{(k,m)}$) = $\{e_0,e_1,e_2,\ldots\ldots\ldots\ldots e_{k*m}\}$.

Case 1: When n =1, we have k=2, m=4, V($H_{(2,4)}$)= $\{v_0,v_1,v_2,v_3\}$, where k is vertex connectivity and m is number of edges. Let $H_{(2,4)}$ be a Harary graphshaving four vertices and edges four of degree two. Let V=$\{v_0,v_1,v_2,v_3\}$ and c_i denotes the color class which has color i, for i=1,2,. . . .n. By definition, a proper k-coloring c of a graph G is a b-coloring if for every color class c_i, there is a vertex with color i which has at least one neighbor in every other adjacent color classes. Let us define a mapping Φ: V→c such that Φ (v_i) =$c_i$for all i. Hence there is a valid b-coloring. In this case we assign c_1=\{1\} =$\{v_0,v_2\}$, c_2=\{2\}= $\{v_1,v_3\}$. Therefore, the b-chromatic number of H $_{(2,4)}$= 2 = (1+1). By the definition of bondage number,we know that the cardinality of the smallest set of edges E such that the domination of the graph with edge removed is greater than the domination number of the original graph. We know that the bondage number for H $_{(2,4)}$is $\gamma_d(\mathrm{H}_{(2,4)}) = 3 \; \forall \; n \; \geq 4$. We need to remove three edges to make the domination number of H$_{(2,4)}$is greater that than the domination number H$_{(2,4)}$– 3e. Therefore, the bondage number of H $_{(2,4)}$ is given by $\gamma_b(\mathrm{H}_{(2,4)}) = 3 \; \forall \; n \; \geq 4$. We know that from the definition of connected domination set whose vertices are connected by edges.

i.e) all vertices of the dominating set lies in the same component. Here we take the vertices $v_0$and v_3 which are connected by an edge. Also v_0 and $v_3$lie in the same component. Thus v_0 and $v_3$are connected. Hence the connected domination set, S = $\{v_0,v_3\}$ for H$_{(2,4)}$. Therefore, $\gamma_c(\mathrm{H}_{(2,4)}) = 2$.

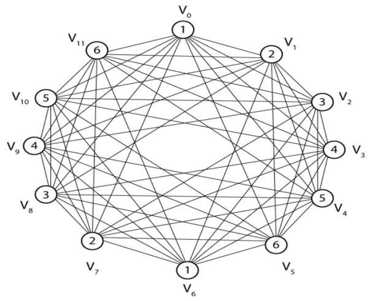
Case 2: When n =2, we have k=4, m=6 and V(H$_{(4,6)}$) = $\{v_0,v_1,v_2,v_3,v_4,v_5\}$. Let H$_{(4,6)}$ be a Harary graphshaving six vertices and twelve edges of degree four. Let V=$\{v_0,v_1,v_2,v_3,v_4,v_5\}$ and$c_i$denotes the color class which has color i, for i=1,2,. . . .n. By definition, a proper k-coloring c of a graph G is a b-coloring if for every color class c_i, there is a vertex with color i which has at least one neighbor in every other adjacent color classes. Let us define a mapping Φ: V→c such that Φ (v_i) =$c_i$for all i. Hence there is a valid b-coloring. In this case we assign c_1= {1} =$\{v_0,v_3\}$, C_2 ={2}=$\{v_1,v_4\}$, C_3= {3} =$\{v_2,v_5\}$. Therefore, the b-chromatic number of H $_{(4,6)}$ =3. By the definition of bondage number,we know that the cardinality of the smallest set of edges E such that the domination of the graph with edge removed is greater than the domination number of the original graph. We know that the bondage number for H $_{(4,6)}$is $\gamma_d(\mathrm{H}_{(4,6)}) = 2 \; \forall \; n \; \geq 4$. We need to remove five edges to make the domination number of H$_{(2,4)}$ to become greater that than the domination number H$_{(4,6)}$-5e.

Therefore, the bondage number of H$_{(4,6)}$ is given by $\gamma_b(\mathrm{H}_{(4,6)}) = 5 \; \forall \; n \; \geq 4$. We know that from the definition of connected domination set whose vertices are connected by edges. i.e) all vertices of the dominating set lies in the same component. Here we take the vertices $v_0$and v_4 which are connected by an edge. Also v_0 and v_{4lie} in the same component. Thus v_0 and v_4 are connected. Hence the connected domination set S = $\{v_0,v_4\}$for H$_{(4,6)}$. Therefore, $\gamma_b(\mathrm{H}_{(2,4)}) = \mathrm{n(s)} = 2$.

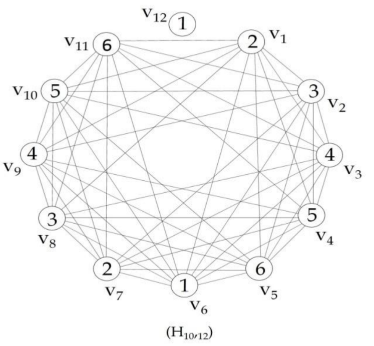
Case 3: When n=3, we have k=6, m=8 and V(H$_{(6,8)}$)= $\{v_0,v_1,v_2,v_3,v_4,v_5,v_6,v_7\}$. Let H$_{(6,8)}$be a Harary graphshaving eight vertices and twenty four edges of degree six. Let V=$\{v_0,v_1,v_2,v_3,v_4,v_5,v_6,v_7\}$ and$c_i$denotes the color class, which has color i, for i=1,2,. . . n. By definition, a proper k-coloring c of a graph G is a b-coloring if for every color class c_i, there is a vertex with color i which has at least one neighbor in every other adjacent color classes. Let us define a mapping Φ: V→c such that Φ (v_i) =$c_i$for all i. Hence there is a valid b-coloring. In this case we assignc_1= {1}=$\{v_0,v_4\}$, c_2={2}=$\{v_1,v_5\}$, c_3={3}=$\{v_2,v_6\}$, c_4={4}=$\{v_3,v_7\}$. Therefore, the b-chromatic number of H$_{(6,8)}$ = 4 = (3+1). By the definition of bondage number, we know that the cardinality of the smallest set of edges E such that the domination of the graph with edge removed is greater than the domination number of the original graph. We know that the bondage number for H$_{(6,8)}$is $\gamma_d(\mathrm{H}_{(6,8)}) = 2 \; \forall \; n \; \geq 4$. We need to remove seven edges to make the domination number of H$_{(6,8)}$ to become greater that than the domination number H$_{(6,8)}$– 7e.

Therefore, the bondage number of H$_{(6,8)}$ is given by $\gamma_b(\mathrm{H}_{(6,8)}) = 7 \; \forall \; n \; \geq 4$. We know that from the definition of connected domination [11] set whose vertices are connected by edges. i.e) all vertices of the dominating set lies in the same component. Here we take the vertices v_0 and v_5 which are connected by an edge. Also v_0 and $v_5$lie in the same component. Thus v_0 and v_5 are connected. Hence the connected domination set S = $\{v_0,v_5\}$for H$_{(6,8)}$. Therefore, $\gamma_c(\mathrm{H}_{(6,8)}) = 2$. Hence in general we conclude that $\gamma_c(\mathrm{H}_{(k,m)}) \leq \chi_b(\mathrm{H}_{(k,m)}) \leq \gamma_b(\mathrm{H}_{(k,m)})$. □

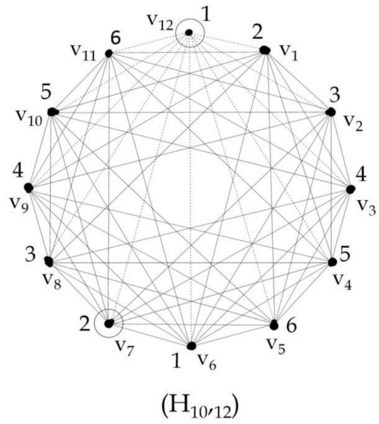
Example 3.4.



(a) $\chi_b(H_{(10,12)}) = 6$



(b) $\gamma_b(\mathrm{H}_{(10,12)}) = 11$



(c) $\gamma_c(\mathrm{H}_{(10,12)}) = 2$

Theorem 3.5. *Let B_n be the book graph. Then the b-chromatic number, bondage number and connected domination number of B_{nis} given below:*

(i). $\chi_b(B_n) = 2 \; \forall \; n = 3$.

(ii). $\gamma_b(B_n) = 1$

(iii). $\gamma_c(B_n) = 2$ for all n.

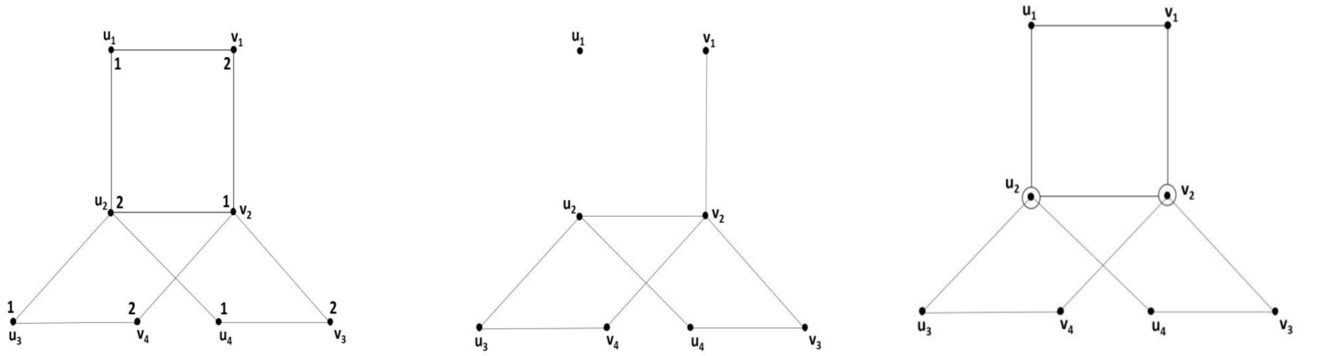
Proof. Let B_n be the book graph having two vertices in common. By definition of Bn, we know that the Bn is defined as the Cartesian product S_{m+1} , $P\mathbf{2}where$ $\mathrm{S}_m$is a star graph and P2 is the path graph on two nodes. Let V = $\{\mathrm{v}_1, \mathrm{v}2 \ldots \mathrm{v}_n\}$ and U = $\{\mathrm{u}_1, \mathrm{u}_2, \ldots \mathrm{u}_n\}$be the vertices of book graph. Let us now determine the b-chromatic number, bondage number and connected domination number of B_n.

Case 1: The b-chromatic number of B_n. It is not a complete graph but each u_i is adjacent to $\mathrm{v}_{i,}$so it is possible to color all the vertices with distinct. i.e) assign colors 1,2,. . . n to the vertices $\mathrm{v}_1, \mathrm{v}_2, \ldots \mathrm{v}_{n.}$ Therefore, $\chi_b(\mathrm{B}_n) = 2$. Note that for the particular vertex u_i which is adjacent to the vertex vi will be assigned with a different color other than the color of u_i. So this is a valid proper coloring and here all the vertices of each color class are adjacent to each other. Hence this is a valid b-coloring. Therefore, $\chi_b(\mathrm{B}_n) = 2 \; \forall \; \mathrm{n} \geq 3$.

Case 2: Let us now determine the bondage number of B_n. From the definition of bondage numbers of Bn. We know that it is the cardinality of the smallest set of edges E such that the domination of the graph with edge removed is greater than the domination number of the original graph. We know that the domination number of B_n is $\gamma_d(\mathrm{B}_n) = 2 \; \forall \; \mathrm{n} \geq 3$. For book graph B_n for $\mathrm{n} \geq 3$ we need to remove one edge to make the domination number of B_n to become greater than the domination number B_n-e. Therefore, the bondage number of B_n is given by $\gamma_b(\mathrm{B}_n) = 1 \; \forall \; \mathrm{n} \geq 3$. The bondage number of B_n is given by $\gamma_b(\mathrm{B}_n) = 1 \; \forall \; \mathrm{n} \geq 3$.

Case 3: Let us now determine the connected domination number of B_n. We know that from the definition of connected domination set whose vertices are connected by edges. i.e) all vertices of the dominating set lies in the same component. Here we take the vertices $\mathrm{v}_i$and $\mathrm{u}_i$which are connected by an edge. Because the vertex v_i is adjacent to all the other vertices of u_i. Also u_i and v_i lie in the same component. Thus u_i and v_i are connected. Hence the connected domination set, S = $\{\mathrm{u}_i, \mathrm{v}_i\}$. Therefore, $\gamma_c(\mathrm{B}_n) = 2$. Hence $\chi_b(\mathrm{B}_n) = \gamma_c(\mathrm{B}_n) \geq \gamma_b(\mathrm{B}_n)$. □

Example 3.6.



(a) $\gamma_c(\mathrm{H}_{(10,12)}) = 2$ (b) $\gamma_b(\mathrm{B}_n) = 1$ (c) $\gamma_c(\mathrm{B}_n)= 2$

Theorem 3.7. *Let G be a Grotzsch graph. Then the b-chromatic number, bondage number and connected domination number of Gis given below:*

(i). $\chi_b(G) = 4$ *for all n.*

(ii). $\gamma_b(G) = 1$

(iii). $\gamma_c(G) = 4$

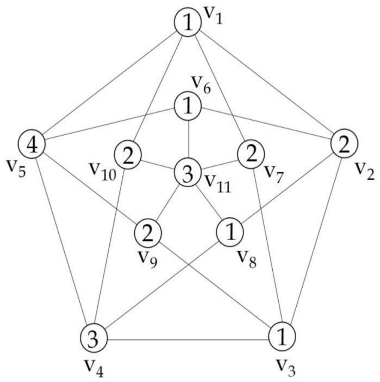
Proof. Let G be the Grotzsch graph with 11 vertices and 20 edges. By the definition of Grotzsch graph, we know that G is the smallest triangle – free graph. Let V = $\{v_1, v_2 \ldots v_n\}$ be the vertices of G. Let us now determine the b-chromatic number, bondage number and connected domination number of G.

Case 1: The b-chromatic number of G. It is non-planar and we know that it is smallest triangle free graph, so it is possible to color all the vertices with distinct colors. i.e assign colors 1,2,. . . n to the vertices $v_1,v_2,\ldots v_n$. Therefore, χ_b(G) = 4. Note that the vertex v_i will be assigned with a different color. So this is a valid proper coloring and here all the vertices of each color class are adjacent to each other. Hence this is a valid b-coloring. Therefore, χ_b(G) = 4.

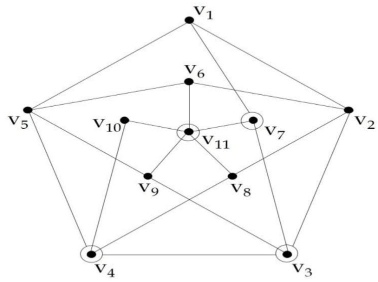
Case 2: Let us now determine the bondage number of G. From the definition of bondage number of G. We know that it is the cardinality of the smallest set of edges E such that the domination of the graph with edge removed is greater than the domination number of the original graph. We know that the domination number of G is γ_d(G) = 1. For Grotzsch book graph G we need to remove one edge to make the domination number of G to become greater than the domination number G-e. Therefore, the bondage number of G is given by γ_b(G) = 1 The bondage number of G is given by γ_b(G) = 1.

Case 3: Let us now determine the connected domination number of G. We know that from the definition of connected domination set whose vertices are connected by edges. i.e all vertices of the dominating set lies in the same component. Here we take the vertices $v_i$which are connected by an edge. Because the vertex v_i is adjacent to the vertices. Hence the connected domination set, S = $\{v_3, v_4, v_7, v_{11}\}$. Therefore, γ_c(G) = 4. Hence we conclude that,χ_b(G) = γ_c(G) $\leq$ γ_b(G). □

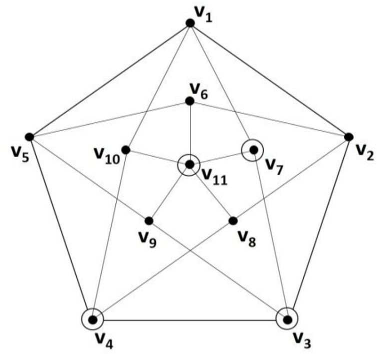
Example 3.8.



(a) χ_b(G) = 4



(b) γ_b(G) = 1



(c) γ_c(G)= 4

Theorem 3.9. *Let G be a Wagner graph. Then the b-chromatic number, bondage number and connected domination number of Gis given below:*

(i). $\chi_b(G) = 3$ *for all n.*

(ii). $\gamma_b(G) = 2$

(iii). $\gamma_c(G) = 4$.

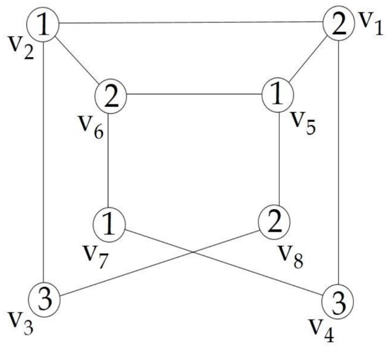
Proof. Let G be the Wagner graph with 8 vertices. By the definition of Wagner graph, we know that G is the cubic Hamiltonian graph. Let V = {v_1, $v_2 \ldots v_n$} be the vertices of G. Let us now determine the b-chromatic number, bondage number and connected domination number of G.

Case 1: The b-chromatic number of G. The Wagner graph is a name sometimes given to the 4-Mobius ladder. The Wagner graph has the most spanning trees among the 8 vertex, so it is possible to color all the vertices with distinct colors. i.e assign colors 1,2,... n to the vertices $v_1, v_2, \ldots v_n$. Therefore, χ_b(G) = 3. Note that the vertex v_i will be assigned with a different color. So this is a valid proper coloring and here all the vertices of each color class are adjacent to each other. Hence this is a valid b-coloring. Therefore, χ_b(G) = 3.

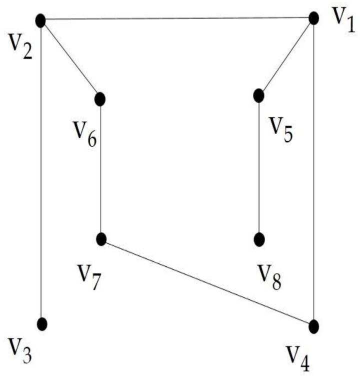
Case 2: Let us now determine the bondage number of G. From the definition of bondage number of G.We know that it is the cardinality of the smallest set of edges E such that the domination of the graph with edge removed is greater than the domination number of the original graph. We know that the domination number of G is γ_d(G) = 2. For Wagner graph G we need to remove two one edge to make the domination number of G to become greater than the domination number G-2e. Therefore, the bondage number of G is given by γ_b(G) = 2. The bondage number of G is given by γ_b(G) =2.

Case 3: Let us now determine the connected domination number of G. We know that from the definition of connected domination set whose vertices are connected by edges. i.e all vertices of the dominating set lies in the same component. Here we take the vertices $v_i$which are connected by an edge. Because the vertex v_i is adjacent to the vertices. Hence the connected domination set, S = {v_1, v_2, v_3, v_4}. Therefore, γ_c(G) = 4. Hence we conclude that γ_b(G) ≤ χ_b(G) ≤γ_c(G) □

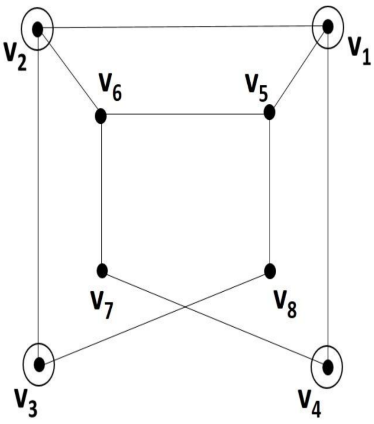
Example 3.10.



(a) χ_b(G) = 3



(b) γ_b(G) = 2



(c) γ_c(G)= 4

4. Conclusion

In this paper we determined the relationship for b-chromatic number, bondage number and connected domination number for some special graphs. This work can be extended for various graphs.

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