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Finitistic Spaces in Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper the concept of intuitionistic fuzzy finitistic space is introduced by means Q-cover of intuitionistic fuzzy open sets. Some interesting results are obtained.

Keywords: Intuitionistic fuzzy topology; locally finite family; Intuitionistic fuzzy finitistic space; Intuitionistic fuzzy paracompact.
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1. Introduction

The notion of intuitionistic fuzzy set was first defined by Atanassov [3,4] as a generalization of Zadeh's [18] fuzzy set, and this notion of intuitionistic fuzzy set has been developed by the same author and appeared in the literature. Using the notion of intuitionistic fuzzy sets, Coker [8] introduced the notion of intuitionistic fuzzy topological spaces as a genralization of Chang [7] fuzzy topological spaces. Recently many concepts of fuzzy topological space have been extended in intuitionistic fuzzy topological spaces such as [1,2,10,12,14,16]. The concept of finitistic spaces in general topology was introduced by R. G. Swan [15] in 1960. Many results related to finitistic spaces also known as boundedly metacompact spaces can be seen in [11,13,17]. In [13], Jamwal and Shakeel introduced the concept of finitistic spaces in fuzzy topological spaces by using the usual open cover in L- topology. Also, in [6], T. Baiju and S. J. John, studied finitistic spaces in L-topological spaces. In [1] we have introduced the concept of covering dimension in intuitionitic fuzzy topological spaces and obtained some results for it. In this paper we introduce the concept of intuitionistic fuzzy finitistic spaces and will study some of its properties. For a paracompactness we prove that every intuitionistic fuzzy paracompact space of finite covering dimension is an intuitionistic fuzzy finitistic, also, we show that for intuitionistic fuzzy finite-dimensional paracompact space the inverse image of an intuitionistic fuzzy closed finitistic sets is an intuitionistic fuzzy finitistic also, the intuitionistic fuzzy finitisticness is a topological property.

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2. Preliminaries

Throughout this paper by (X, τ) or simply by X we mean an intuitionistic fuzzy topological space (IFTS, Shorty). First, we present the basic definitions:

Definition 2.1 ([3,4]). *Let X be a non empty fixed set. An intuitionistic fuzzy set A (IFS for short)in X is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, where the function $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$, for each $x \in X$.*

Definition 2.2 ([3,4]). *Let $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ be an intuitionistic fuzzy set in X. For any ordered pair α, β, where $\alpha, \beta \in [0,1]$, and $0 \le \alpha + \beta \le 1$. Let $A_{\langle \alpha,\beta \rangle} = \{\langle x, \mu_A(x) \ge \alpha, \gamma_A(x) \le \beta\rangle : x \in X\}$ which is called $\langle \alpha, \beta\rangle$-cut set (or level set) of the intuitionistic fuzzy set A. The cut set with $\alpha = 1$ and $\beta = 0$ i.e., $A_{\langle 0,1\rangle}$ is called the support of the intuitionistic fuzzy set A.*

Definition 2.3 ([3,4]). *Let A and B be IF sets of the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Then*

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(c) $A^c = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;

(d) $A \bigcap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;

(e) $A \bigcup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

(f) $0_X^{\sim} = \{\langle x, , 0, 1\rangle : x \in X\}$ and $1_X^{\sim} = \{\langle x, 1, 0\rangle : x \in X\}$.

Definition 2.4 ([3]). *Let $\{A_i : i \in I\}$ be an arbitrary family of IFS in X. Then*

(a) $\bigcap A_i = \{\langle x, \wedge \mu_{A_i}(x), \vee \gamma_{A_i}(x) : x \in X\}$;

(b) $\bigcup A_i = \{x, \vee \mu_{A_i}(x), \wedge \gamma_{A_i}(x) : x \in X\}$.

Definition 2.5 ([9]). *Let $\alpha, \beta \in [0,1], \alpha + \beta \le 1$ An intuitionistic fuzzy point (IFP for short) of nonempty set X is an IFS of X denoted by $P = x_{(\alpha,\beta)}$ and defined by*

$$P = x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta), & \text{if } x = y; \\ (0, 1), & \text{if } x \neq y. \end{cases} \tag{1}$$

In this case, x is called the support of $x(\alpha, \beta)$ and α, β are called the value and no value of $x(\alpha, \beta)$ respectively.

Clearly an intuitionistic fuzzy point can be represented by an ordered pair of fuzzy point as follows: $x_{(\alpha,\beta)} = (x_\alpha, 1 - x_{(1-\beta)})$. An IFP $x_{(\alpha,\beta)}$ is said to belong to an IFS $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$

denoted by $P = x_{(\alpha,\beta)} \in A(or P \subseteq A), if \alpha \leq \mu_A(x)$ and $\beta \geq \gamma_A(x)$. We identify a fuzzy point x_r in X by the intuitionistic fuzzy point $x_{(r,(1-r))}$ in X.

Proposition 2.6. *An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points belonging to A.*

Definition 2.7 ([3])**.** *Let X and Y be two nonempty sets and $f : X \to Y$ be a function. Then*

(a) *If $B = \{\langle x, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$ is an intuitionistic fuzzy set in Y, then the pre-image of B under f denoted by $f^{-}1(B)$ is the IFS in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$;*

(b) *If $B = \{\langle x, \lambda_B(y), \nu_B(y)\rangle : x \in X\}$ is an intuitionistic fuzzy set in X, then the image of A under f denoted by $f(A)$ is the intuitionistic fuzzy set in Y defined by $f(A) = \{\langle y, f(\lambda_A(y)), 1 - f(1-\nu_A)(y)\rangle : y \in Y\}$, where,*

$$f(\lambda_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \{\lambda_A(x)\}, & \text{if } f^{-1}(y) \neq \emptyset; \\ 0, & \text{otherwise.} \end{cases}$$

$$1 - f(1 - v_A)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \{\nu_A(x)\}, & \text{if } f^{-1}(y) \neq \emptyset; \\ 1, & \text{otherwise.} \end{cases}$$

Proposition 2.8 ([8])**.** *Let A, A_i $(i = 1, 2)$ be IFSs in X, and B, B_j $(j = 1, 2)$ be IFSs in Y*

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$;

(b) $B_1 \subseteq B_2 \Rightarrow f^{-}1(B_1) \subseteq f^{-1}(B_2)$;

(c) $A \subseteq f^{-}(f(A))$ *and if f is injective, then* $A = f(f^{-1}(A))$;

(d) $f^{-}(f(B)) \subseteq B$ *and if f is surjective, then* $f(f^{-1}(B)) = B$;

(e) $f^{-1}(1_{\tilde{Y}}) = 1_{\tilde{X}}$;

(f) $f^{-1}(0_{\tilde{Y}}) = 0_{\tilde{X}}$;

(g) $f(1_{\tilde{X}}) = 1_{\tilde{Y}}$ *if f is surjective;*

(h) $f(0_{\tilde{X}}) = 0_{\tilde{Y}}$.

Definition 2.9. *Let (X, τ) be an IFTS.*

(1) *An intuitionistic fuzzy point $x(\alpha, \beta)$ is said to be quasi-coincident with the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle\}$ denoted by $x(\alpha, \beta)qA$, if $\alpha > \mu_A(x)$ or $\beta < \gamma_A(x)$;*

(2) *Let* $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle\}$ *and* $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle\}$ *be two intuitionistic fuzzy sets in X. Then A and B are said to be quasi-coincident, denoted by* AqB *, if there exists an element* $x \in X$ *such that* $\mu_A(x) > \gamma_B(x)$ *or* $\gamma_A(x) < \mu_B(x)$*.*

Definition 2.10 ([8]). *An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family of intuitionistic fuzzy sets in X satisfy the following axioms:*

(T1) $0_X^\sim, 1_X^\sim \in \tau$.

(T2) If $A_1, A_2 \in \tau$, *then* $A_1 \bigcap A_2 \in \tau$.

(T3) If $A_\lambda \in \tau$ *for each* $\lambda in \Lambda$, *then* $\bigcup_{\lambda \in \Lambda} A_\lambda \in \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short)and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set (IFOS for short) of X, and the complement of an intuitionistic fuzzy closed set (IFCS for short).

Example 2.11 ([2]). *Let* $X = \{a, b, c\}$ *and* M_1, M_2, M_3 *and* M_4 *be an intuitionistic fuzzy sets on X defined as follows:*

$$M_1 = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\rangle$$
$$M_2 = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.3}, \frac{c}{0.3}\right)\right\rangle$$
$$M_3 = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}\right)\right\rangle$$
$$M_4 = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\rangle$$

Then the family $\tau = \{0_X^\sim, 1_X^\sim, M_1, M_2, M_3, M_4\}$ *is an IFT on X.*

Definition 2.12 ([8]). *Let A any intuitionistic fuzzy set in an IFTS* (X, τ). *Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior of A are defined by* $IF - cl(A) = \bigcap\{F : A \subseteq F, F \text{ is } IFCS \text{ in} X\}$ *is called intuitionistic fuzzy closure of A, and denote by* $\bar{A}$. $IF - int(A) = \bigcup\{U : U \subseteq A, U \text{ is } IFOS \text{ in } X\}$ *is called intuitionistic fuzzy interior of A denoted by* A°.

Example 2.13. *In Example 2.11, if* $A = \langle x, (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.3})\rangle$, *then* $int(A) = \{U : U \subseteq A, U \text{ is } IFOS \text{ in } X\} = M_4$ *and* $cl(A) = \bigcap\{F : A \subseteq F, F \text{ is } IFCS \text{ in } X\} = 1_X^\sim$.

Definition 2.14 ([2]). *Let* (X, τ) *be an intuitionistic fuzzy topological space, and let* $M \subseteq X$, *the intuitionistic fuzzy collection* $\tau_M = \{O \bigcap M : O \in \tau\}$ *is an intuitionistic fuzzy topology on X, and the pair* (M, τ_M) *is called an intuitionistic fuzzy subspace of an intuitionistic fuzzy topological space of* (X, τ).

Definition 2.15. *A family of intuitionistic fuzzy sets* $A = \{A_\lambda : \lambda \in \Lambda\}$ *is said to be refinement of the family of intuitionistic fuzzy sets* $B = \{B_\gamma : \gamma \in \Gamma\}$ *in IFTS* (X, τ) *if for every* $A_\lambda \in A$ *there exists* $B_\gamma \in B$ *such that* $A_\lambda \subset B_\gamma$, $\forall\ \lambda \in \Lambda$ *and for some* $\gamma \in \Gamma$.

Definition 2.16. *An intuitionistic fuzzy set A in IFTS (X, τ) is called a Q-neighborhood of intuitionistic fuzzy point $P = x_{(\alpha,\beta)}$ if and only if there exists intuitionistic fuzzy set $B \in \tau$ such that $PqB \subset A$ i.e. PqB, $B \subset A$. A is called a Q-neighborhood of intuitionistic fuzzy set B iff there exist intuitionistic fuzzy $C \in \tau$ such that BqC, $B \subset A$. We denote by $Q(A)$, the Q-neighborhood system of intuitionistic fuzzy set A.*

Remark 2.17. *Clearly Q-neighborhood of of intuitionistic fuzzy point $P = x_{(\alpha,\beta)}$ (respectively IFset A) is the neighborhood of its dual of intuitionistic fuzzy point $P = x_{(\alpha,\beta)}$ fuzzy (point $x_{1-(\alpha,\beta)}$, respectively set A^c).*

Definition 2.18. *Let $\mathbb{A}$ be a family of intuitionistic fuzzy set and B be an intuitionistic fuzzy set in IFTS (X, τ), w say that $\mathbb{A}$ is locally finite in B if for each intuitionistic fuzzy point $P = x_{(\alpha,\beta)}$ in B there exists $U \in Q(P = x(\alpha, \beta))$ such that U quasi-coincident with at most a finite numbers of a set $\mathbb{A}$, when $B = 1_X^\sim$, we say that A is locally finite only.*

Theorem 2.19. *If a family of intuitionistic fuzzy sets $\mathbb{A} = \{A_\lambda : \lambda \in \Lambda\}$ of IFTS (X, τ) is locally finite in an IFS A (respectively $1_X^\sim$) then the family of intuitionistic fuzzy sets $\{\bar{A}_\lambda : \lambda \in \Lambda\}$ is also locally finite in A (respectively $1_X^\sim$).*

Proof. Since $A \subseteq \bar{A}$, and $A_i q U \ \ \forall\, x(\alpha, \beta)$ in A (respectively X) and a finite set of the Ifs $\{A_\lambda : \lambda \in \Lambda\}$, $U \in Q(P = x_{(\alpha,\beta)}$, then from the properties of quasi-coincident we get$\bar{A}_i q U$ also. Then $\{\bar{A}_\lambda : \lambda \in \Lambda\}$ is locally finite in A (respectively X). □

Definition 2.20 ([5]). *A family $\mathbb{A}$ of intuitionistic fuzzy sets in IFTS (X, τ) is called an intuitionistic fuzzy Q-cover of an intuitionistic fuzzy set B if $B \subseteq \bigcup A_\lambda$, $\mu_B(x) + \mu_{A_\lambda}(x) \geq 1$ for each μ_{A_λ} and some $x \in X$. i.e. $\mathbb{A}$ is an intuitionistic fuzzy Q-cover of an intuitionistic fuzzy set B if for each $x \in supp(B)$, there exists an $A \in \mathbb{A}$ such that A and B are quasi-coincident at x, when $B = X$ then $\mathbb{A}$ is Q-cover of $1_X^\sim$.*

Definition 2.21. *Let A be an IFS in an IFTS (X, τ), we say that A is paracompact if for each intuitionistic fuzzy open Q-cover of A there exists an intuitionistic fuzzy open refinement of it which is both locally finite in A, and an Q-cover of A. We say that (X, τ) is intuitionistic fuzzy - paracompact if the set $1_X^\sim$ is an intuitionistic fuzzy-paracompact.*

Definition 2.22 ([1]). *Let X be a nonempty set. A family $\mathbb{U} = \{U_\gamma : \gamma \in \Gamma\}$ of intuitionistic fuzzy sets in X is said to be of order n $(n > 1)$ written $ord_{IF} U = n$, if n is largest integer such that there exists an overlapping subfamily of U having $n + 1$ elements.*

Definition 2.23 ([1]). *The covering dimension of an IFTS (X, τ) denoted $dim_{If}(X)$ is the least integer n such that every finite open cover of $1_X^\sim$ has a finite open refinement of order not exceeding n or$+\infty$ if there exists no such integer. Thus it follows that $dim_{If}(X) = -1$ if and only if $X = \varnothing$ and $dim_{If}(X) \leq n$ if every finite open cover of $1_X^\sim$ has a finite open refinement of order $\leq n$. We have $dim_{If}(X) = n$ if it is true that $dim_{If}(X) \leq n$, but it is false that $dim_{If}(X) \leq n - 1$. Finally $dim_{If}(X) = +\infty$ if for every positive integer n it is false that $dim_{If}(X) \leq n$.*

3. Intuitionistic Fuzzy Finitistic Spaces

Definition 3.1. *Let (X,τ) be an IFTS, and let A be an IFS. Then A is said to be intuitionistic fuzzy finitistic if every intuitionistic fuzzy Q-cover of A has an intuitionistic fuzzy Q- cover refinement by intuitionistic fuzzy open sets with finite order. (X,τ) is intuitionistic fuzzy finitistic if $1_X^{\sim}$ is finite.*

Example 3.2. *Let $X = \{a,b\}$ and M be an intuitionistic fuzzy set on X defined as follows: $M = \langle x, (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5})\rangle$. Then the family $\tau = \{0_X^{\sim}, 1_X^{\sim}, M\}$ is an IFT on X, let $\mathbb{A} = \{A_\lambda : \lambda \in \Lambda\}$ $\bigcup\{M, 1_X^{\sim}\}$ be a family of IFOS in X for any intuitionistic fuzzy open set A_λ, then clearly it is an intuitionistic fuzzy open Q-cover of $1_X^{\sim}$ which has an intuitionistic fuzzy open Q- cover refinement of finite order, then (X,τ) is intuitionistic fuzzy finitistic.*

Theorem 3.3. *Every closed subspace of an intuitionistic fuzzy finitistic space is intuitionistic fuzzy finitistic.*

Proof. Suppose that (X,τ) is an intuitionistic fuzzy finitistic space and (Y,τ_Y) is an intuitionistic fuzzy closed subspaces of (X,τ). Then Y^c is an intuitionistic fuzzy open set in X. Let $\mathbb{U} = \{U_\lambda : \lambda \in \Lambda\}$ be an intuitionistic fuzzy open-Q-cover of $1_Y^{\sim}$. Then for every $U_\lambda \in \mathbb{U}$ there exist intuitionistic fuzzy open sets V_λ of X such that $U_\lambda = V_\lambda \bigcap Y$. Now, consider $\mathbb{V} = \{V_\lambda : U_\lambda = V_\lambda \bigcap Y, \forall\ U_\lambda \in \mathbb{U}\} \bigcup Y^c$. Then clearly $\mathbb{V}$ is an intuitionistic fuzzy open Q-cover of $1_X^{\sim}$ by intuitionistic fuzzy open sets. Since (X,τ) is an intuitionistic fuzzy finitistic $\mathbb{V}$ has a finite order -Q-cover refinement say $\mathbb{C}$ by intuitionistic fuzzy open sets. Then, we let $\mathbb{U}_{\mathbb{C}} = \{C \bigcap Y : C \in \mathbb{C}\}$ is an intuitionistic fuzzy finite order Q-cover refinement of intuitionistic fuzzy open sets of $\mathbb{U}$ and hence (Y,τ_Y) is intuitionistic fuzzy finitistic. □

Theorem 3.4. *Let (X,τ) be an finitely intuitionistic fuzzy topological space, then (X,τ) is intuitionistic fuzzy finitistic.*

Proof. Let $\mathbb{A} = \{A_\lambda : \lambda \in \Lambda\}$ be an be an intuitionistic fuzzy open Q-cover of (X,τ), then for each intuitionistic fuzzy point $P = x_{(\alpha,\beta)}$ in X there is an intuitionistic fuzzy $A \in \mathbb{A}$ containing $P = x_{(\alpha,\beta)}$. Put $A_P = \{\bigcup A_P : P \in X\}$, since (X,τ) is a finitely IFTS, it follows that the collection $\mathbb{M} = \{A_x : x \in Supp(X)\}$ is an IF finitely ordered open Q-cover refinement of $\mathbb{A}$ and hence (X,τ) is an IF finitistic. □

Theorem 3.5. *Every IF Q-Compact space (X,τ) is an intuitionistic fuzzy finitistic.*

Proof. Let (X,τ) be an intuitionistic fuzzy Q-Compact space and Let $\mathbb{A} = \{A_\lambda : \lambda \in \Lambda\}$ be an intuitionistic fuzzy Q-cover of $1_X^{\sim}$. Since (X,τ) is intuitionistic fuzzy Q-compact, $\mathbb{A}$ has a finite intuitionistic fuzzy Q -subcover say $\mathbb{B} = \{B_i : i = 1,2,...,n\}$. Then it is clear that $\mathbb{B}$ is a finite order open refinement of $\mathbb{A}$. Hence (X,τ) is intuitionistic fuzzy finitistic. □

Theorem 3.6. *Every intuitionistic fuzzy paracompact space (X,τ) of finite covering dimension is an intuitionistic fuzzy finitistic.*

Proof. Let (X, τ) be a intuitionistic fuzzy paracompact space with finite covering dimension. Let $\mathbb{U} = \{U_\lambda : \lambda \in \Lambda\}$ be an intuitionistic fuzzy open Q-cover of $1_X^\sim$. Since (X, τ) is intuitionistic fuzzy paracompact, then $1_X^\sim$ paracompact. Then for every intuitionistic fuzzy Q-cover $\mathbb{U}$ of $1_X^\sim$, there exist an intuitionistic fuzzy open refinement $\mathbb{V}$ of $\mathbb{U}$ such that $\mathbb{V}$ is locally finite in $1_X^\sim$ and $\mathbb{V}$ is an intuitionistic fuzzy Q-cover of $1_X^\sim$. Now, let $dim_{If}(X) = n$. Then it is clear that $\mathbb{V}$ has an intuitionistic fuzzy open Q-cover refinement which is $\mathbb{M}$ of order not exceeding n. Hence it follows that $\mathbb{U}$ has an intuitionistic fuzzy open Q-cover refinement $\mathbb{M}$ with finite order. Therefore (X, τ) is an intuitionistic fuzzy finitistic. □

Lemma 3.7. *Let (X, τ) be an intuitionistic fuzzy normal finitistic space. Then there is an intuitionistic fuzzy compact subspace Q of X such that $dim_{If}(F) < \infty$ for every closed set F with $F \bigcap Q = 0_X^\sim$.*

Theorem 3.8. *An intuitionistic fuzzy paracompact space (X, τ) is an intuitionistic fuzzy finitistic if and only if there is an intuitionistic fuzzy compact set Q in X so that $1_X^\sim \setminus U$ is an intuitionistic fuzzy finite-dimensional for every intuitionistic fuzzy open neighborhood U of Q in X.*

Corollary 3.9. *Let (X, τ) be an intuitionistic fuzzy paracompact space with $1_X^\sim = A_1 \bigcup A_2$. If both A_1 and A_2 are IF paracompact and finitistic, then (X, τ) is an IF finitistic if one of the following conditions are satisfied:*

(a) A_1 is an IF closed in X,

(b) (X, τ) is an metrizable space.

Theorem 3.10. *Let $f : (X, \tau) \to (Y, \delta)$ be an IF closed function of IF paracompact spaces such that $f^{-1}(A)$ is an IF finite-dimensional for all IF finite-dimensional closed sets A in Y. If B is an IF closed finitistic set in(Y, δ), then $f^{-1}(B)$ is an IF finitistic.*

Proof. Let Q be an IF compact set of B so that $B \setminus U$ is finite-dimensional for all IF open neighborhoods U of Q in B (see Theorem 3.7). Notice that $f^{-1}(Q)$ is an IF compact. Suppose V is an IF open neighborhood of $f^{-1}(Q)$ in $f^{-1}(B)$. On can find an IF open neighborhood U of Q in B with $f^{-1}(U) \subseteq V$. Since $B \setminus U$ is finite-dimensional, so is $f^{-1}(B \setminus U)$. Now, $f^{-1}(B \setminus V$ is an IF closed subset of $f^{-1}(B \setminus U)$ and, therefore is finite dimensional. By Corollary 3.9, $f^{-1}(B)$ is an IF finitistic in (X, τ). □

Theorem 3.11. *An intuitionistic fuzzy finitisticness is a topological property.*

Proof. Let $f : (T, \psi) \to (Y, \delta)$ be an intuitionistic fuzzy homeomorphism, and (X, τ) be an intuitionistic fuzzy finitistic we need to show that (Y, δ) is an IF finitistic. Let $\mathbb{U} = \{U_\lambda : \lambda \in \Lambda\}$ be an intuitionistic fuzzy open Q-cover of $1_Y^\sim$ in (Y, δ), then $\mathbb{V} = \{f^{-1}(U_\lambda) : U_\lambda \in \mathbb{U}\}$ is an IFopen Q-cover of $1_X^\sim$ in (T, ψ), since (T, ψ) is an intuitionistic fuzzy finitistic, there exist an intuitionistic fuzzy Q-cover refinement $\mathbb{V} = \{V_\gamma : \gamma \in \Gamma\}$ of $\mathbb{V}$, such that for each $V_\gamma \in \mathbb{V}$ there exist some $f^{-1}(U_\lambda) \in \mathbb{U}$ such that $V_\gamma \subseteq f^{-1}(U_\lambda)$, since f is an intuitionistic fuzzy homeomorphism, then it can be easily seen that $\mathbb{V}_\gamma = \{f(V_\gamma) : V_\gamma \in \mathbb{V}\}$ is an intuitionistic fuzzy finite order subfamily of $1_Y^\sim$ such that $f(V_\gamma) \subseteq U_\lambda$. Hence (Y, δ) is an intuitionistic fuzzy finitistic. □

4. Conclusion

In this study, we introduced the concept of intuitionistic fuzzy finitistic space as a generalization of finitistic space this concept based on IF fuzzy Q-cover. Some properties of the intuitionistic fuzzy finitistic space were presented. In the future, based on some recent intuitionistic fuzzy finitistic spaces studies, we will expand the research content of this paper further. Also, the entire content will be a successful tool for the researchers for finding the path to obtain the results in the context of intuitionistic fuzzy finitistic spaces and other spaces.

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