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An Application of Interval Rough Pythagorean Fuzzy Set in a Multi Attribute Decision Making Problem

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Abstract

This paper aims to define the cosine, sine and cotangent similarity measures of interval rough pythagorean sets and investigate some properties. Furthermore, based on these proposed measures, a multi attribute decision making problem is solved. Finally, through an example to demonstrate the applicability of the proposed measures.

Keywords: Sine hamming similarity measure; cosine hamming similarity measure; cotangent hamming similarity measure; interval rough pythagorean set.

1. Introduction

The basic concept of fuzzy sets was introduced by Zadeh [9]. Here some membership grade is assigned to an element of a fuzzy set. In many situations of real world, apart from the grade of membership, the grade of non-membership is also required. To handle such conditions, the concept of pythagorean fuzzy set was introduced by Yagar [8], as a generalization of intuitionistic fuzzy set. Rough set theory was introduced by Z. Pawlak [6] in 1980. Dubois and Prade [4] combine the rough sets and fuzzy sets. In this paper we introduce the multi-attribute decision making based on hamming similarity measure under interval rough pythagorean environment.

2. Preliminaries

For basic definitions let us see [1–7] and [8]. Throughout this paper let us denote R as complete congruence relation on the universe U. The following abbreviations are used in this work:

- Cosine Hamming Similarity - **CHS**
- Sine Hamming Similarity - **SHS**
- Cotangent Hamming Similarity - **CTHS**

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- Interval Rough Pythagorean Fuzzy - $\mathcal{I}_{RPF}$

3. CHS Measure of $\mathcal{I}_{RPF}$ Set

This section deals with **CHS** measure of $\mathcal{I}_{RPF}$ set.

Definition 3.1. *Let* $\mathtt{P}^i = \{\langle \alpha_1, \mu_{\mathtt{P}^i}(\alpha_1), \nu_{\mathtt{P}^i}(\alpha_1)/\alpha_1 \in U\rangle\}$ *be an* $\mathcal{I}_{PF}$ *set of* U. *The lower and upper-approximations of* $\mathcal{I}_{PF}$ *is defined as follows:*

$$\underline{R}(\mathtt{P}^i) = \{\langle \alpha_1, \underline{R}(\mu_{\mathtt{P}^i}), \underline{R}(\nu_{\mathtt{P}^i})\rangle, \alpha_1 \in U\}$$
$$\overline{R}(\mathtt{P}^i) = \left\{\left\langle \alpha_1, \overline{R}(\mu_{\mathtt{P}^i}), \overline{R}(\nu_{\mathtt{P}^i})\right\rangle, \alpha_1 \in U\right\},$$

where

$$\underline{R}(\mu_{\mathtt{P}^i})(\alpha_1) = \bigwedge_{\alpha_2 \in [\alpha_1]_R} \mu_{\mathtt{P}^i}(\alpha_2), \quad \underline{R}(\nu_{\mathtt{P}^i})(\alpha_1) = \bigvee_{\alpha_2 \in [\alpha_1]_R} \nu_{\mathtt{P}^i}(\alpha_2)$$
$$\overline{R}(\mu_{\mathtt{P}^i})(\alpha_1) = \bigvee_{\alpha_2 \in [\alpha_1]_R} \mu_{\mathtt{P}^i}(\alpha_2), \quad \overline{R}(\nu_{\mathtt{P}^i})(\alpha_1) = \bigwedge_{\alpha_2 \in [\alpha_1]_R} \nu_{\mathtt{P}^i}(\alpha_2)$$

with the condition that

$$0 \leq \sup\{\underline{R}(\mu_{\mathtt{P}^i}(\alpha_1))\}^2 + \sup\{\underline{R}(\nu_{\mathtt{P}^i}(\alpha_1))\}^2 \leq 1$$
$$0 \leq \sup\left\{\overline{R}(\mu_{\mathtt{P}^i}(\alpha_1))\right\}^2 + \sup\left\{\overline{R}(\nu_{\mathtt{P}^i}(\alpha_1))\right\}^2 \leq 1.$$

The pair $R(\mathtt{P}^i) = \left(\underline{R}(\mathtt{P}^i), \overline{R}(\mathtt{P}^i)\right)$ *is called the* $\mathcal{I}_{RPF}$ *set of* U.

Let $\mathtt{P}^i_1$ and $\mathtt{P}^i_2$ be two $\mathcal{I}_{RPF}$ set in $U = \{u_1, u_2 \ldots u_n\}$. A **CHS** measure between $\mathtt{P}^i_1$ and $\mathtt{P}^i_2$ is defined as follows:

$$\mathbf{CHS}(\mathtt{P}^i_1, \mathtt{P}^i_2) = \frac{1}{n}\sum_{k=1}^{n} cos\left[\frac{\pi}{6}\left(\left|\delta\mu_{\mathtt{P}^i_1}(u_k) - \delta\mu_{\mathtt{P}^i_2}(u_k)\right| + \left|\delta\nu_{\mathtt{P}^i_1}(u_k) - \delta\nu_{\mathtt{P}^i_2}(u_k)\right|\right)\right]$$

where

$$\delta\mu_{\mathtt{P}^i_1}(u_k) = \frac{\left(\underline{R}(\mu^-) + \underline{R}(\mu^+) + \overline{R}(\mu^-) + \overline{R}(\mu^+)\right)}{4}$$
$$\delta\nu_{\mathtt{P}^i_1}(u_k) = \frac{\left(\underline{R}(\nu^-) + \underline{R}(\nu^+) + \overline{R}(\nu^-) + \overline{R}(\nu^+)\right)}{4}$$
$$\delta\mu_{\mathtt{P}^i_2}(u_k) = \frac{\left(\underline{R}(\mu^-) + \underline{R}(\mu^+) + \overline{R}(\mu^-) + \overline{R}(\mu^+)\right)}{4}$$
$$\delta\nu_{\mathtt{P}^i_2}(u_k) = \frac{\left(\underline{R}(\nu^-) + \underline{R}(\nu^+) + \overline{R}(\nu^-) + \overline{R}(\nu^+)\right)}{4}$$

Proposition 3.2. *A* **CHS** *measure between* $\mathtt{P}^i_1$ *and* $\mathtt{P}^i_2$ *satisfies the following properties*

1. $0 \leq \mathbf{CHS}(\mathtt{P}^i_1, \mathtt{P}^i_2) \leq 1$.

2. $\textbf{CHS}(\mathtt{P}_1^i, \mathtt{P}_2^i)$=*1* $\mathtt{P}_1^i = \mathtt{P}_2^i$.

3. $\textbf{CHS}(\mathtt{P}_1^i, \mathtt{P}_2^i)$= $\textbf{CHS}(\mathtt{P}_1^i, \mathtt{P}_2^i)$.

Proof.

1. According to the cosine value it is obvious.

2. For any two $\mathcal{I}_{RPF}$ sets $\mathtt{P}_1^i$ and $\mathtt{P}_2^i$, if $\mathtt{P}_1^i = \mathtt{P}_2^i$ then, $\delta\mu_{\mathtt{P}_1^i}(u_k) = \delta\mu_{\mathtt{P}_2^i}(u_k)$ and $\delta\nu_{\mathtt{P}_1^i}(u_k) = \delta\mu_{\mathtt{P}_2^i}(u_k)$. Hence $\left|\delta\mu_{\mathtt{P}_1^i}(u_k) - \delta\mu_{\mathtt{P}_2^i}(u_k)\right| = 0$ and $\left|\delta\nu_{\mathtt{P}_1^i}(u_k) - \delta\nu_{\mathtt{P}_2^i}(u_k)\right| = 0$. Thus, $cos(\mathtt{P}_1^i, \mathtt{P}_2^i) = 1$. Conversely, if $cos(\mathtt{P}_1^i, \mathtt{P}_2^i) = 1$, then $\left|\delta\mu_{\mathtt{P}_1^i}(u_k) - \delta\mu_{\mathtt{P}_2^i}(u_k)\right| = 0$ and $\left|\delta\nu_{\mathtt{P}_1^i}(u_k) - \delta\nu_{\mathtt{P}_2^i}(u_k)\right| = 0$. Since $cos(0) = 1$. So we can write $\delta\mu_{\mathtt{P}_1^i}(u_k) = \delta\mu_{\mathtt{P}_2^i}(u_k)$ and $\delta\nu_{\mathtt{P}_1^i}(u_k) = \delta\mu_{\mathtt{P}_2^i}(u_k)$. Hence $\mathtt{P}_1^i = \mathtt{P}_2^i$.

3. It is obvious.

□

4. SHS Measure of $\mathcal{I}_{RPF}$ Set

This section deals with **SHS** measure of an $\mathcal{I}_{RPF}$.

Let $\mathtt{P}_1^i$ and $\mathtt{P}_2^i$ be two $\mathcal{I}_{RPF}$ in $U = \{u_1, u_2 \ldots u_n\}$. A **SHS** between $\mathtt{P}_1^i$ and $\mathtt{P}_2^i$ is defined as follows:

$$\textbf{SHS}(\mathtt{P}_1^i, \mathtt{P}_2^i) = 1 - \left[\frac{1}{n}\sum_{k=1}^{n} sin\left[\frac{\pi}{6}\left(\left|\delta\mu_{\mathtt{P}_1^i}(u_k) - \delta\mu_{\mathtt{P}_2^i}(u_k)\right| + \left|\delta\nu_{\mathtt{P}_1^i}(u_k) - \delta\nu_{\mathtt{P}_2^i}(u_k)\right|\right)\right]\right]$$

where

$$\begin{aligned}
\delta\mu_{\mathtt{P}_1^i}(u_k) &= \frac{(\underline{R}(\mu^-) + \underline{R}(\mu^+) + \overline{R}(\mu^-) + \overline{R}(\mu^+))}{4} \\
\delta\nu_{\mathtt{P}_1^i}(u_k) &= \frac{(\underline{R}(\nu^-) + \underline{R}(\nu^+) + \overline{R}(\nu^-) + \overline{R}(\nu^+))}{4} \\
\delta\mu_{\mathtt{P}_2^i}(u_k) &= \frac{(\underline{R}(\mu^-) + \underline{R}(\mu^+) + \overline{R}(\mu^-) + \overline{R}(\mu^+))}{4} \\
\delta\nu_{\mathtt{P}_2^i}(u_k) &= \frac{(\underline{R}(\nu^-) + \underline{R}(\nu^+) + \overline{R}(\nu^-) + \overline{R}(\nu^+))}{4}
\end{aligned}$$

Proposition 4.1. *The* $\mathcal{I}_{RPF}$ **SHS** *measure between* $\mathtt{P}_1^i$ *and* $\mathtt{P}_2^i$ *satisfies the following properties*

1. $0 \leq \textbf{SHS}(\mathtt{P}_1^i, \mathtt{P}_2^i) \leq 1$.

2. $\textbf{SHS}(\mathtt{P}_1^i, \mathtt{P}_2^i)$=*1 if and only if* $\mathtt{P}_1^i = \mathtt{P}_2^i$.

3. $\textbf{SHS}(\mathtt{P}_1^i, \mathtt{P}_2^i)$= $\textbf{SHS}(\mathtt{P}_1^i, \mathtt{P}_2^i)$.

Proof. Proof is similar to Proposition 3.2. □

5. CTHS Measure of $\mathcal{I}_{RPF}$ Set

In this section we introduce the notion of **CTHS** measure of $\mathcal{I}_{RPF}$ set.

Let $\mathtt{P}_1^i$ and $\mathtt{P}_2^i$ be two $\mathcal{I}_{RPF}$ set in $U = \{u_1, u_2 \dots u_n\}$. A **CTHS** measure between $\mathtt{P}_1^i$ and $\mathtt{P}_2^i$ is defined as follows:

$$\mathbf{CTHS}(\mathtt{P}_1^i, \mathtt{P}_2^i) = \frac{1}{n}\sum_{k=1}^{n} cot\left[\frac{\pi}{4} + \frac{\pi}{12}\left(\left|\delta\mu_{\mathtt{P}_1^i}(u_k) - \delta\mu_{\mathtt{P}_2^i}(u_k)\right| + \left|\delta\nu_{\mathtt{P}_1^i}(u_k) - \delta\nu_{\mathtt{P}_2^i}(u_k)\right|\right)\right]$$

where

$$\delta\mu_{\mathtt{P}_1^i}(u_k) = \frac{(\underline{R}(\mu^-) + \underline{R}(\mu^+) + \overline{R}(\mu^-) + \overline{R}(\mu^+))}{4}$$
$$\delta\nu_{\mathtt{P}_1^i}(u_k) = \frac{(\underline{R}(\nu^-) + \underline{R}(\nu^+) + \overline{R}(\nu^-) + \overline{R}(\nu^+))}{4}$$
$$\delta\mu_{\mathtt{P}_2^i}(u_k) = \frac{(\underline{R}(\mu^-) + \underline{R}(\mu^+) + \overline{R}(\mu^-) + \overline{R}(\mu^+))}{4}$$
$$\delta\nu_{\mathtt{P}_2^i}(u_k) = \frac{(\underline{R}(\nu^-) + \underline{R}(\nu^+) + \overline{R}(\nu^-) + \overline{R}(\nu^+))}{4}$$

Proposition 5.1. *The $\mathcal{I}_{RPF}$* **CTHS** *measure between $\mathtt{P}_1^i$ and $\mathtt{P}_2^i$ satisfies the following properties*

1. **CTHS**$(\mathtt{P}_1^i, \mathtt{P}_2^i)$*=1 if and only if* $\mathtt{P}_1^i = \mathtt{P}_2^i$.

2. **CTHS**$(\mathtt{P}_1^i, \mathtt{P}_2^i)$= **CTHS**$(\mathtt{P}_2^i, \mathtt{P}_1^i)$.

Proof. Proof is similar to Proposition 3.2. □

6. MADM Problem Under $\mathcal{I}_{RPF}$ Hamming Similarity Measure

In this section, we apply $\mathcal{I}_{RPF}$ **CHS**, **SHS** and **CTHS** measures between $\mathcal{I}_{RPF}$ sets to the *MADM* problem. Consider $K = \{K_1, K_2 \dots K_m\}$ be the set of attributes and $R = \{\tilde{Q}_1, \tilde{Q}_2 \dots \tilde{Q}_n\}$ be a set of alternatives.

Algorithm:

Step 1: Nomination of decision matrix with n alternatives and m attributes.

Step 2: Definition of ideal alternative:

For benefit type attribute:

$$Z^* = \left\{\left(min\underline{R}(\mu_{\tilde{Q}_i}), max\underline{R}(\nu_{\tilde{Q}_i})\right), \left(max\overline{R}(\mu_{\tilde{Q}_i}), min\overline{R}(\nu_{\tilde{Q}_i})\right)\right\}.$$

For cost type attribute:

$$Z^* = \left\{\left(max\underline{R}(\mu_{\tilde{Q}_i}), min\underline{R}(\nu_{\tilde{Q}_i})\right), \left(min\overline{R}(\mu_{\tilde{Q}_i}), max\overline{R}(\nu_{\tilde{Q}_i})\right)\right\}.$$

Step 3: We calculate $\mathcal{I}_{RP}$ similarity measure between the ideal alternative Z^* and each alternative $Q_i, i = 1, 2 \ldots n$.

Step 4: The best alternative is opted with the highest similarity value.

7. Numerical Example

Let us consider a decision maker wants to select the bike for random use from $\{\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3\}$, by considering mileage K_1, reasonable price K_2, features K_3 and the risk factor K_4. By the above proposed measure, problem is solved by the following steps:

Step 1: The decision maker construct the decision matrix with respect to the three alternatives in terms of $\mathcal{I}_{RPF}$ number.

	K_1	K_2	K_3	K_4
$\tilde{Q}_1$	([.3,.4],[.5,.7]),([.3,.4],[.5,.7])	([.5,.6],[.8,.9]),([.5,.6],[.8,.9])	([.1,.2],[.7,.8]),([.5,.8],[.4,.6])	([.1,.2],[.7,.8]),([.5,.8],[.4,.6])
$\tilde{Q}_2$	([.7,.8],[.6,.7]),([.7,.8],[.6,.7])	([.7,.8],[.6,.7]),([.8,.9],[.4,.5])	([.5,.6],[.4,.5]),([.5,.6],[.4,.5])	([.7,.8],[.6,.7]),([.8,.9],[.4,.5])
$\tilde{Q}_3$	([.5,.7],[.3,.4]),([.8,.9],[.1,.2])	([.5,.7],[.3,.4]),([.8,.9],[.1,.2])	([.5,.7],[.3,.4]),([.8,.9],[.1,.2])	([.8,.9],[.1,.2]),([.8,.9],[.1,.2])

Table 1:

Step 2: The benefit type attributes are K_1, K_2, K_3 and cost type attribute is K_4. Then the ideal alternative is,

$$Z^* = \begin{cases} \langle([.3,.4],[.6,.7]),([.8,.9],[.1,.2])\rangle \\ \langle([.5,.6],[.8,.9]),([.8,.9],[.1,.2])\rangle \\ \langle([.1,.2],[.7,.8]),([.8,.9],[.1,.2])\rangle \\ \langle([.1,.2],[.7,.8]),([.8,.9],[.1,.2])\rangle \end{cases}$$

Step 3: Calculate the $\mathcal{I}_{RPF}$ hamming similarity measure of the alternatives

$$\mathbf{CHS}(Q_1, Z^*) = .249975$$

$$\mathbf{CHS}(Q_2, Z^*) = .249988$$

$$\mathbf{CHS}(Q_3, Z^*) = .249972$$

$$\mathbf{SHS}(Q_1, Z^*) = .9966$$

$$\mathbf{SHS}(Q_2, Z^*) = .9975$$

$$\mathbf{SHS}(Q_3, Z^*) = .9963$$

$$\mathbf{CTHS}(Q_1, Z^*) = 12.2243$$

$$\mathbf{CTHS}(Q_2, Z^*) = 13.3690$$

$$\mathbf{CTHS}(Q_3, Z^*) = 11.8483$$

Stop 4: Select the highest value.

$$\mathbf{CHS}(Q_2, Z^*) > \mathbf{CHS}(Q_1, Z^*) > \mathbf{CHS}(Q_3, Z^*)$$
$$\mathbf{SHS}(Q_2, Z^*) > \mathbf{SHS}(Q_1, Z^*) > \mathbf{SHS}(Q_3, Z^*)$$
$$\mathbf{CTHS}(Q_2, Z^*) > \mathbf{CTHS}(Q_1, Z^*) > \mathbf{CTHS}(Q_3, Z^*)$$

Hence $\mathbf{Q_2}$ is the best alternative for random use.

References

[1] K. Attanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20(1986), 87-96.

[2] Azmat Hussain, Tahir mahmood and Muhammad Irfan Ali, *Rough Pythagorean Fuzzy Ideals in Semigroups*, Computational and Applied Mathematics.

[3] V. Chinnadurai, *Contributions to the study of some fuzzy algebraic structures Doctoral Thesis*, Annamalai University, (2010).

[4] D. Dubois and H. Prade, *Rough fuzzy sets and fuzzy rough sets*, Internationl Journal of General Systems, 17(23)(1990), 191-209.

[5] Z. Gong, B. Sun and D. Chen, *Rough set theory for the interval-valued fuzzy information systems*, Information Sciences, 178(2008), 1968-1985.

[6] Z. Pawlak, *Rough sets*, International Journal of Computer and Information Sciences, 11(1982), 341-356.

[7] Z. Pawlak, *Rough sets and Intelligent data analysis*, Information Sciences, 147(2002), 1-12.

[8] R. R. Yager, *Pythagorean Fuzzy subsets*, In IFSA World congress and NAFIPS annual meeting, (2013) Joint, IEEE, 57-61.

[9] L. A. Zadeh, *The Concept of a linguistic variable and its application to approximation reasoning I*, Information Sciences, 8(1975), 199-249.

[10] L. A. Zadeh, *Fuzzy sets*, Information and Control, 8(1965), 338-353.