International Journal of
Mathematics And its Applications
ISSN: 2347-1557

Int. J. Math. And Appl., **12(2)**(2024), 61–71
Available Online: http://ijmaa.in

On Absolute Mean Graceful Labeling of Disjoint Union of Some Graphs

P. Z. Akbari[1], J. M. Shah[1,*], V. J. Kaneria[1]

[1]*Department of Mathematics, Saurashtra University, Rajkot, Gujarat, India*

Abstract

In this paper, we prove that the disjoint union of cycle and complete bipartite graph, jewel graph and sunlet graph, cycle and alternate helm, jewel graph and complete bipartite graph, swastik graph and path, and gear graph and sunlet graph are absolute mean graceful graphs.

Keywords: Union of graphs; graph labeling; absolute mean graceful labeling.
2020 Mathematics Subject Classification: 05C78, 05C76.

1. Introduction

We begin with simple, finite and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. For all standard terminology and notations we follow Harary [7]. A *graph labeling* is an assignment of values to the vertices or edges or both subject to certain conditions [10]. For various graph labeling problems and references we refer to Gallian [5]. Labeling of graphs is a potential area of research due to its diversified applications. Some of the applications are reported in Bloom and Golomb [3] and Yegnanarayanan and Vaidhyanathan [11]. The concept of β-valuation was introduced by Rosa [10]. Later on, Golomb [6] referred to this as graceful labeling. Kaneria and Chudasama [8] have introduced absolute mean graceful labeling with the flavour of graceful labeling, which is defined as follows. A function f is called an *absolute mean graceful labeling* of a graph G with q edges, if $f : V(G) \to \{0, \pm 1, \pm 2, \dots, \pm q\}$ is one-one and the edge labeling function $f^*\colon E(G) \to \{1, 2, \dots, q\}$ defined as $f^*(uv) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ is bijective, for every edge $uv \in E(G)$. If a graph G admits absolute mean graceful labeling, then it is called *absolute mean graceful graph* [8]. Kaneria and Chudasama [4,8] have proved some standard graphs are absolute mean graceful and also investigated some absolute mean graceful graphs in the context of duplication of graph elements. While Kaneria *et al.* [9] have investigated absolute mean graceful graphs in the context of path union of graphs. Akbari *et al.* [1] have proved that jewel and jelly fish related graphs are absolute mean graceful. While the same authors in [2] have investigated some absolute mean graceful graphs in the context of barycentric subdivision.

*Corresponding author (jaydeepshah1998@gmail.com)

In this paper we discuss absolute mean graceful labeling of some disconnected graphs. For all the undefined terminologies reader can refer [7] and [5].

2. Main Results

Theorem 2.1. *The disjoint union of cycle C_m and complete bipartite graph $K_{n,r}$ is absolute mean graceful graph, $\forall m > 2, n, r \in \mathbb{N}$.*

Proof. Let cycle be C_m and complete bipartite graph be $K_{n,r}$. Let $V(C_m) = \{u_1, u_2, u_3, \ldots, u_m\}$, $E(C_m) = \{u_i u_{i+1}/1 \leq i < m\} \cup \{u_1 u_m\}$, and $V(K_{n,r}) = \{v_1, v_2, v_3, \ldots, v_n, w_1, w_2, w_3, \ldots, w_r\}$, $E(K_{n,r}) = \{v_i w_j/1 \leq i \leq n, 1 \leq j \leq r\}$. Let $G = C_m \cup K_{n,r}$ be disjoint union of these two graphs. To obtain vertex labeling function $f\colon V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$ we take following cases.

Case 1 $m \equiv 0 \pmod 2$

$$f(u_i) = \begin{cases} (-1)^{i+1}(q - 2i + 2), & \text{if } i = 1, 2, \ldots, \frac{m+2}{2}; \\ (-1)^{i+1}(q - 2m + 2i - 1), & \text{if } i = \frac{m+4}{2}, \frac{m+6}{2}, \ldots, m. \end{cases}$$

Subcase 1 $m \equiv 0 \pmod 4$

$$f(v_i) = \begin{cases} nr - 1, & \text{if } i = 1; \\ nr - 2ri + 2r, & \text{if } i = 2, 3, \ldots, n. \end{cases}$$

$$f(w_i) = -nr + 2i - 2, \text{ for } i = 1, 2, \ldots, r.$$

Subcase 2 $m \equiv 2 \pmod 4$

$$f(v_i) = nr - 2ri + 2r, \text{ for } i = 1, 2, \ldots, n.$$

$$f(w_i) = \begin{cases} -nr + 1, & \text{if } i = 1; \\ -nr + 2i - 2, & \text{if } i = 2, 3, \ldots, r. \end{cases}$$

Case 2 $m \equiv 1 \pmod 2$

$$f(u_i) = \begin{cases} (-1)^{i+1}(q - 2i + 2), & \text{if } i = 1, 2, \ldots, \frac{m+1}{2}; \\ (-1)^{i+1}(q - 2m + 2i - 1), & \text{if } i = \frac{m+3}{2}, \frac{m+5}{2}, \ldots, m. \end{cases}$$

Subcase 1 $m \equiv 1 \pmod 4$

$$f(v_i) = \begin{cases} nr, & \text{if } i = 1; \\ nr - 2ri + 2r + 1, & \text{if } i = 2, 3, \ldots, n. \end{cases}$$

$$f(w_i) = -nr + 2i - 3, \text{ for } i = 1, 2, \ldots, r.$$

Subcase 2 $m \equiv 3 \pmod 4$

$$f(v_i) = nr - 2ri + 2r + 1, \text{ for } i = 1, 2, \ldots, n.$$

$$f(w_i) = \begin{cases} -nr, & \text{if } i = 1; \\ -nr + 2i - 3, & \text{if } i = 2, 3, \ldots, r. \end{cases}$$

By defined pattern of vertex labeling function f, it can be seen that f is one-one. It is easy to check that the edge labeling function f^* is bijective. Therefore, the disjoint union of cycle and complete bipartite graph $G = C_m \cup K_{n,r}$ is absolute mean graceful graph. □

Example 2.2. *Absolute mean graceful labeling of the graph $C_8 \cup K_{3,5}$ is shown in Figure 1.*

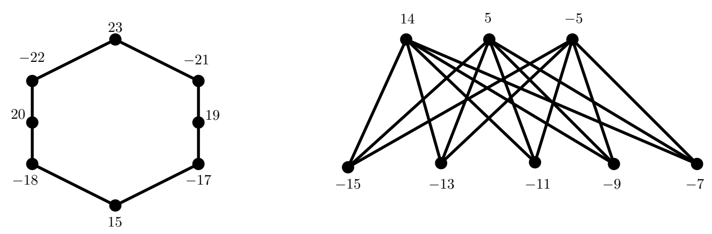

Figure 1: $C_8 \cup K_{3,5}$ and its absolute mean graceful labeling

Theorem 2.3. *The disjoint union of jewel graph J_m and sunlet graph graph S_n is absolute mean graceful graph, $\forall m \in \mathbb{N}, n > 2$.*

Proof. Let jewel graph be J_m and sunlet graph be S_n. Let $V(J_m) = \{x, y, u, v, u_i / 1 \leq i \leq m\}$, $E(J_m) = \{xy, ux, vx, uy, uu_i, vu_i / 1 \leq i \leq m\}$, and $V(S_n) = \{v_1, v_2, v_3, \ldots, v_{2n}\}$, $E(S_n) = \{v_i v_{i+1} / 1 \leq i < n\} \cup \{v_1 v_n\} \cup \{v_i v_{n+i} / 1 \leq i \leq n\}$. Let $G = J_m \cup S_n$ be disjoint union of these two graphs. We define vertex labeling function $f \colon V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$ as follows.

$$f(x) = -q;$$

$$f(y) = q - 4;$$

$$f(u) = q - 3;$$

$$f(v) = q - 1;$$

$$f(u_i) = -q + 4i + 2, \text{ for } i = 1, 2, \ldots, m.$$

Case 1 $n \equiv 0 \pmod 2$

$$f(v_i) = \begin{cases} 2n, & \text{if } i = 1; \\ (-1)^{i+1}(2n - 2i + 4), & \text{if } i = 2, 3, \ldots, n; \\ (-1)^{i}(4n - 2i + 6), & \text{if } i = n+1, n+2; \\ (-1)^{i}(4n - 2i + 4), & \text{if } i = n+3, n+4, \ldots, \frac{n}{2}; \\ (-1)^{i}(4n - 2i), & \text{if } i = \frac{n+2}{2}, \frac{n+4}{2}, \ldots, 2n. \end{cases}$$

Case 2 $n \equiv 1 \pmod 2$

$$f(v_i) = \begin{cases} (-1)^{i+1}(2n - 2i + 4), & \text{if } i = 1, 2, \ldots, n; \\ (-1)^{i+1}(4n - 2i + 4), & \text{if } i = n+1, n+2, \ldots, \frac{3n+3}{2}; \\ (-1)^{i+1}(4n - 2i), & \text{if } i = \frac{3n+5}{2}, \frac{3n+7}{2}, \ldots, 2n. \end{cases}$$

By defined pattern of vertex labeling function f, it can be seen that f is one-one. It is easy to check that the edge labeling function f^* is bijective. Therefore, the disjoint union of jewel and sunlet graph $G = J_m \cup S_n$ is absolute mean graceful graph. □

Example 2.4. *Absolute mean graceful labeling of the graph $J_4 \cup S_9$ is shown in Figure 2.*

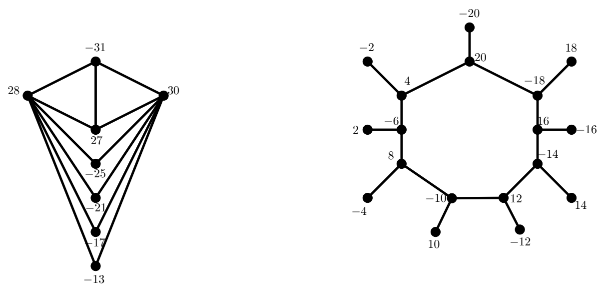

Figure 2: $J_4 \cup S_9$ and its absolute mean graceful labeling

Theorem 2.5. *The disjoint union of cycle C_m and alternate helm AH_n is absolute mean graceful graph, $\forall m, n > 2$.*

Proof. Let cycle be C_m and alternate helm be AH_n. Let $V(C_m) = \{u_1, u_2, u_3, \ldots, u_m\}$, $E(C_m) = \{u_i u_{i+1} / 1 \leq i < m\} \cup \{u_1 u_m\}$, and $V(AH_n) = \{v_0, v_1, v_2, v_3, \ldots, v_{3n}\}$, $E(AH_n) = \{v_i v_{i+1} / 1 \leq i < 2n\} \cup \{v_1 v_{2n}\} \cup \{v_0 v_i / i = 1, 3, \ldots, 2n-1\} \cup \{v_i v_{2n+\frac{i}{2}} / i = 2, 4, \ldots, 2n\}$. Let $G = C_m \cup AH_n$ be disjoint union of these two graphs. To obtain vertex labeling function $f \colon V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$ we take following cases.

Case 1 $m \equiv 0 \pmod 2$

$$f(u_i) = \begin{cases} (-1)^{\frac{m-2}{2}+i}(q - 2i + 2), & \text{if } i = 1, 2, \ldots, \frac{m+2}{2}; \\ (-1)^{\frac{m-2}{2}+i}(q - 2m + 2i - 1), & \text{if } i = \frac{m+4}{2}, \frac{m+6}{2}, \ldots, m. \end{cases}$$

Subcase 1 $n \equiv 0 \pmod 2$

$$f(v_i) = \begin{cases} 4, & \text{if } i = 0; \\ (-1)^i(4n-2i+2), & \text{if } i = 1, 2, \ldots, n; \\ -2, & \text{if } i = n+1; \\ (-1)^i(2i-1), & \text{if } i = n+2, n+3, \ldots, 2n; \\ 6n-2i-2, & \text{if } i = 2n+1, 2n+2, \ldots, \frac{5n}{2}; \\ 2i-3n-7, & \text{if } i = \frac{5n+2}{2}, \frac{5n+4}{2}, \ldots, 3n-2; \\ 4n-8, & \text{if } i = 3n-1; \\ 4n-3, & \text{if } i = 3n. \end{cases}$$

Subcase 2 $n \equiv 1 \pmod 2$

$$f(v_i) = \begin{cases} 4, & \text{if } i = 0; \\ (-1)^i(4n-2i+2), & \text{if } i = 1, 2, \ldots, n; \\ 0, & \text{if } i = n+1; \\ (-1)^i(2i-1), & \text{if } i = n+2, n+3, \ldots, 2n; \\ 6n-2i, & \text{if } i = 2n+1, 2n+2, \ldots, \frac{5n-1}{2}; \\ n, & \text{if } i = \frac{5n+1}{2}; \\ 2i-3n-4, & \text{if } i = \frac{5n+3}{2}, \frac{5n+5}{2}, \ldots, 3n-2; \\ 4n-8, & \text{if } i = 3n-1; \\ 4n-3, & \text{if } i = 3n. \end{cases}$$

Case 2 $m \equiv 1 \pmod 2$

$$f(u_i) = \begin{cases} (-1)^{\frac{m+1}{2}+i}(q-2i+2), & \text{if } i = 1, 2, \ldots, \frac{m+1}{2}; \\ (-1)^{\frac{m-1}{2}+i}(q-2m+2i-1), & \text{if } i = \frac{m+3}{2}, \frac{m+5}{2}, \ldots, m. \end{cases}$$

Subcase 1 $n \equiv 0 \pmod 2$

$$f(v_i) = \begin{cases} 5, & \text{if } i = 0; \\ (-1)^i(4n+3-2i), & \text{if } i = 1, 2, \ldots, n; \\ -3, & \text{if } i = n+1; \\ (-1)^i(2i), & \text{if } i = n+2, n+3, \ldots, 2n; \\ 6n-2i-3, & \text{if } i = 2n+1, 2n+2, \ldots, \frac{5n}{2}; \\ 2i-3n-8, & \text{if } i = \frac{5n+2}{2}, \frac{5n+4}{2}, \ldots, 3n-2; \\ 4n-10, & \text{if } i = 3n-1; \\ 4n-3, & \text{if } i = 3n. \end{cases}$$

Subcase 2 $n \equiv 1 \pmod 2$

$$f(v_i) = \begin{cases} 5, & \text{if } i = 0; \\ (-1)^i(4n+3-2i), & \text{if } i = 1,2,\ldots,n; \\ 1, & \text{if } i = n+1; \\ (-1)^i(2i), & \text{if } i = n+2, n+3, \ldots, 2n; \\ 6n-2i-1, & \text{if } i = 2n+1, 2n+2, \ldots, \frac{5n-1}{2}; \\ 8, & \text{if } i = \frac{5n+1}{2}; \\ 2i-3n-7, & \text{if } i = \frac{5n+3}{2}, \frac{5n+5}{2}, \ldots, 3n-2; \\ 4n-10, & \text{if } i = 3n-1; \\ 4n-3, & \text{if } i = 3n. \end{cases}$$

By defined pattern of vertex labeling function f, it can be seen that f is one-one. It is easy to check that the edge labeling function f^* is bijective. Therefore, the disjoint union of cycle and alternate helm $G = C_m \cup AH_n$ is absolute mean graceful graph. □

Example 2.6. *Absolute mean graceful labeling of the graph $C_7 \cup AH_8$ is shown in Figure 3.*

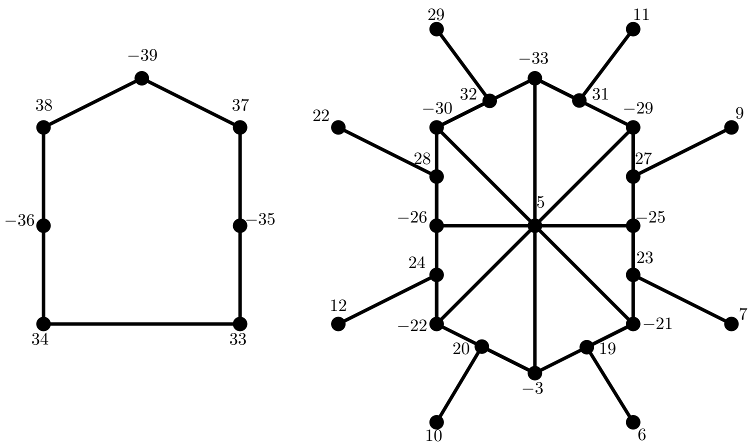

Figure 3: $C_7 \cup AH_8$ and its absolute mean graceful labeling

Theorem 2.7. *The disjoint union of jewel graph J_m and complete bipartite graph $K_{n,r}$ is absolute mean graceful graph, $\forall m, n, r \in \mathbb{N}$.*

Proof. Let jewel graph be J_m and complete bipartite graph be $K_{n,r}$. Let $V(J_m) = \{x, y, u, v, u_i/1 \leq i \leq m\}$, $E(J_m) = \{xy, ux, vx, uy, uu_i, vu_i/1 \leq i \leq m\}$, and $V(K_{n,r}) = \{v_1, v_2, v_3, \ldots, v_n, w_1, w_2, w_3, \ldots, w_r\}$, $E(K_{n,r}) = \{v_i w_j/1 \leq i \leq n, 1 \leq j \leq r\}$. Let $G = J_m \cup K_{n,r}$ be disjoint union of these two graphs. We define vertex labeling function $f \colon V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$ as follows.

$$f(x) = -q;$$

$$f(y) = q - 4;$$

$$f(u) = q - 3;$$

$$f(v) = q - 1;$$

$$f(u_i) = -q + 4i + 2, \text{ for } i = 1, 2, \ldots, m;$$

$$f(w_i) = nr - 2i + 4, \text{ for } i = 1, 2, \ldots, r;$$

$$f(v_i) = -nr + 2ri - 2r - 1, \text{ for } i = 1, 2, \ldots, n.$$

By defined pattern of vertex labeling function f, it can be seen that f is one-one. It is easy to check that the edge labeling function f^* is bijective. Therefore, the disjoint union of jewel and complete bipartite graph $G = J_m \cup K_{n,r}$ is absolute mean graceful graph. □

Example 2.8. *Absolute mean graceful labeling of the graph $J_4 \cup K_{4,3}$ is shown in Figure 4.*

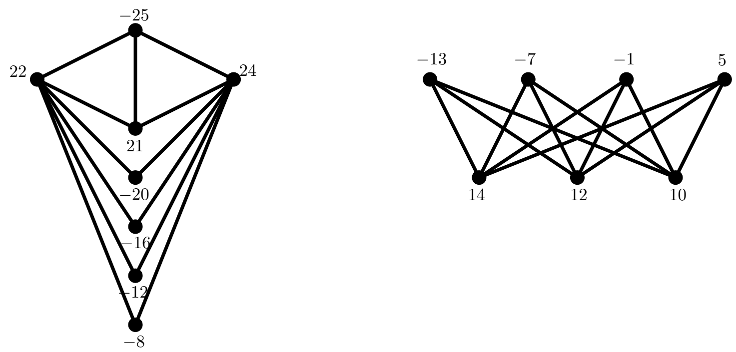

Figure 4: $J_4 \cup K_{4,3}$ and its absolute mean graceful labeling

Theorem 2.9. *The disjoint union of Swastik graph SW_n and path P_m is absolute mean graceful graph, $\forall m, n \geq 2$.*

Proof. Let Swastik graph be SW_n and path be P_m. Let $V(P_m) = \{u_1, u_2, u_3, \ldots, u_m\}$, $E(P_m) = \{u_i u_{i+1} / 1 \leq i < m\}$, and $V(SW_n) = \{v_{i,j} / i = 1, 2, 3, 4; j = 1, 2, \ldots, 4n\}$. Let $G = P_m \cup SW_n$ be disjoint union of these two graphs. We define vertex labeling function $f \colon V(G) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$ as follows.

$$f(u_i) = (-1)^i (16n + i - 1), \text{for } i = 1, 2, \ldots, m.$$

$$f(v_{1,j}) = \begin{cases} (-1)^{j+1}(12n + 2j - 2), & \text{if } j = 1, 2, \ldots, 2n + 1; \\ (-1)^{j+1}(20n - 2j + 3), & \text{if } j = 2n + 2, 2n + 3, \ldots, 4n. \end{cases}$$

$$f(v_{2,j}) = \begin{cases} (-1)^j (12n + 3), & \text{if } j = 1; \\ (-1)^{j+1}(j - 2), & \text{if } j = 2, 3, \ldots, 2n; \\ (-1)^{j+1}(j - 1), & \text{if } j = 2n + 1, 2n + 2, \ldots, 4n - 2; \\ (-1)^{j+1}(2j - 4n + 1), & \text{if } j = 4n - 1, 4n. \end{cases}$$

$$f(v_{3,j}) = \begin{cases} (-1)^j(4n+j), & \text{if } j = 1,2,\ldots,2n; \\ (-1)^j(4n+j+1), & \text{if } j = 2n+1,2n+2,\ldots,4n-1; \\ (-1)^{j+1}(12n-1), & \text{if } j = 4n. \end{cases}$$

$$f(v_{4,j}) = \begin{cases} (-1)^j(12n-2j+1), & \text{if } j = 1,2,\ldots,2n-1; \\ (-1)^j(12n-2j), & \text{if } j = 2n,2n+1,2n+2,\ldots,4n. \end{cases}$$

By defined pattern of vertex labeling function f, it can be seen that f is one-one. It is easy to check that the edge labeling function f^* is bijective. Therefore, the disjoint union of path and swastik graph $G = P_m \cup SW_n$ is absolute mean graceful graph. □

Example 2.10. *Absolute mean graceful labeling of the graph $SW_2 \cup P_4$ is shown in Figure 5.*

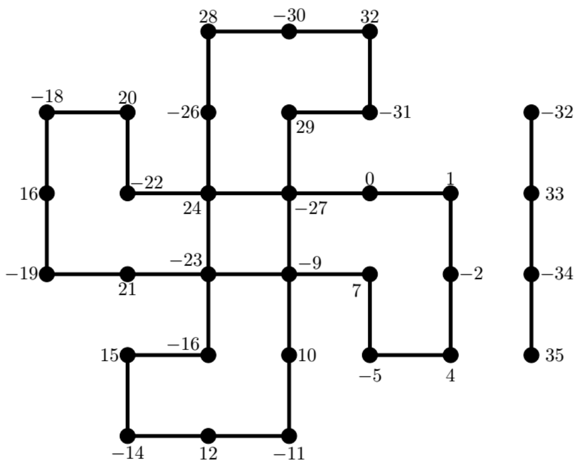

Figure 5: $SW_2 \cup P_4$ and its absolute mean graceful labeling

Theorem 2.11. *The disjoint union of gear graph G_m and sunlet graph S_n is absolute mean graceful graph, $\forall m,n > 2$.*

Proof. Let gear graph be G_m and sunlet graph be S_n. Let $V(G_m) = \{u_0,u_1,u_2,u_3,\ldots,u_{2m}\}$, $E(G_m) = \{u_iu_{i+1}/1 \le i < 2m\} \cup \{u_1u_{2m}\}$, and $V(S_n) = \{v_1,v_2,v_3,\ldots,v_{2n}\}$, $E(S_n) = \{v_iv_{i+1}/1 \le i < n\} \cup \{v_1v_n\} \cup \{v_iv_{n+i}/1 \le i \le n\}$. Let $G = G_m \cup S_n$ be disjoint union of these two graphs. We define vertex labeling function $f\colon V(G) \to \{0,\pm1,\pm2,\ldots,\pm q\}$ as follows.

$$f(u_i) = \begin{cases} (-1)^i(q-2i+2), & \text{if } i = 1,2,\ldots,m+1; \\ q-4m, & \text{if } i = 0; \\ (-1)^i(q-4m+2i-1), & \text{if } i = m+2,m+3,\ldots,2m. \end{cases}$$

Case 1 $m \equiv 0 \pmod 2$

<u>**Subcase 1**</u> $n \equiv 0 \pmod 2$

$$f(v_i) = \begin{cases} 2n+1, & \text{if } i=1; \\ (-1)^i(2n-2i+3), & \text{if } i=2,3,\dots,n; \\ -(2n+1), & \text{if } i=n+1; \\ (-1)^{i+1}(4n-2i+3), & \text{if } i=n+2,n+3,\dots,\frac{3n+2}{2}; \\ (-1)^i(n-4), & \text{if } i=\frac{3n+4}{2}; \\ (-1)^{i+1}(4n-2i+3), & \text{if } i=\frac{3n+6}{2},\frac{3n+8}{2},\dots,2n. \end{cases}$$

Subcase 2 $n \equiv 1 \pmod 2$

$$f(v_i) = \begin{cases} 2n+1, & \text{if } i=1; \\ (-1)^i(2n-2i+3), & \text{if } i=2,3,\dots,n; \\ -(2n+1), & \text{if } i=n+1; \\ (-1)^i(4n-2i+3), & \text{if } i=n+2,n+3,\dots,\frac{3n-1}{2}; \\ (-1)^{i+1}(n-1), & \text{if } i=\frac{3n+1}{2}; \\ (-1)^i(4n-2i+3), & \text{if } i=\frac{3n+3}{2},\frac{3n+5}{2},\dots,2n. \end{cases}$$

Case 2 $m \equiv 1 \pmod 2$

Subcase 1 $n \equiv 0 \pmod 2$

$$f(v_i) = \begin{cases} (-1)^{i+1}(2n-2i+2), & \text{if } i=1,2,\dots,\frac{n+2}{2}; \\ (-1)^i(2n-2i+2), & \text{if } i=\frac{n+4}{2},\frac{n+6}{2},\dots,n; \\ (-1)^i(4n-2i+2), & \text{if } i=n+1,n+2,\dots,\frac{3n+2}{2}; \\ (-1)^{i+1}(4n-2i+2), & \text{if } i=\frac{3n+4}{2},\frac{3n+6}{2},\dots,2n. \end{cases}$$

Subcase 2 $n \equiv 1 \pmod 2$

$$f(v_i) = \begin{cases} (-1)^{i+1}(2n-2i+2), & \text{if } i=1,2,\dots,n; \\ (-1)^{i+1}(4n-2i+2), & \text{if } i=n+1,n+2,\dots,\frac{3n+2}{2}; \\ (-1)^i(n-2), & \text{if } i=\frac{3n+3}{2}; \\ (-1)^{i+1}(4n-2i+2), & \text{if } i=\frac{3n+5}{2},\frac{3n+7}{2},\dots,2n. \end{cases}$$

By defined pattern of vertex labeling function f, it can be seen that f is one-one. It is easy to check that the edge labeling function f^* is bijective. Therefore, the disjoint union of gear graph and sunlet graph $G = G_m \cup S_n$ is absolute mean graceful graph. □

Example 2.12. *Absolute mean graceful labeling of the graph $G_9 \cup S_9$ is shown in Figure 6.*

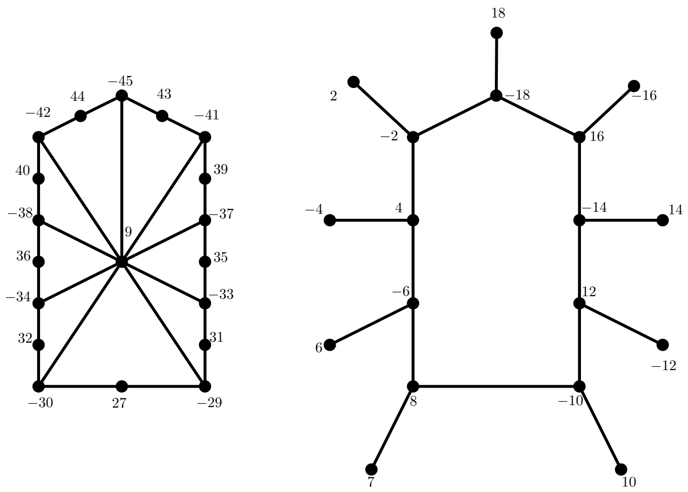

Figure 6: $G_9 \cup S_9$ and its absolute mean graceful labeling

3. Conclusion

In this paper, some disconnected absolute mean graceful graphs are investigated. To establish general characterizations for disconnected absolute mean graceful graphs is an open area of research.

Acknowledgment

The first author is supported by a Senior Research Fellowship from CSIR-UGC, India (UGC-Ref.No.:1060/(CSIR-UGC NET JUNE 2019)).

References

[1] P. Z. Akbari, V. J. Kaneria and N. A. Parmar, *Absolute Mean Graceful Labeling of Jewel Graph and Jelly Fish Graph*, International Journal of Mathematics Trends and Technology, **68** (1) (2022), 86–93.

[2] P. Z. Akbari, V. J. Kaneria and N. A. Parmar, *Some Absolute Mean Graceful Graphs in the Context of Barycentric Subdivision*, International Journal of Mathematics Trends and Technology, **68** (5) (2022), 51–60.

[3] G. S. Bloom and S. W. Golomb, *Applications of Numbered Undirected Graphs*, In: Proceedings of IEEE, **65**, 1977, pp. 562–570.

[4] H. P. Chudasama and V. J. Kaneria, *Duplication of Some Graph Elements and Absolute Mean Graceful Labeling*, International Journal of Mathematics Trends and Technology, **65** (4) (2019), 107–115.

[5] J. A. Gallian, *A Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinatorics, **26** (2023), #DS6.

[6] S. W. Golomb, *How to Number a Graph*, In: R. C. Read (Ed.), Graph Theory and Computing, Academic Press, New York, 1972, pp. 23–37.

[7] F. Harary, *Graph Theory*, Addison-Wesley, Reading, Massachusetts, 1969.

[8] V. J. Kaneria and H. P. Chudasama, *Absolute Mean Graceful Labeling in Various Graphs*, International Journal of Mathematics and its Applications, **5** (4-E) (2017), 723–726.

[9] V. J. Kaneria, H. P. Chudasama and P. P. Andharia, *Absolute Mean Graceful Labeling in Path Union of Various Graphs*, Mathematical Journal of Interdisciplinary Sciences, **7** (1) (2018), 51–56.

[10] A. Rosa, *On Certain Valuations of the Vertices of a Graph*, In: Theory of Graphs (International Symposium, Rome, July 1966), Gordon and Breach, New York and Dunod Paris, 1967, pp. 349–355.

[11] V. Yegnanarayanan and P. Vaidhyanathan, *Some Interesting Applications of Graph Labellings*, Journal of Mathematical and Computational Science, **2** (5) (2012), 1522–1531.