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# Existence of Fixed Points for $\gamma$-$FG$-contractive Condition via Cyclic $(\alpha, \beta)$-admissible Mappings in $b$-metric Like Spaces

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**Abstract**

This paper extends and generalizes the results of paper Padhan [14]. We show various fixed point theorems for such mappings in a complete $b$-metric like space, and propose the novel ideas of cyclic $(\alpha, \beta)$-admissible mapping utilising $\gamma$-$FG$-contractive mapping. Adequate illustrations are provided to validate the findings, along with the implications of the primary findings.

**Keywords:** $b$-metric like space; cyclic $(\alpha, \beta)$-admissible; complete $b$-metric like space; $\gamma$-$FG$-contractive.

**2020 Mathematics Subject Classification:** 47H09, 54H25, 47H10.

## 1. Introduction

The most well-known conclusion in fixed point theory is the Banach contraction principle, which shows that every contractive mapping in a full metric space has a distinct fixed point. Many applications of this theory have been made by employing diverse contractive circumstances in different kinds of inconsistencies. There have been a lot of intriguing but distinct generalisations of the Banach-contraction principle in recent years have been provided by Wardowski [18] and Samet et al. [17]. Wardowski [18] first proposed this idea in 2012 of an F-contraction mapping and looked into whether fixed points for these mappings exist. Wardowski and Van Dung [19], in addition to Piri and Kumam [16], expanded upon the notion of F-contraction and demonstrated certain fixed and common fixed point results. Parvaneh et al. [15] recently generalised the Wardowski fixed point findings in $b$-metric and ordered $b$-metric spaces using a slightly modified family of functions, shown by $\Delta_{G,\beta}$. However, Samet et al. [17] generalised BCP by introducing the idea of $\alpha$-admissible mappings and providing the idea of $\alpha$-$\psi$-contractive mapping. Following then, a number of additional writers obtained different fixed point conclusions by using $\alpha$-admissible mappings. In keeping with this vein, Alizadeh et al.

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[2], Padhan et al. [7,13] established the concept of cyclic $(\alpha,\beta)$-admissible mapping and demonstrated fundamental fixed point outcomes. In this work, we continue this line of inquiry by introducing new ideas for cyclic $(\alpha,\beta)$-type $\gamma$-$FG$-contractive mapping and proving some fixed point theorems pertaining to such contractive mapping, supported by several instances. The cyclic mapping findings are presented with some implications. A nonlinear integral equation's solution is provided as an application, along with an example to illustrate it.

## 2. Preliminaries

Throughout this paper, we denote by $\mathbb{N}, \mathbb{R}_+$ and $\mathbb{R}$ the sets of positive integers, nonnegative real numbers and real numbers, respectively.

**Definition 2.1** ([5]). *Let $X$ be a nonempty set, let $k \geq 1$ be a given real number. A function $d : X \times X \to [0,\infty)$ is called a b-metric if for all $x,y,z \in X$ the following conditions holds:*

$(S_1)$ $d(x,y) = 0$ *if and only if* $x = y$;

$(S_2)$ $d(x,y) = d(y,x)$;

$(S_3)$ $d(x,y) \leq k[d(x,z) + d(z,y)]$.

*Then $(X,d)$ is said to be a b-metric space. The coefficient of $(X,d)$ is $k \geq 1$.*

**Definition 2.2** ([20]). *Let $\mathcal{F}$ be a nonempty set and a mapping $\sigma : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_+$ is such that $\forall\ u,v,z \in \mathcal{F}$, it satisfies*

$(\sigma_1)$ $\sigma(u,v) = 0$ *implies* $u = v$

$(\sigma_2)$ $\sigma(u,v) = \sigma(v,u)$;

$(\sigma_3)$ $\sigma(u,v) \leq \sigma(u,z) + \sigma(z,v)$.

*Then $(\mathcal{F},\sigma)$ is said to be a metric-like space.*

Examples of metric-like spaces are as follows.

**Example 2.3** ([23]). *Let $\mathcal{F} = \mathbb{R}$; then the mappings $\sigma_i : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_+ (i \in \{2,3,4\})$, defined by*

$$\sigma_2(u,v) = |u| + |v| + a, \qquad \sigma_3(u,v) = |u-b| + |v-b|, \qquad \sigma_4(u,v) = u^2 + v^2, \tag{1}$$

*are metric-like on $\mathcal{F}$, where $a \geq 0$ and $b \in \mathbb{R}$.*

**Definition 2.4** ([21]). *Let $\mathcal{F}$ be a nonempty set and $k \geq 1$ be a real number. A function $\sigma_b : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_+$ is b-metric-like if $\forall\ u,v,z \in \mathcal{F}$, the following assertions hold:*

$(\sigma_b 1)$ $\sigma_b(u,v) = 0$ *implies* $u = v$

($\sigma_b$2) $\sigma_b(u,v) = \sigma_b(v,u)$

($\sigma_b$3) $\sigma_b(u,v) \leq k[\sigma_b(u,z) + \sigma_b(z,v)]$.

*The pair $(\mathcal{F}, \sigma_b)$ is called a b-metric-like space with the coefficient k.*

In a $b$-metric-like space $(\mathcal{F}, \sigma_b)$ if $u, v \in \mathcal{F}$ and $\sigma_b(u,v) = 0$, then $u = v$, but the converse may not be true and $\sigma_b(u,u)$ may be positive for $u \in \mathcal{F}$. Clearly, every $b$-metric and every partial $b$-metric is a $b$-metric-like with the same coefficient $k$. However, the converses of these facts need not hold [22]. Every $b$-metric-like $\sigma_b$ on $\mathcal{F}$ generates a topology $\tau_{\sigma_b}$ on $\mathcal{F}$ whose base is the family of all open $\sigma_b$-balls $\{B_{\sigma_b}(u,\delta) : u \in \mathcal{F}, \delta > 0\}$, where $B_{\sigma_b}(u,\delta) = \{v \in \mathcal{F} : |\sigma_b(u,v) - \sigma_b(u,u)| < \delta\}$, $\forall\, u \in \mathcal{F}$ and $\delta > 0$.

**Definition 2.5** ([21,22]). *Let $(\mathcal{F}, \sigma_b)$ be a b-metric-like space with coefficient k, let $\{u_n\}$ be a sequence in $\mathcal{F}$ and $u \in \mathcal{F}$. Then*

*(i) $\{u_n\}$ is called convergent to u w.r.t. $\tau_{\sigma_b}$, if $\lim_{n\to\infty} \sigma_b(u_n, u) = \sigma_b(u,u)$;*

*(ii) $\{u_n\}$ is called a Cauchy sequence in $(\mathcal{F}, \sigma_b)$ if $\lim_{n,m\to\infty} \sigma_b(u_n, u_m)$ exists (and is finite).*

*(iii) $(\mathcal{F}, \sigma_b)$ is called a complete b-metric-like space if for every Cauchy sequence $\{u_n\}$ in $\mathcal{F}$ there exists $u \in \mathcal{F}$ such that*

$$\lim_{n,m\to\infty} \sigma_b(u_n, u_m) = \lim_{n\to\infty} \sigma_b(u_n, u) = \sigma_b(u,u). \tag{2}$$

It is clear that the limit of a sequence is usually not unique in a $b$-metric-like space (already partial metric spaces have this property).

**Proposition 2.6** ([12]). *Every partial ordered b-metric-like $\sigma_b$ defines a b-metric-like $d_{\sigma_b}$, where*

$$d_{\sigma_b}(x,y) = 2\sigma_b(x,y) - \sigma_b(x,x) - \sigma_b(y,y), \text{ for all } x, y \in \mathcal{F} \tag{3}$$

**Definition 2.7** ([12]). *Let $(\mathcal{F}, \preceq)$ be a partially ordered set and $\mathcal{P} \colon \mathcal{F} \to \mathcal{F}$ be a mapping. We say that $\mathcal{P}$ is nondecreasing with respect to $\preceq$ if*

$$x, y \in \mathcal{F}, \qquad x \preceq y \qquad \Rightarrow \mathcal{P}x \preceq \mathcal{P}y.$$

**Definition 2.8** ([12]). *Let $(\mathcal{F}, \preceq)$ be a partially ordered set. A sequence $\{x_n\}$ is said to be a nondecreasing with respect to $\preceq$ if $x_n \preceq x_{n+1}$. for all $n \in$ ♮.*

**Definition 2.9** ([12]). *A triple $(\mathcal{F}, \preceq, \sigma_b)$ is called an ordered b-metric-like space if $(\mathcal{F}, \preceq)$ is a partially ordered set and $\sigma_b$ is a b-metric-like on $\mathcal{F}$.*

**Lemma 2.10** ([6]). *Let $(\mathcal{F}, \sigma_b)$ be a partial b-metric-like space with the coefficient $s > 1$ and suppose that $\{x_n\}$ and $\{y_n\}$ are convergent to x and y, respectively. Then we have*

$$\frac{1}{s^2}\sigma_b(x,y) - \frac{1}{s}\sigma_b(x,x) - \sigma_b(y,y) \leq \liminf_{n\to\infty} p_b(x_n, y_n)$$

$$\leq \limsup_{n\to\infty} \sigma_b(x_n, y_n)$$
$$\leq s\sigma_b(x,x) + s^2\sigma_b(y,y) + s^2\sigma_b(x,y).$$

Alizadeh et al. [2] introduced the concept of cyclic $(\alpha,\beta)$-admissible mapping as follows:

**Definition 2.11** ([2]). *Let $X$ be a nonempty set, $f$ be a self-mapping on $X$ and $\alpha,\beta : X \to [0,\infty)$ be two mappings. We say that the mapping $f$ is a cyclic $(\alpha,\beta)$-admissible mapping if*

$$x \in X,\ with\ \alpha(x) \geq 1 \Rightarrow \beta(fx) \geq 1.$$
$$x \in X,\ with\ \beta(x) \geq 1 \Rightarrow \alpha(fx) \geq 1.$$

## 3. Main Results

In this section, we extends and generalizes the results of paper Padhan et al. [14] and investigate some fixed point results for cyclic $(\alpha,\beta)$-type $\gamma$-$FG$-contractive mappings and then we prove some fixed point results in $b$-metric like and partially ordered $b$-metric like spaces. To prove our main result we will use the following notations cited in Parvaneh et al. [15]. We will consider the following classes of functions. $\Delta_F$ will denote the set of all functions $F : \mathbb{R}_+ \to \mathbb{R}$ such that

$(\Delta_1)$ $F$ is continuous and strictly increasing;

$(\Delta_2)$ for each sequence $\{t_n\} \subseteq \mathbb{R}_+$, $\lim_{n\to\infty} t_n = 0$ iff $\lim_{n\to\infty} F(t_n) = -\infty$.

$\Delta_{G,\gamma}$ will denote the set of pairs $(G,\gamma)$, where $G : \mathbb{R}_+ \to \mathbb{R}$ and $\gamma : [0,\infty) \to [0,1)$, such that

$(\Delta_3)$ for each sequence $\{t_n\} \subseteq \mathbb{R}_+$, $\limsup_{n\to\infty} G(t_n) \geq 0$ iff $\limsup_{n\to\infty} t_n \geq 1$;

$(\Delta_4)$ for each sequence $\{t_n\} \subseteq [0,\infty)$, $\limsup_{n\to\infty} \gamma(t_n) = 1$ implies $\lim_{n\to\infty} t_n = 0$;

$(\Delta_5)$ for each sequence $\{t_n\} \subseteq \mathbb{R}_+$, $\sum_{n=1}^{\infty} G(\gamma(t_n)) = -\infty$.

**Example 3.1** ([15]). *If $F(t) = G(t) = \ln t$ and $\gamma(t) = k \in (0,1)$, then $F \in \Delta_F$ and $(G,\gamma) \in \Delta_{G,\gamma}$. Let $F(t) = -\frac{1}{\sqrt{t}}$, $G(t) = \ln t$ and $\gamma(t) = \frac{1}{k}e^{-t}$ for $t > 0$ and $\gamma(t) = 0$. Then $F \in \Delta_F$ and $(G,\gamma) \in \Delta_{G,\gamma}$.*

**Definition 3.2.** *Let $(X,\sigma)$ be a $b$-metric like space with coefficient $k \geq 1$. Also suppose that $\alpha,\beta$ and $f : X \times X \to [0,\infty)$ are mappings. Then $f$ is called cyclic $(\alpha,\beta)$-type $\gamma$-FG- contractive mapping if there exist $F \in \Delta_F$, $(G,\gamma) \in \Delta_{G,\gamma}$ such that the following condition holds:*

$$\alpha(x)\beta(y) \geq 1, \sigma(fx,fy) > 0 \Rightarrow \alpha(x)\beta(y)F(k^3\sigma(fx,fy)) \leq F(M_k(x,y)) + G(\gamma(M_k(x,y))) \tag{4}$$

*for all $x,y \in X$*

where
$$M_k(x,y) = \max\left\{\sigma(x,y), \sigma(y,fy), \sigma(x,fx), \frac{\sigma(x,fy)+\sigma(y,fx)}{2k}\right\}. \tag{5}$$

**Theorem 3.3.** *Let $(X, \sigma)$ be a $\sigma_b$-complete b-metric like space with coefficient $k \geq 1$, let $\alpha, \beta : X \to [0, \infty)$ and $f : X \to X$ be a cyclic $(\alpha, \beta)$-type $\gamma$-$FG$-contractive mapping satisfying the following conditions:*

*(1) one of the following conditions holds:*

*(a) There exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$;*

*(b) There exists $y_0 \in X$ such that $\beta(y_0) \geq 1$;*

*(2) $f$ is $\sigma_b$-continuous;*

*(3) $f$ is a cyclic $(\alpha, \beta)$-admissible mapping.*

*Then $f$ has a unique fixed point. Moreover, if the sequence $\{x_n\}$ in $X$ defined by $x_n = fx_{n-1}$ for all $n \in \mathbb{N}$ is such that $x_0$ is an initial point in condition $(a)$ and the sequence $\{y_n\}$ in $X$ defined by $y_n = fy_{n-1}$ for all $n \in \mathbb{N}$ is such that $y_0$ is an initial point in condition $(b)$, then $\{x_n\}$ and $\{y_n\}$ converges to a fixed point of $f$.*

*Proof.*
**Case 1:** Let $x_0 \in X$ such that $\alpha(x_0) \geq 1$. Define the sequence $\{x_n\}$ by $x_{n+1} = fx_n$. If there exists $n_0 \in \mathbb{N}$, such that $x_{n_0} = x_{n_0+1}$, then $x_{n_0}$ is the fixed point of $f$, and hence the proof is completed. So we assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. It follows that

$$\sigma(x_n, x_{n+1}) > 0, \ \forall\, n \in \mathbb{N}.$$

Now we need to prove that

$$\lim_{n\to\infty} \sigma(x_n, x_{n+1}) = 0. \tag{6}$$

Since $f$ is cyclic $(\alpha, \beta)$-admissible mapping, we have

$$\alpha(x_0) \geq 1 \Rightarrow \beta(x_1) = \beta(fx_0) \geq 1 \Rightarrow \alpha(x_2) = \alpha(fx_1) \geq 1. \tag{7}$$

By induction, we obtain

$$\alpha(x_{2k}) \geq 1 \text{ and } \beta(x_{2k+1}) \geq 1 \tag{8}$$

for all $k \in \mathbb{N}$. Since $\alpha(x_0)\beta(x_1) \geq 1$, we get

$$\begin{aligned} F(\sigma(fx_0, fx_1)) &\leq \alpha(x_0)\beta(x_1)F(k^3\sigma(fx_0, fx_1)) \\ &\leq F(M_k(x_0, x_1)) + G(\gamma(M_k(x_0, x_1))). \end{aligned}$$

Proceeding in the same manner, we get $\alpha(x_n)\beta(x_{n+1}) \geq 1$, for all $n \in \mathbb{N}$.

$$\begin{aligned} F(\sigma(fx_n, fx_{n+1})) &\leq \alpha(x_n)\beta(x_{n+1})F(k^3\sigma(fx_n, fx_{n+1})) \\ &\leq F(M_k(x_n, x_{n+1})) + G(\gamma(M_k(x_n, x_{n+1}))). \end{aligned} \tag{9}$$

Note that for each $n \in \mathbb{N}$, we have

$$\begin{aligned}
M_k(x_n, x_{n+1}) &= \max\left\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, fx_{n+1}), \sigma(x_n, fx_n), \frac{\sigma(x_n, fx_{n+1}) + \sigma(x_{n+1}, fx_n)}{2k}\right\} \\
&= \max\left\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, x_{n+2}), \frac{\sigma(x_n, x_{n+2})}{2k}\right\} \\
&\leq \max\left\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, x_{n+2})\frac{k[\sigma(x_n, x_{n+1}) + \sigma(x_{n+1}, x_{n+2})]}{2k}\right\} \\
&\leq \max\left\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, x_{n+2}), \frac{\sigma(x_n, x_{n+1}) + \sigma(x_{n+1}, x_{n+2})}{2}\right\} \\
&\leq \max\left\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, x_{n+2})\right\}.
\end{aligned} \tag{10}$$

If $M_k(x_n, x_{n+1}) = \sigma(x_{n+1}, x_{n+2})$ for some $n \in \mathbb{N}$, then inequality (9) implies that

$$\begin{aligned}
F(\sigma(x_{n+1}, x_{n+2})) &\leq \alpha(x_n)\beta(x_{n+1})F(k^3\sigma(x_{n+1}, x_{n+2})) \\
&< F(\sigma(x_{n+1}, x_{n+2})) + G(\gamma(M_k(x_n, x_{n+1}))).
\end{aligned}$$

So, $G(\gamma(M_k(x_n, x_{n+1}))) \geq 0$, which implies that $\gamma(M_k(x_n, x_{n+1})) \geq 1$, a contradiction. Therefore, for all $n \in \mathbb{N}$.

$$M_k(x_n, x_{n+1}) = \sigma(x_n, x_{n+1}).$$

From (4), we have

$$\begin{aligned}
F(\sigma(x_{n+1}, x_{n+2})) =&\alpha(x_n)\beta(x_{n+1})F(k^3\sigma(x_{n+1}, x_{n+2})) \\
\leq&F(\sigma(x_n, x_{n+1})) + G(\gamma(M_k(x_n, x_{n+1})))
\end{aligned} \tag{11}$$

for all $n \in \mathbb{N}$. Consequently, we deduce that

$$F(\sigma(x_{n+1}, x_{n+2})) \leq F(\sigma(x_{n-1}, x_n)) + G(\gamma(M_k(x_{n-1}, x_n))) + G(\gamma(M_k(x_n, x_{n+1}))).$$

Iteratively, we find that

$$F(\sigma(x_n, x_{n+1})) \leq F(\sigma(x_0, x_1)) + \sum_{i=1}^{n} G(\gamma(M_k(x_{i-1}, x_i))). \tag{12}$$

By taking $n \to \infty$ in above equation we obtain $\lim_{n\to\infty} F(\sigma(x_n, x_{n+1})) = -\infty$, since $(G, \gamma) \in \Delta_{G,\gamma}$ and since, $F \in \Delta_F$ gives

$$\lim_{n\to\infty} \sigma(x_n, x_{n+1}) = 0. \tag{13}$$

Next, we prove that $\{x_n\}$ is a $b$-Cauchy sequence in $X$. Arguing by contradiction, then there exists $\epsilon_0 > 0$ for which we can find subsequences $\{x_{p(r)}\}$ and $\{x_{q(r)}\}$ of $\{x_n\}$ such that $p(r) > q(r) \geq r$ and

$$\sigma(x_{p(r)}, x_{q(r)}) \geq \epsilon_0 \tag{14}$$

and $q(r)$ is the smallest number such that (14) holds.

$$\sigma(x_{p(r)}, x_{q(r)-1}) < \epsilon_0. \tag{15}$$

By $(S_3)$, (14) and (15), we get

$$\begin{aligned} \epsilon_0 \leq \sigma(x_{p(r)}, x_{q(r)}) &\leq k\sigma(x_{p(r)}, x_{q(r)-1}) + k\sigma(x_{q(r)-1}, x_{q(r)}) \\ &< k\epsilon_0 + k\sigma(x_{q(r)-1}, x_{q(r)}). \end{aligned} \tag{16}$$

Taking the limit supremum as $r \to \infty$ in above inequality, which together with (13) shows

$$\limsup_{r\to\infty} \sigma(x_{p(r)}, x_{q(r)}) < k\epsilon_0, \ \forall\ \mathbb{N}, \tag{17}$$

using the triangular inequality and we deduce,

$$\sigma(x_{p(r)}, x_{q(r)}) \leq k[\sigma(x_{p(r)}, x_{q(r)+1}) + \sigma(x_{q(r)+1}, x_{q(r)})] \tag{18}$$

and

$$\sigma(x_{p(r)}, x_{q(r)+1}) \leq k[\sigma(x_{p(r)}, x_{q(r)}) + \sigma(x_{q(r)}, x_{q(r)+1})]. \tag{19}$$

Letting $r \to +\infty$ in (18),(19) by (13) and (17) we obtain

$$\epsilon_0 \leq k \limsup_{r\to\infty} \sigma(x_{p(r)}, x_{q(r)+1}) \tag{20}$$

and

$$\limsup_{r\to\infty} \sigma(x_{p(r)}, x_{q(r)+1}) \leq k^2\epsilon_0. \tag{21}$$

This implies that

$$\frac{\epsilon_0}{k} \leq \limsup_{r\to\infty} \sigma(x_{p(r)}, x_{q(r)+1}) \leq k^2\epsilon_0. \tag{22}$$

Similarly, we obtain

$$\frac{\epsilon_0}{k} \leq \limsup_{r\to\infty} \sigma(x_{q(r)}, x_{p(r)+1}) \leq k^2\epsilon_0. \tag{23}$$

Finally, we obtain that

$$\sigma(x_{q(r)}, x_{p(r)+1}) \leq k[\sigma(x_{q(r)}, x_{q(r)+1}) + \sigma(x_{q(r)+1}, x_{p(r)+1})]. \tag{24}$$

Taking the limit supremum as $r \to \infty$ in (24), from (13) and (22), we obtain that

$$\frac{\epsilon_0}{k^2} \leq \limsup_{r\to\infty} \sigma(x_{q(r)+1}, x_{p(r)+1}) \leq k^3\epsilon_0. \tag{25}$$

Using the cyclic property of $\alpha$, $\beta$ we get

$$\alpha(x_{p(r)})\beta(x_{q(r)}) \geq 1.$$

Now

$$\begin{aligned} F(\sigma(fx_{p(r)}, fx_{q(r)})) &\leq \alpha(x_{p(r)})\beta(x_{q(r)})F(k^3\sigma(x_{p(r)+1}, x_{q(r)+1})) \\ &\leq F(M_k(x_{p(r)}, x_{q(r)})) + G(\gamma(M_k(x_{p(r)}, x_{q(r)}))). \end{aligned} \tag{26}$$

where

$$\begin{aligned} &M_k(x_{p(r)}, x_{q(r)}) \\ &= \max\left\{\sigma(x_{p(r)}, x_{q(r)}), \sigma(x_{p(r)}, fx_{p(r)}), \sigma(x_{q(r)}, fx_{q(r)}), \frac{\sigma(x_{p(r)}, fx_{q(r)}) + \sigma(x_{q(r)}, fx_{p(r)})}{2k}\right\} \\ &= \max\left\{\sigma(x_{p(r)}, x_{q(r)}), \sigma(x_{p(r)}, x_{p(r)+1}), \sigma(x_{q(r)}, x_{q(r)+1}), \frac{\sigma(x_{p(r)}, x_{q(r)+1}) + \sigma(x_{q(r)}, x_{p(r)+1})}{2k}\right\} \end{aligned} \tag{27}$$

for all $k \in \mathbb{N}$. Letting limit supremum as $r \to +\infty$ in (27) and using (13),(17),(22), and (23), we obtain

$$M_k(x_{p(r)}, x_{q(r)}) = \max\left\{k\epsilon_0, \frac{k^2\epsilon_0 + k^2\epsilon_0}{2k}\right\} = k\epsilon_0. \tag{28}$$

Now

$$\begin{aligned} F(k\epsilon_0) &\leq F(k^3\frac{\epsilon_0}{k^2}) \\ &\leq F(k^3 \limsup_{r\to\infty} \sigma(x_{q(r)+1}, x_{p(r)+1}) \\ &\leq \limsup_{r\to\infty} F(M_k(x_{p(r)}, x_{q(r)})) + \limsup_{r\to\infty} G(\gamma(M_k(x_{p(r)}, x_{q(r)}))) \\ &\leq F(k\epsilon_0) + \limsup_{r\to\infty} G(\gamma(M_k(x_{p(r)}, x_{q(r)}))) \end{aligned} \tag{29}$$

which implies that

$$\limsup_{r\to\infty} G(\gamma(M_k(x_{p(r)}, x_{q(r)}))) \geq 0.$$

This yields to $\limsup_{k\to\infty} \gamma(M_k(x_{p(r)}, x_{q(r)})) \geq 1$, and since $\gamma(t) < 1$ for all $t \geq 0$, we have

$$\limsup_{k\to\infty} \gamma(M_k(x_{p(r)}, x_{q(r)})) = 1.$$

Therefore,

$$\limsup_{k\to\infty} M_k(x_{p(r)}, x_{q(r)}) = 0,$$

a contradiction because of (14) and (27). Therefore $\{x_n\}$ is a $b$-Cauchy sequence in $X$. Now by using the $b$-completeness of $b$-metric like space, there exists $x^* \in X$ such that

$$\sigma(x^*, x^*) = \lim_{n\to\infty} \sigma(x_n, x^*) = \lim_{n,m\to\infty} \sigma(x_n, x_m) = 0. \tag{30}$$

By $\sigma_b$-continuity of $f$, we get

$$\lim_{n\to\infty} \sigma(fx_n, fx) = 0.$$

Using $(S_3)$, we have

$$\sigma(x, fx) \leq k[\sigma(x, fx_n) + \sigma(fx_n, fx)] \tag{31}$$

for all $n \in \mathbb{N}$. Taking the limit as $n \to \infty$ in the above inequality, we obtain

$$\sigma(x, fx) = 0.$$

and then $fx = x$. Let $x, y$ are fixed points of $f$, where $x \neq y$. Now using (7), we have $\alpha(x)\beta(y) \geq 1$, and then from

$$\begin{aligned} F(\sigma(fx, fy)) &\leq \alpha(x)\beta(y)F(k^3\sigma(fx, fy)) \\ &\leq F(M_k(x, y)) + G(\gamma(M_k(x, y))) \end{aligned} \tag{32}$$

where,

$$M_k(x, y) = \left\{\sigma(x, y), \sigma(x, fx), \sigma(y, fy), \frac{\sigma(x, fy) + \sigma(fx, y)}{2k}\right\} = \sigma(x, y)$$

we get

$$F(\sigma(x, y)) \leq F(\sigma(x, y)) + G(\gamma(\sigma(x, y)))$$

so $G(\gamma(\sigma(x, y))) \geq 0$ which yields that $\gamma(\sigma(x, y)) \geq 1$, a contradiction. Hence $x = y$. Therefore, $f$ has unique fixed point.

**Case 2:** Assume that there exists $y_0 \in X$ such that $\beta(y_0) \geq 1$. Proceeding in a similar manner as above, we get the conclusion. □

Taking $G(t) = \ln t, \gamma(t) = k$ where $k \in (0, 1)$ and putting $\tau = -\ln k$ in the above theorem, we obtain a generalization of the results from [18,19] in the setup of $b$-metric spaces.

**Corollary 3.4.** *Let $(X, \sigma)$ be a $\sigma_b$-complete b-metric like space with coefficient $k \geq 1$, let $\alpha, \beta : X \to [0, \infty)$ and $f : X \to X$ be a mapping such that the mapping $f$ satisfying the following conditions:*

*(1) one of the following conditions holds:*

*(a) There exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$;*

*(b) There exists $y_0 \in X$ such that $\beta(y_0) \geq 1$;*

*(2) $\alpha(x)\beta(y) \geq 1, \sigma(fx, fy) > 0 \Rightarrow \tau + \alpha(x)\beta(y)F(k^3\sigma(fx, fy)) \leq F(M_k(x, y))$ for some $\tau > 0$, for all $x, y \in X$ and $M_k$ is defined as earlier;*

*(3) $f$ is $\sigma_b$-continuous;*

*(4) $f$ is a cyclic $(\alpha, \beta)$-admissible mapping.*

*Then $f$ has a unique fixed point. Moreover, if the sequence $\{x_n\}$ in $X$ defined by $x_n = fx_{n-1}$ for all $n \in \mathbb{N}$ is such that $x_0$ is an initial point in condition $(a)$ and the sequence $\{y_n\}$ in $X$ defined by $y_n = fy_{n-1}$ for all $n \in \mathbb{N}$ is such that $y_0$ is an initial point in condition $(b)$, then $\{x_n\}$ and $\{y_n\}$ converges to a fixed point of $f$.*

Taking $F(t) = G(t) = \ln(t)$, and $\alpha(x)\beta(y) = 1$ in the above theorem, we obtain the following result.

**Corollary 3.5.** *Let $(X, \sigma)$ be a $\sigma_b$-complete b-metric like space with coefficient $k \geq 1$, let $\alpha, \beta : X \to [0, \infty)$, and $f : X \to X$ be a mapping such that the mapping $f$ satisfying the following conditions:*

*(1) one of the following conditions holds:*

  *(1) There exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$;*

  *(2) There exists $y_0 \in X$ such that $\beta(y_0) \geq 1$;*

*(2) $k^3\sigma(fx, fy) \leq \gamma(M_k(x, y))M_k(x, y)$; $\sigma(fx, fy) > 0$ for all $x, y \in X$, and $M_k$ is defined as earlier;*

*(3) $f$ is $\sigma_b$-continuous;*

*(4) $f$ is a cyclic $(\alpha, \beta)$-admissible mapping.*

*Then $f$ has a unique fixed point. Moreover, if the sequence $\{x_n\}$ in $X$ defined by $x_n = fx_{n-1}$ for all $n \in \mathbb{N}$ is such that $x_0$ is an initial point in condition $(a)$ and the sequence $\{y_n\}$ in $X$ defined by $y_n = fy_{n-1}$ for all $n \in \mathbb{N}$ is such that $y_0$ is an initial point in condition $(b)$, then $\{x_n\}$ and $\{y_n\}$ converges to a fixed point of $f$.*

Taking $F(t) = -\frac{1}{\sqrt{t}}$ and $G(t) = \ln(t)$, and $\alpha(x)\beta(y) = 1$ in the above theorem, we obtain the following result.

**Corollary 3.6.** *Let $(X, \sigma)$ be a $\sigma_b$-complete b-metric like space with coefficient $k \geq 1$, let $\alpha, \beta : X \to [0, \infty)$, and $f : X \to X$ be a mapping such that the mapping $f$ satisfying the following conditions:*

*(1) one of the following conditions holds:*

  *(a) There exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$;*

  *(b) There exists $y_0 \in X$ such that $\beta(y_0) \geq 1$;*

*(2) $k^3\sigma(fx, fy) \leq \frac{M_k(x,y)}{[1-\sqrt{M_k(x,y)}\ln\gamma(M_k(x,y))]^2}$ for some $\sigma(fx, fy) > 0$ for all $x, y \in X$, where $(\ln t, \gamma) \in \Delta_{G,\gamma}$, and $M_k$ is defined as earlier;*

*(3) $f$ is $\sigma_b$-continuous;*

*(4) $f$ is a cyclic $(\alpha, \beta)$-admissible mapping.*

*Then $f$ has a unique fixed point. Moreover, if the sequence $\{x_n\}$ in $X$ defined by $x_n = fx_{n-1}$ for all $n \in \mathbb{N}$ is such that $x_0$ is an initial point in condition $(a)$ and the sequence $\{y_n\}$ in $X$ defined by $y_n = fy_{n-1}$ for all $n \in \mathbb{N}$ is such that $y_0$ is an initial point in condition $(b)$, then $\{x_n\}$ and $\{y_n\}$ converges to a fixed point of $f$.*

Taking $\gamma(t) = r$, where $r \in (0,1)$ and $\alpha(x)\beta(y) = 1$ in the above corollary and denoting $k' = -k$, we obtain the following result.

**Corollary 3.7.** *Let $(X, \sigma)$ be a $\sigma_b$-complete b-metric like space with coefficient $k \geq 1$, let $\alpha, \beta : X \to [0, \infty)$, and $f : X \to X$ be a mapping such that the mapping $f$ satisfying the following conditions:*

*(1) one of the following conditions holds:*

*(a) There exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$;*

*(b) There exists $y_0 \in X$ such that $\beta(y_0) \geq 1$;*

*(2) $k^3\sigma(fx, fy) \leq \frac{M_k(x,y)}{[1+k'\sqrt{M_k(x,y)}]^2}$ for some $\sigma(fx, fy) > 0$ for all $x, y \in X$, where $k' > 0$, and $M_k$ is defined as earlier.*

*(3) $f$ is $\sigma_b$-continuous;*

*(4) $f$ is a cyclic $(\alpha, \beta)$-admissible mapping.*

*Then $f$ has a unique fixed point. Moreover, if the sequence $\{x_n\}$ in $X$ defined by $x_n = fx_{n-1}$ for all $n \in \mathbb{N}$ is such that $x_0$ is an initial point in condition $(a)$ and the sequence $\{y_n\}$ in $X$ defined by $y_n = fy_{n-1}$ for all $n \in \mathbb{N}$ is such that $y_0$ is an initial point in condition $(b)$, then $\{x_n\}$ and $\{y_n\}$ converges to a fixed point of $f$.*

Taking $F(t) = t, G(t) = (r-1)t, \gamma(t) = r$ where $r \in [0, \infty)$ and putting $k = 1$, $\alpha(x) = 1, \beta(x) = 1$ in the above theorem, we obtain a following result.

**Corollary 3.8.** *Let $(X, \sigma)$ be a $\sigma_b$-complete b-metric like space with coefficient $k$ let $\alpha, \beta : X \to [0, \infty)$, and $f : X \to X$ be a mapping such that*

$$\sigma(fx, fy) \leq rM(x, y))$$

*for some $r \in [0,1)$ and for all $x, y \in X$, and $M$ is defined as earlier. Then $f$ has a fixed point. Moreover, if the sequence $\{x_n\}$ in $X$ defined by $x_n = fx_{n-1}$ for all $n \in \mathbb{N}$ is such that $x_0$ is an initial point then $\{x_n\}$ converges to a fixed point of $f$.*

Taking $k = k^3$ and $\alpha(x)\beta(y) = 1$ in Theorem 3.3, we obtain the result of Parvaneh et al. [15].

**Corollary 3.9.** *Let $(X, \sigma)$ be a $\sigma_b$-complete b-metric space with coefficient $k > 1$, let $\alpha, \beta : X \to [0, \infty)$, and $f : X \to X$ be a mapping such that the mapping $f$ satisfying the following conditions:*

*(1) one of the following conditions holds:*

*(a) There exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$;*

*(b) There exists $y_0 \in X$ such that $\beta(y_0) \geq 1$;*

*(2)*

$$\alpha(x)\beta(y) \geq 1, \sigma(fx, fy) > 0 \Rightarrow F(k\sigma(fx, fy)) \leq F(M(x, y)) + G(\gamma(M(x, y))) \tag{33}$$

*for all $x, y \in X$, and*

$$M(x,y) = \max\left\{\sigma(x,y), \sigma(y,fy), \sigma(x,fx), \frac{\sigma(x,fy)+\sigma(y,fx)}{2}\right\};$$

(3) *$f$ is $\sigma_b$-continuous;*

(4) *$f$ is a cyclic $(\alpha,\beta)$-admissible mapping.*

*Then $f$ has a unique fixed point. Moreover, if the sequence $\{x_n\}$ in $X$ defined by $x_n = fx_{n-1}$ for all $n \in \mathbb{N}$ is such that $x_0$ is an initial point in condition $(a)$ and the sequence $\{y_n\}$ in $X$ defined by $y_n = fy_{n-1}$ for all $n \in \mathbb{N}$ is such that $y_0$ is an initial point in condition $(b)$, then $\{x_n\}$ and $\{y_n\}$ converges to a fixed point of $f$.*

Taking $F(t) = t, G(t) = (1-k)t, \gamma(t) = k$ where $k \in [0,\infty)$ and putting $\alpha(x) = 1, \beta(x) = 1$ in the above theorem, we obtain a following result.

**Corollary 3.10.** *Let $(X,\sigma)$ be a $\sigma_b$-complete b-metric like space with coefficient $k \geq 1$, let $\alpha, \beta : X \to [0,\infty)$, and $f : X \to X$ be a mapping such that*

$$k^3\sigma_b(fx,fy) \leq rM_k(x,y))$$

*for some $r \in [0,1)$ and for all $x, y \in X$, and $M_k$ is defined earlier. Then $f$ has a fixed point. Moreover, if the sequence $\{x_n\}$ in $X$ defined by $x_n = fx_{n-1}$ for all $n \in \mathbb{N}$ is such that $x_0$ is an initial point then $\{x_n\}$ converges to a fixed point of $f$.*

**Example 3.11.** *Let $X = [0,\infty)$ and let $\sigma : X \times X \to [0,\infty)$ be defined by $\sigma(x,y) = |x-y|^2$ for all $x, y \in X$. Then $(X,\sigma_b)$ is a complete b-metric like space with $k = 2$. Define the mappings $\alpha, \beta, : X \to [0,\infty)$, $\gamma : [0,\infty) \to [0,1)$ and $f : X \to X$ as follows:*

$$\alpha(x) = \begin{cases} \frac{x+7}{2}, & x \in [0,\frac{1}{2}], \\ 0, & otherwise, \end{cases} \quad and\ \beta(x) = \begin{cases} \frac{x+6}{2}, & x \in [0,\frac{1}{2}], \\ 1, & otherwise \end{cases}$$

*and*

$$f(x) = \begin{cases} \frac{x^2}{3}, & x \in [0,\frac{1}{2}], \\ x+0.01, & otherwise \end{cases} \quad and\ \gamma(t) = \frac{2}{9}.$$

*Now, we will prove that $f$ is a cyclic $(\alpha,\beta)$-admissible mapping. For $x \in [0,\frac{1}{2}]$, we have*

$$\alpha(x) \geq 1 \Rightarrow \beta(fx) = \beta\left(\frac{x^2}{3}\right) = \left(\frac{\left(\frac{x+6}{2}\right)^2}{3}\right) \geq 1$$

*and*

$$\beta(x) \geq 1 \Rightarrow \alpha(fx) = \alpha\left(\frac{x^2}{3}\right) = \left(\frac{\left(\frac{x+7}{2}\right)^2}{3}\right) \geq 1.$$

*Therefore, $f$ is a cyclic $(\alpha, \beta)$-admissible mapping. Next, we will prove that $f$ satisfy the contractive condition* (33)*, with the mappings $F, G : \mathbb{R}^+ \to \mathbb{R}$ as $F(t) = G(t) = \ln t$, for all $t \in [0, \infty)$, Assume that $x, y \in X$ are such that $\alpha(x)\beta(y) \geq 1$. Then we have $x, y \in [0, \frac{1}{2}]$ and hence*

$$\begin{aligned} k\sigma(fx, fy) &= 2\left|\frac{x^2}{3} - \frac{y^2}{3}\right|^2 \\ &\leq \frac{2}{9}|x^2 - y^2|^2 \\ &\leq \frac{2}{9}(|x - y|^2) \\ &\leq \gamma(M(x, y))\sigma(x, y) \\ &\leq \gamma(M(x, y))M(x, y) \end{aligned}$$

*and hence,*

$$F(k\sigma(fx, fy)) \leq F(M(x, y)) + G(\gamma(M(x, y))).$$

*Therefore, $f$ satisfies all the conditions of Corollary 3.9, hence $f$ has a unique fixed point $x^* = 0$.*

**Example 3.12.** *Let $X = [0, \infty)$ and let $\sigma : X \times X \to [0, \infty)$ be defined by $\sigma(x, y) = |x - y|^2$ for all $x, y \in X$. Then $(X, \sigma)$ is a complete b-metric like space with $k = 2$. Define the mappings $\alpha, \beta, : X \to [0, \infty)$, $\gamma : [0, \infty) \to [0, 1)$ and $f : X \to X$ as follows:*

$$\alpha(x) = \begin{cases} \frac{x^2+3}{2}, & x \in [0, 1], \\ 0, & otherwise\ , \end{cases} \quad and\ \beta(x) = \begin{cases} \frac{2x^2+5}{4}, & x \in [0, 1], \\ 1, & otherwise \end{cases}$$

*and*

$$f(x) = \begin{cases} \frac{x}{3\sqrt{3+x^2}}, & x \in [0, 1], \\ 2x, & otherwise \end{cases} \quad and\ \gamma(t) = \frac{8}{9}.$$

*Now, we will prove that $f$ is a cyclic $(\alpha, \beta)$-admissible mapping. For $x \in [0, 1]$, we have*

$$\alpha(x) \geq 1 \Rightarrow \beta(fx) = \beta\left(\frac{x}{3\sqrt{3 + x^2}}\right) = \left(\frac{\left(2\frac{x^2}{9(3+x^2)}\right) + 5}{4}\right) \geq 1$$

*and*

$$\beta(x) \geq 1 \Rightarrow \alpha(fx) = \alpha\left(\frac{x}{3\sqrt{3 + x^2}}\right) = \left(\frac{\left(\frac{x^2}{9(3+x^2)}\right) + 3}{2}\right) \geq 1.$$

*Therefore, $f$ is a cyclic $(\alpha, \beta)$-admissible mapping. Next, we will prove that $f$ satisfy the contractive condition* (4)*, with the mappings $F, G : \mathbb{R}^+ \to \mathbb{R}$ as $F(t) = G(t) = \ln t$, for all $t \in [0, \infty)$. Assume that $x, y \in X$ are such that $\alpha(x)\beta(y) \geq 1$. Then we have $x, y \in [0, 1]$ and hence*

$$k^3\sigma(fx, fy) = 8\left|\frac{x}{3\sqrt{3 + x^2}} - \frac{y}{3\sqrt{3 + y^2}}\right|^2$$

$$\leq \frac{8}{9}|x-y|^2$$
$$\leq \gamma(M(x,y))\sigma(x,y)$$
$$\leq \gamma(M(x,y))M(x,y)$$

*and hence,*

$$F(k^3\sigma(fx,fy)) \leq F(M(x,y)) + G(\gamma(M(x,y))).$$

*Therefore, $f$ satisfies all the conditions of Theorem 3.3, hence $f$ has a unique fixed point $x^* = 0$.*

In the following, we give some fixed point results involving cyclic mappings which can be regarded as consequences of the previous results.

**Definition 3.13.** *[9] Let $A$ and $B$ be nonempty subsets of a set $X$. A mapping $f : A \cup B \to A \cup B$ is called cyclic if $f(A) \subseteq B$ and $f(B) \subseteq A$.*

**Definition 3.14.** *Let $(X, \sigma)$ be a b-metric like space with coefficient $k \geq 1$. We say that a mapping $f : A \cup B \to A \cup B$ is a $(A, B)$-$\gamma$-FG- contractive mapping if there exist $F \in \Delta_F$, $(G, \gamma) \in \Delta_{G,\gamma}$ such that the following condition holds:*

$$A(x)B(y) \geq 1, \sigma(fx, fy) > 0 \Rightarrow A(x)B(y)F(k^3\sigma(fx, fy)) \leq F(M_k(x,y)) + G(\gamma(M_k(x,y))) \tag{34}$$

*for all $x \in A$ and $y \in B$, where,*

$$M_k(x,y) = \max\left\{\sigma(x,y), \sigma(y,fy), \sigma(x,fx), \frac{\sigma(x,fy)+\sigma(y,fx)}{2k}\right\} \tag{35}$$

**Theorem 3.15.** *Let $A$ and $B$ be two nonempty subsets of the complete b-metric like space $(X, \sigma)$ with coefficient $k \geq 1$ and $f : A \cup B \to A \cup B$ is a $(A, B)$-$\gamma$-FG-contractive mapping. Then $f$ has a fixed point in $A \cap B$.*

*Proof.* Define mappings $\alpha, \beta : A \cup B \to [0, \infty)$ by

$$\alpha(x) = \begin{cases} 1, & x \in A \\ 0, & \text{otherwise} \end{cases} \quad \text{and } \beta(x) = \begin{cases} 1, & x \in B \\ 0, & \text{otherwise} . \end{cases}$$

For $x, y \in A \cup B$ such that $\alpha(x)\beta(y) \geq 1$, we get $x \in A$ and $y \in B$. Then we have

$$\alpha(x)\beta(y) \geq 1, \sigma(fx, fy) > 0 \Rightarrow \alpha(x)\beta(y)F(k^3\sigma(fx, fy)) \leq F(M_k(x,y)) + G(\gamma(M_k(x,y)))$$

and thus the condition (4) holds. Therefore, $f$ is an $(\alpha, \beta)$-$\gamma$-$FG$-contractive mapping. It is easy to see that $f$ is a cyclic $(\alpha, \beta)$-admissible mapping. Since $A$ and $B$ are nonempty subsets, there exists $x_0 \in A$ such that $\alpha(x_0) \geq 1$ and there exists $y_0 \in B$ such that $\beta(y_0) \geq 1$. Now, all conditions of Theorem 3.3

holds, so $f$ has a fixed point in $A \cup B$, say $z$. If $z \in A$, then $z = fz \in B$. Similarly, if $z \in B$, then $z \in A$. Hence $z \in A \cup B$. □

Similarly, by replacing $M_k(x,y) = \sigma(x,y)$ we obtain the following corollary.

**Corollary 3.16.** *Let $A$ and $B$ be two nonempty subsets of the complete b-metric like space $(X,\sigma)$ with coefficient $k \geq 1$ and $f : A \cup B \to A \cup B$ be a mapping such that*

$$A(x)B(y) \geq 1, \sigma(fx,fy) > 0 \Rightarrow A(x)B(y)F(k^3\sigma(fx,fy)) \leq F(\sigma(x,y)) + G(\gamma(\sigma(x,y))), \quad (36)$$

*Then $f$ has a fixed point in $A \cap B$.*

Taking $F(t) = G(t) = \ln(t)$, and $\alpha(x)\beta(y) = 1$ in theorem 3.15, we obtain the following Corollary.

**Corollary 3.17.** *Let $A$ and $B$ be two nonempty subsets of the complete b-metric like space $(X,\sigma)$ with coefficient $k \geq 1$ and $f : A \cup B \to A \cup B$ be a mapping such that*

$$k^3\sigma(fx,fy) \leq M_k(x,y))\gamma(M_k(x,y)), \quad (37)$$

*for all $x \in A$, $y \in B$ and $M_k$ is defined as earlier. Then $f$ has a fixed point in $A \cap B$.*

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