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L-valued Intuitionistic L-fuzzy Generalised Lattices of the Type 3

Parimi Radhakrishna Kishore[1,2,*], Sisay Tadesse Taye[3]

[1] *Department of Mathematics, Arba Minch University, Arba Minch, Ethiopia*

[2] *Department of Mathematics, SRM University, Andhra Pradesh, India*

[3] *Ph.D. Research Scholar, Department of Mathematics, Arba Minch University, Arba Minch, Ethiopia*

Abstract

This article deals with the concept of L-valued intuitionistic L-fuzzy subgeneralised lattice of the type 3 (IFsubgl of type 3) of a generalised lattice. Introduced the concepts L-valued intuitionistic L-fuzzy subgeneralised lattice of the type 3 (IFsubgl of type 3), L-valued intuitionistic L-fuzzy ideal of the type 3 (IFideal of type 3), L-valued intuitionistic L-fuzzy filter of the type 3 (IFfilter of type 3), L-valued intuitionistic L-fuzzy prime ideal of the type 3 (IF prime ideal of type 3), L-valued intuitionistic L-fuzzy prime filter of the type 3 (IF prime filter of type 3) and L-valued intuitionistic L-fuzzy convex subgeneralised lattice of the type 3 (IF convex subgl of type 3) of a generalised lattice. Characterized them by their $(\alpha, \beta)-$level subsets, discussed some equivalent conditions and their intersections.

Keywords: poset; lattice; generalised lattice; fuzzy set; fuzzy lattice.
2020 Mathematics Subject Classification: 06-XX.

1. Introduction

The theory related to the concepts fuzzy set (L-fuzzy set, intuitionistic L-fuzzy set) and fuzzy lattice (L-fuzzy lattice, intuitionistic L-fuzzy lattice) are known from [2, 3, 4, 5] and [12, 13, 14, 15, 16, 17, 18]. Mellacheruvu Krishna Murty and U. Madana Swamy [6] (Professors of Andhra University) introduced the concept of generalised lattice and the theory of generalised lattices developed by the author P.R.Kishore in [7, 8] that can play an intermediate role between the theories of lattices and posets. The concepts and the corresponding theory of fuzzy generalised lattices and fuzzy generalised lattice ordered groups [9, 10, 11] introduced and developed by the author P.R.Kishore. In [20] Gerstenkorn and Tepavcevic introduced the concept L-valued intuitionistic L-fuzzy set of type 3. This paper deals with the concept of L-valued intuitionistic L-fuzzy subgeneralised lattice of the type 3 (IFsubgl of type 3) of a generalised lattice. Section 2 contains some preliminaries from the references. In Section 3,

*Corresponding author (parimirkk@gmail.com)

introduced the concept L-valued intuitionistic L-fuzzy subgeneralised lattice of the type 3 (IFsubgl of type 3), L-valued intuitionistic L-fuzzy ideal of the type 3 (IFideal of type 3), L-valued intuitionistic L-fuzzy filter of the type 3 (IFfilter of type 3), L-valued intuitionistic L-fuzzy prime ideal of the type 3 (IF prime ideal of type 3), and L-valued intuitionistic L-fuzzy prime filter of the type 3 (IF prime filter of type 3) Characterized them by their $(\alpha,\beta)-$level subsets, discussed some equivalent conditions and their intersections. In section 4 introduced the concept of L-valued intuitionistic L-fuzzy convex subgeneralised lattice of the type 3 (IF convex subgl of type 3) of a generalised lattice. Characterized them by their $(\alpha,\beta)-$level subsets, discussed some equivalent conditions and their intersections.

2. Preliminaries

This section contains some preliminaries from the references those are useful in the next sections.
A lattice L is said to be a complete lattice if for any subset of L the infimum and supremum exists in L. Every complete lattice is bounded and the least element is denoted by 0, the greatest element is denoted by 1. Let X be a non-empty set and L is a complete lattice satisfying the infinite meet distributive law, then any mapping from X into L is called a $L-$fuzzy subset (or L-fuzzy set or L-set) of X. Let μ be a L-fuzzy subset of X, then for any $\alpha \in L$, the set $\mu_\alpha = \{x \in X \mid \mu(x) \geq \alpha\}$ is called a level subset of μ.

Definition 2.1 ([12]). *Let X be a non-empty set. A collection of objects in the set form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ is called an intuitionistic fuzzy set of X if (i) $\mu_A : X \to [0,1]$ is a fuzzy set in X called degree of membership function on X, (ii) $\nu_A : X \to [0,1]$ is a fuzzy set in X called degree of non-membership function on X and (iii) for each $x \in X$, we have $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.*

Note 2.2. *If ν_A is complement of μ_A (that is $\nu_A(x) = 1 - \mu_A(x)$ for all $x \in X$), then the intuitionistic fuzzy set A will be fuzzy set in X.*

Definition 2.3 ([16]). *Let L be a lattice and $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in L\}$ be an intuitionistic fuzzy set of L. Then A is called an intuitionistic fuzzy sublattice of L if the following conditions satisfied: for all $x, y \in L$; (i) $\mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}$ (ii) $\mu_A(x \wedge y) \geq min\{\mu_A(x), \mu_A(y)\}$ (iii) $\nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}$ and (iv) $\nu_A(x \wedge y) \leq max\{\nu_A(x), \nu_A(y)\}$.*

Definition 2.4 ([14]). *Let X be a set and $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ be an intuitionistic fuzzy set of X. Let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. Then the $(\alpha,\beta)-$cut of A defined by the set $C_{\alpha,\beta}(A) = \{x \in X \mid \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$.*

In [14] it is observed that If A and B are two intuitionistic fuzzy sets of a set X, then we have $C_{\alpha,\beta}(A) \subseteq C_{\delta,\theta}(A)$ if $\alpha \geq \delta$ and $\beta \leq \theta$.

Definition 2.5 ([20]). *Let L be a complete lattice with least element 0_L and greatest element 1_L. Let $[0,1]$ be the interval in real line. Let $h : L \to [0,1]$ be a lattice homomorphism, that is $h(\alpha \wedge \beta) = min\{h(\alpha), h(\beta)\}$ and $h(\alpha \vee \beta) = Max\{h(\alpha), h(\beta)\}$. Let X be a non-empty set. A collection of objects in the set form $A =$*

$\{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ *is called a lattice valued intuitionistic fuzzy (L-valued intuitionistic fuzzy) set of type-3 of* X *if (i)* $\mu_A : X \to L$ *is a* $L-$*fuzzy set in* X *called degree of membership function on* X*, (ii)* $\nu_A : X \to L$ *is a* $L-$*fuzzy set in* X *called degree of non-membership function on* X *and (iii) for each* $x \in X$*, we have* $0 \leq h(\mu_A(x)) + h(\nu_A(x)) \leq 1$.

The definitions of generalised lattice, subgeneralised lattice, strong ideal, prime ideal, convex subgl and homomorphism of generalised lattices are known from [6, 7, 8, 9]. Throughout this article we shall denote by P a generalised lattice.

3. L-valued Intuitionistic L-fuzzy Subgls of the Type 3 of a Generalised Lattice

In this section defined the concepts IFset of type 3, IFsubgl of type 3, IFideal of type 3, IFfilter of type 3, IF prime ideal of type 3 and IF prime filter of type 3. Discussed some equivalent conditions for IFideal of type 3, IFfilter of type 3, IF prime ideal of type 3 and IF prime filter of type 3. Characterized the IFsubgls of type 3, IFideals of type 3, IFfilters of type 3, IF prime ideals of type 3 and IF prime filters of type 3 by their $(\alpha, \beta)-$level subsets. Finally proved that the intersection of any collection of IFsubgls of type 3 (IFideals of type 3, IFfilters of type 3) is again a IFsubgl of type 3 (IFideal of type 3, IFfilter of type 3).

Definition 3.1. *Let* L *be a complete lattice with least element* 0_L *and greatest element* 1_L*. Let* $[0,1]$ *be the interval in real line. Let* $h : L \to [0,1]$ *be a lattice homomorphism, that is* $h(\alpha \wedge \beta) = min\{h(\alpha), h(\beta)\}$ *and* $h(\alpha \vee \beta) = Max\{h(\alpha), h(\beta)\}$*. A collection of objects in the set form* $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in P\}$ *is called a L-valued intuitionistic L-fuzzy set of the type-3 (IFset of type 3) of* P *if (i)* $\mu_A : P \to L$ *is a* $L-$*fuzzy set in* P *called degree of membership function on* P*, (ii)* $\nu_A : P \to L$ *is a* $L-$*fuzzy set in* P *called degree of non-membership function on* P *and (iii) for each* $x \in P$*, we have* $0 \leq h(\mu_A(x)) + h(\nu_A(x)) \leq 1$.

Definition 3.2. *Let* $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in P\}$ *be an L-valued intuitionistic L-fuzzy set of the type 3 (IFset of type 3) of* P*. Then* A *is called an L-valued intuitionistic L-fuzzy subgeneralised lattice of type 3 (IFsubgl of type 3) of* P *if the following conditions satisfied: for any finite subset* X *of* P*; (i)* $\mu_A(s) \geq \bigwedge_{x \in X} \mu_A(x)$ *for all* $s \in mu(X)$ *(ii)* $\mu_A(t) \geq \bigwedge_{x \in X} \mu_A(x)$ *for all* $t \in ML(X)$ *(iii)* $\nu_A(s) \leq \bigvee_{x \in X} \nu_A(x)$ *for all* $s \in mu(X)$ *and (iv)* $\nu_A(t) \leq \bigvee_{x \in X} \nu_A(x)$ *for all* $t \in ML(X)$.

Definition 3.3. *Let* A *be an IFsubgl of type 3 of* P*. Then* A *is called an L-valued intuitionistic L-fuzzy ideal of type 3 (IFideal of type 3) of* P *if for any* $p, q \in P$*;* $p \leq q$ *in* P *implies* $\mu_A(p) \geq \mu_A(q)$ *and* $\nu_A(p) \leq \nu_A(q)$.

Definition 3.4. *Let* A *be an IFsubgl of type 3 of* P*. Then* A *is called an L-valued intuitionistic L-fuzzy filter of type 3 (IFfilter of type 3) of* P *if for any* $p, q \in P$*;* $p \leq q$ *in* P *implies* $\mu_A(p) \leq \mu_A(q)$ *and* $\nu_A(p) \geq \nu_A(q)$.

Definition 3.5. *An IFideal of type 3* A *of* P *is said to be an L-valued intuitionistic L-fuzzy prime ideal of type 3 (IF prime ideal of type 3) of* P *if for any finite subset* X *of* P *(i)* $\mu_A(t) \leq \bigvee_{x \in X} \mu_A(x)$ *for all* $t \in ML(X)$ *and (ii)* $\nu_A(t) \geq \bigwedge_{x \in X} \nu_A(x)$ *for all* $t \in ML(X)$.

Definition 3.6. *An IFfilter of type 3 A of P is said to be an L-valued intuitionistic L-fuzzy prime filter of type 3 (IF prime filter of type 3) of P if for any finite subset X of P (i) $\mu_A(s) \leq \bigvee_{x\in X} \mu_A(x)$ for all $s \in mu(X)$ and (ii) $\nu_A(s) \geq \bigwedge_{x\in X} \nu_A(x)$ for all $s \in mu(X)$.*

Theorem 3.7. *Let A be an IFset of type 3 of P. Then for any $p, q \in P$ and for any finite subset X of P the following conditions are equivalent: (i) $p \leq q$ in P implies $\mu_A(p) \geq \mu_A(q)$ and $\nu_A(p) \leq \nu_A(q)$ (ii) $\mu_A(t) \geq \bigvee_{x\in X} \mu_A(x)$ and $\nu_A(t) \leq \bigwedge_{x\in X} \nu_A(x)$ for all $t \in ML(X)$ (iii) $\mu_A(s) \leq \bigwedge_{x\in X} \mu_A(x)$ and $\nu_A(s) \geq \bigvee_{x\in X} \nu_A(x)$ for all $s \in mu(X)$.*

Proof. $(i) \Rightarrow (ii)$: Suppose the condition (i) and to prove (ii): Let $t \in ML(X)$. Then for all $x \in X$, we have $t \leq x$ and by (i) we get $\mu_A(t) \geq \mu_A(x)$, $\nu_A(t) \leq \nu_A(x)$. Therefore $\mu_A(t) \geq \bigvee_{x\in X} \mu_A(x)$ and $\nu_A(t) \leq \bigwedge_{x\in X} \nu_A(x)$. $(ii) \Rightarrow (i)$: Suppose the condition (ii) and to prove (i): Suppose $p \leq q$, that is $ML\{p, q\} = \{p\}$. Then by (ii) we get $\mu_A(p) \geq \mu_A(p) \vee \mu_A(q) \geq \mu_A(q)$ and $\nu_A(p) \leq \nu_A(p) \wedge \nu_A(q) \leq \nu_A(q)$. $(i) \Rightarrow (iii)$: Suppose the condition (i) and to prove (iii): Let $s \in mu(X)$. Then for all $x \in X$, we have $s \geq x$ and by (i) we get $\mu_A(s) \leq \mu_A(x)$, $\nu_A(s) \geq \nu_A(x)$. Therefore $\mu_A(s) \leq \bigwedge_{x\in X}\{\mu_A(x)\}$ and $\nu_A(s) \geq \bigvee_{x\in X}\{\nu_A(x)\}$. $(iii) \Rightarrow (i)$: Suppose the condition (iii) and to prove (i): Suppose $p \leq q$, that is $mu\{p, q\} = \{q\}$. Then by (iii) we get $\mu_A(q) \leq \mu_A(p) \wedge \mu_A(q) \leq \mu_A(p)$ and $\nu_A(q) \geq \nu_A(p) \vee \nu_A(q) \geq \nu_A(p)$. □

Theorem 3.8. *Let A be an IFset of type 3 of P. Then for any $p, q \in P$ and for any finite subset X of P the following conditions are equivalent: (i) $p \leq q$ in P implies $\mu_A(p) \leq \mu_A(q)$ and $\nu_A(p) \geq \nu_A(q)$ (ii) $\mu_A(s) \geq \bigvee_{x\in X} \mu_A(x)$ and $\nu_A(s) \leq \bigwedge_{x\in X} \nu_A(x)$ for all $s \in mu(X)$ (iii) $\mu_A(t) \leq \bigwedge_{x\in X} \mu_A(x)$ and $\nu_A(t) \geq \bigvee_{x\in X} \nu_A(x)$ for all $t \in ML(X)$.*

Note 3.9. *By Definition 3.3 and Theorem 3.7 we can say that, an IFsubgl of type 3 of P is an IFideal of type 3 of P if it satisfies any one of the three conditions of Theorem 3.7. Similarly by Definition 3.4 and Theorem 3.8 we can say that, an IFsubgl of type 3 of P is an IFfilter of type 3 of P if it satisfies any one of the three conditions of Theorem 3.8.*

Note 3.10. *By Definition 3.2 and Note 3.9 we have, an IFset of type 3 of P is an IFideal of type 3 of P if and only if for any finite subset X of P we have $\mu_A(s) = \bigwedge_{x\in X} \mu_A(x)$ and $\nu_A(s) = \bigvee_{x\in X} \nu_A(x)$ for all $s \in mu(X)$. Similarly we have, an IFset of type 3 of P is an IFfilter of type 3 of P if and only if for any finite subset X of P we have $\mu_A(t) = \bigwedge_{x\in X} \mu_A(x)$ and $\nu_A(t) = \bigvee_{x\in X} \nu_A(x)$ for all $t \in ML(X)$.*

Theorem 3.11. *Let A be an IFideal of type 3 of P. Then for any finite subset X of P, the following conditions are equivalent: (i) A is an IF prime ideal of type 3 of P (ii) $\mu_A(t) = \bigvee_{x\in X} \mu_A(x)$ and $\nu_A(t) = \bigwedge_{x\in X} \nu_A(x)$ for all $t \in ML(X)$ (iii) there exists $x, y \in X$ such that $h(\mu_A(t)) = h(\mu_A(x))$ and $h(\nu_A(t)) = h(\nu_A(y))$ for all $t \in ML(X)$.*

Proof. $(i) \Rightarrow (ii)$: Suppose A is an IF prime ideal of type 3 of P. By Definition 3.5 and Theorem 3.7, we get (ii). $(ii) \Rightarrow (i)$: Clear. $(iii) \Rightarrow (ii)$: Suppose the condition (iii) and to prove (ii). Let $t \in ML(X)$.

Then by (iii) there exists $x, y \in X$ such that $\mu_A(t) = \mu_A(x)$ and $\nu_A(t) = \nu_A(y)$. Since $t \leq z$ for all $z \in X$ and A is an IFideal of type 3, we have $\mu_A(t) \geq \mu_A(z)$ for all $z \in X$. This implies $\mu_A(t) \geq \bigvee_{z\in X} \mu_A(z)$. Since $x \in X$, it is clear that $\bigvee_{z\in X} \mu_A(z) \geq \mu_A(x) = \mu_A(t)$. Therefore $\mu_A(t) = \bigvee_{z\in X} \mu_A(z)$. Similarly we can prove $\nu_A(t) = \bigwedge_{z\in X} \nu_A(z)$. $(ii) \Rightarrow (iii)$: Suppose the condition (ii) and to prove (iii). Let $t \in ML(X)$. Then by (ii) we have $\mu_A(t) = \bigvee_{x\in X} \mu_A(x)$ and $\nu_A(t) = \bigwedge_{x\in X} \nu_A(x)$. This implies, since X is finite and by definition 3.1, we have $h(\mu_A(t)) = h(\bigvee_{x\in X} \mu_A(x)) = \bigvee_{x\in X} h(\mu_A(x)) = Max_{x\in X}\{h(\mu_A(x))\}$ and $h(\nu_A(t)) = h(\bigwedge_{x\in X} \nu_A(x)) = \bigwedge_{x\in X} h(\nu_A(x)) = min_{x\in X}\{h(\nu_A(x))\}$. Therefore for all $t \in ML(X)$ there exists $x \in X$ such that $h(\mu_A(t)) = \mu_A(x)$ and for all $t \in ML(X)$ there exists $y \in X$ such that $h(\nu_A(t)) = \nu_A(y)$. □

Theorem 3.12. *Let A be an IFideal of type 3 of P. Then for any finite subset X of P, the following conditions are equivalent: (i) A is an IF prime filter of type 3 of P (ii) $\mu_A(s) = \bigvee_{x\in X} \mu_A(x)$ and $\nu_A(s) = \bigwedge_{x\in X} \nu_A(x)$ for all $s \in mu(X)$ (iii) there exists $x, y \in X$ such that $h(\mu_A(s)) = h(\mu_A(x))$ and $h(\nu_A(s)) = h(\nu_A(y))$ for all $s \in mu(X)$.*

Definition 3.13. *Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in P\}$ be an IFset of type 3 of P. Let $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$. Then the $(\alpha, \beta)-$cut of A defined by the set $C_{\alpha,\beta}(A) = \{x \in P \mid \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$.*

Theorem 3.14. *Let A be an IFset of type 3 of P. Then A is an IFsubgl of type 3 of P if and only if $C_{\alpha,\beta}(A)$ is a subgl of P for all $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$.*

Proof. Suppose A is an IFsubgl of type 3 of P and let $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$. To show that $C_{\alpha,\beta}(A)$ is a subgeneralised lattice of P : Let X be a finite subset of $C_{\alpha,\beta}(A)$. Let $s \in mu(X)$ and $t \in ML(X)$. By Definition 3.2 we have $\mu_A(s), \mu_A(t) \geq \bigwedge_{x\in X} \mu_A(x) \geq \alpha$ and $\nu_A(s), \nu_A(t) \leq \bigvee_{x\in X} \nu_A(x) \leq \beta$. This implies $s, t \in C_{\alpha,\beta}(A)$. Then $mu(X), ML(X) \subseteq C_{\alpha,\beta}(A)$. Therefore $C_{\alpha,\beta}(A)$ is a subgeneralised lattice of P. Conversely suppose the condition. To show that A is an IFsubgl of type 3 of P : Let X be a finite subset of P. Then $\alpha = \bigwedge_{x\in X} \mu_A(x) \in L$, $\beta = \bigvee_{x\in X} \nu_A(x) \in L$, $h(\alpha) = h(\bigwedge_{x\in X} \mu_A(x)) = min\{h(\mu_A(x)) \mid x \in X\}$ and $h(\beta) = h(\bigvee_{x\in X} \nu_A(x)) = Max\{h(\nu_A(x)) \mid x \in X\}$. By Definition 3.1, we have $0 \leq h(\mu_A(x)) + h(\nu_A(x)) \leq 1$, that is $h(\mu_A(x)) \leq 1 - h(\nu_A(x))$ for all $x \in X$. Consider $h(\alpha) = min\{h(\mu_A(x)) \mid x \in X\} \leq min\{1 - h(\nu_A(x)) \mid x \in X\} = 1 - Max\{h(\nu_A(x)) \mid x \in X\} = 1 - h(\beta)$. This implies $h(\alpha) + h(\beta) \leq 1$. Since $\alpha \leq \mu_A(x)$ and $\beta \geq \nu_A(x)$ for all $x \in X$, by Definition 3.13 we have $x \in C_{\alpha,\beta}(A)$ for all $x \in X$, that is $X \subseteq C_{\alpha,\beta}(A)$. Since by hypothesis $C_{\alpha,\beta}(A)$ is a subgeneralised lattice of P, we have $ML(X), mu(X) \subseteq C_{\alpha,\beta}(A)$. This implies $\mu_A(s) \geq \alpha$ and $\nu_A(s) \leq \beta$ for all $s \in mu(X)$. Similarly $\mu_A(t) \geq \alpha$ and $\nu_A(t) \leq \beta$ for all $t \in ML(X)$. Therefore by Definition 3.2 A is an IFsubgl of type 3 of P. □

Theorem 3.15. *Let A be an IFset of type 3 of P. Then A is an IFideal of type 3 of P if and only if $C_{\alpha,\beta}(A)$ is a strong ideal of P for all $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$.*

Proof. Suppose A is an IFideal of type 3 of P. Let $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$. To show that $C_{\alpha,\beta}(A)$ is a strong ideal of P : Since A is an IFsubgl of type 3 of P, by Theorem 3.14 we have $C_{\alpha,\beta}(A)$ is a subgl of P. Then for any finite subset X of $C_{\alpha,\beta}(A)$ we have $mu(X) \subseteq C_{\alpha,\beta}(A)$. To show that $C_{\alpha,\beta}(A)$ is an initial segment of P : Let $p \in C_{\alpha,\beta}(A)$, that is $\mu_A(p) \geq \alpha, \nu_A(p) \leq \beta$. Let $q \in P$ and suppose $q \leq p$. Since A is an IFideal of type 3 we have, $\mu_A(q) \geq \mu_A(p) \geq \alpha$ and $\nu_A(q) \leq \nu_A(p) \leq \beta$, that is $q \in C_{\alpha,\beta}(A)$. Therefore $C_{\alpha,\beta}(A)$ is a strong ideal of P. Conversely suppose the condition. Then by Theorem 3.14 we have A is an IFsubgl of type 3 of P. To show that A is an IFideal of type 3 of P : Let $p, q \in P$ and $p \leq q$. Let $\mu_A(p) = \alpha_1, \nu_A(p) = \beta_1, \mu_A(q) = \alpha_2$ and $\nu_(q) = \beta_2$. Then $p \in C_{\alpha_1,\beta_1}(A), q \in C_{\alpha_2,\beta_2}(A), h(\mu_A(p)) = h(\alpha_1), h(\nu_A(p)) = h(\beta_1), h(\mu_A(q)) = h(\alpha_2)$ and $h(\nu_A(q)) = h(\beta_2)$. By Definition 3.1 we have $0 \leq h(\mu_A(p)) + h(\nu_A(p)) \leq 1$ and $0 \leq h(\mu_A(q)) + h(\nu_A(q)) \leq 1$. Then $h(\alpha_1) + h(\beta_1) \leq 1$ and $h(\alpha_2) + h(\beta_2) \leq 1$. By hypothesis we have $C_{\alpha_1,\beta_1}(A), C_{\alpha_2,\beta_2}(A)$ are strong ideals of P, that is $C_{\alpha_2,\beta_2}(A)$ is an initial segments of P. Since $q \in C_{\alpha_2,\beta_2}(A)$ and $p \leq q$, we have $p \in C_{\alpha_2,\beta_2}(A)$. Then $\mu_A(p) \geq \alpha_2 = \mu_A(q)$ and $\nu_A(p) \leq \beta_2 = \nu_A(q)$. Therefore A is an IFideal of type 3 of P. □

Theorem 3.16. *Let A be an IFset of type 3 of P. Then A is an IFfilter of type 3 of P if and only if $C_{\alpha,\beta}(A)$ is a strong filter of P for all $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$.*

Theorem 3.17. *Let A be an IFset of type 3 of P. Then A is an IF prime ideal of type 3 of P if and only if $C_{\alpha,\beta}(A)$ is a prime strong ideal of P for all $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$.*

Proof. Suppose A is an IF prime ideal of type 3 of P. Let $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$. Since by Definition 3.5 A is an IF ideal of type 3 of P, by Theorem 3.15 we have $C_{\alpha,\beta}(A)$ is a strong ideal of P. To show that $C_{\alpha,\beta}(A)$ is a prime strong ideal of P : Let $p, q \in P - C_{\alpha,\beta}(A)$. Then $\mu_A(p) \ngeq \alpha$ or $\nu_A(p) \nleq \beta$ and $\mu_A(q) \ngeq \alpha$ or $\nu_A(q) \nleq \beta$. This implies clearly $\mu_A(p) \vee \mu_A(q) \ngeq \alpha$ and $\nu_A(p) \wedge \nu_A(q) \nleq \beta$. Let $r \in ML\{p, q\}$. Then by Theorem 3.11 we have $\mu_A(r) = \mu_A(p) \vee \mu_A(q) \ngeq \alpha$ and $\nu_A(r) = \nu_A(p) \wedge \nu_A(q) \nleq \beta$. That is $r \leq p, q$ and $r \in P - C_{\alpha,\beta}(A)$. Therefore $C_{\alpha,\beta}(A)$ is a prime strong ideal of P. Conversely suppose the condition. Then by Theorem 3.15 A is an IF ideal of type 3 of P. To show that A is an IF prime ideal of type 3 of P : Assume that A is not prime. Then by Theorem 3.11 there exists $p, q \in P$ and $r \in ML\{p, q\}$ such that $h(\mu_A(r)) \neq h(\mu_A(p))$ and $h(\mu_A(r)) \neq h(\mu_A(q))$, or $h(\nu_A(r)) \neq h(\nu_A(q))$ and $h(\nu_A(r)) \neq h(\nu_A(p))$. Let $\alpha = \mu_A(r)$ and $\beta = \nu_A(r)$. Then $r \in C_{\alpha,\beta}(A)$, $h(\alpha) + h(\beta) \leq 1$, $h(\mu_A(p)) \ngeq h(\alpha)$ or $h(\nu_A(p)) \nleq h(\beta)$, and $h(\mu_A(q)) \ngeq h(\alpha)$ or $h(\nu_A(q)) \nleq h(\beta)$. This implies $\mu_A(p) \ngeq \alpha$ or $\nu_A(p) \nleq \beta$, and $\mu_A(q) \ngeq \alpha$ or $\nu_A(q) \nleq \beta$. That is $p, q \in P - C_{\alpha,\beta}(A)$. Now we have (by hypothesis) $C_{\alpha,\beta}(A)$ is a prime strong ideal of $P, r \in C_{\alpha,\beta}(A), r \leq p, q$, and $p, q \in P - C_{\alpha,\beta}(A)$. This leads contradiction to the prime concept of $C_{\alpha,\beta}(A)$. Therefore the assumption is false. Therefore A is an IF prime ideal of type 3 of P. □

Theorem 3.18. *Let A be an IFset of type 3 of P. Then A is an IF prime filter of type 3 of P if and only if $C_{\alpha,\beta}(A)$ is a prime strong filter of P for all $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$.*

Theorem 3.19. *Let $\{A_i\}$ be any family of IFsubgls of type 3 of P. Then $\bigcap A_i$ is an IFsubgl of type 3 of P.*

Theorem 3.20. *Let $\{A_i\}$ be any family of IFideals of type 3 of P. Then $\bigcap A_i$ is an IFideal of type 3 of P.*

Theorem 3.21. *Let $\{A_i\}$ be any family of IFfilters of type 3 of P. Then $\bigcap A_i$ is an IFfilter of type 3 of P.*

4. L-valued Intuitionistic L-fuzzy Convex Subgeneralised Lattices of the Type 3 of a Generalised Lattice

In this section defined the concept IF convex subgl of type 3, characterized it by its $(\alpha, \beta)-$level sets and observed that every IFideal of type 3 (IFfilter of type 3) is a IF convex subgl of type 3. Finally proved that the intersection of any collection of IF convex subgls of type 3 is again a IF convex subgl of type 3.

Definition 4.1. *Let A be an L-valued intuitionistic L-fuzzy subgeneralised lattice of type 3 (IFsubgl of type 3) of P. Then A is said to be an L-valued intuitionistic L-fuzzy convex subgeneralised lattice of type 3 (IF convex subgl of type 3) of P if for every interval $[a, b] \subseteq P$, we have $\mu_A(x) \geq \mu_A(a) \wedge \mu_A(b)$ and $\nu_A(x) \leq \nu_A(a) \vee \nu_A(b)$ for all $x \in [a, b]$.*

Theorem 4.2. *Let A be an IFset of type 3 of P. Then A is an L-valued intuitionistic L-fuzzy convex subgeneralised lattice of type 3 (IF convex subgl of type 3) of P if and only if $C_{\alpha,\beta}(A)$ is a convex subgeneralised lattice of P for all $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$.*

Proof. Suppose A is an L-valued intuitionistic L-fuzzy convex subgeneralised lattice of type 3 (IF convex subgl of type 3) of P. Then by Definition 4.1 A is an L-valued intuitionistic L-fuzzy subgeneralised lattice of type 3 (IFsubgl of type 3) of P. Let $\alpha, \beta \in L$ with $h(\alpha) + h(\beta) \leq 1$. Then by Theorem 3.14 $C_{\alpha,\beta}(A)$ is a subgeneralised lattice of P. To show that $C_{\alpha,\beta}(A)$ is a convex subgeneralised lattice of P : Let $a, b \in C_{\alpha,\beta}(A)$ and $a < b$. Then $\mu_A(a) \geq \alpha, \nu_A(a) \leq \beta, \mu_A(b) \geq \alpha, \nu_A(b) \leq \beta$. This implies by Definition 4.1 we have $\mu_A(x) \geq \mu_A(a) \wedge \mu_A(b) \geq \alpha$ and $\nu_A(x) \leq \nu_A(a) \vee \nu_A(b) \leq \beta$ for all $x \in [a, b]$, that is $[a, b] \subseteq C_{\alpha,\beta}(A)$. Therefore $C_{\alpha,\beta}(A)$ is a convex subgeneralised lattice of P. Conversely suppose the condition. Then by Theorem 3.14 we have A is an IF subgl of type 3 of P. To show that A is an IF convex subgl of type 3 of P : Let $[a, b]$ be an interval in P and let $\alpha = \mu_A(a) \wedge \mu_A(b), \beta = \nu_A(a) \vee \nu_A(b)$. By Definition 3.1 we have $0 \leq h(\mu_A(a)) + h(\nu_A(a)) \leq 1$ and $0 \leq h(\mu_A(b)) + h(\nu_A(b)) \leq 1$. Consider $h(\alpha) = h(\mu_A(a) \wedge \mu_A(b)) = min\{h(\mu_A(a)), h(\mu_A(b))\} \leq min\{1 - h(\nu_A(a)), 1 - h(\nu_A(b))\} = 1 - Max\{h(\nu_A(a)), h(\nu_A(b))\} = 1 - h(\nu_A(a) \vee \nu_A(b)) = 1 - h(\beta)$. Then $h(\alpha) + h(\beta) \leq 1$ and that implies $a, b \in C_{\alpha,\beta}(A)$. Since by hypothesis $C_{\alpha,\beta}(A)$ is a convex subgeneralised lattice of P, we have $[a, b] \subseteq C_{\alpha,\beta}(A)$. Then for any $x \in [a, b]$, we have $\mu_A(x) \geq \alpha = \mu_A(a) \wedge \mu_A(b)$ and $\nu_A(x) \leq \beta = \nu_A(a) \vee \nu_A(b)$. Therefore A is an IF convex subgl of type 3 of P. □

Theorem 4.3. *In a genralised lattice, every L-valued intuitionistic L-fuzzy ideal of type 3 (IF ideal of type 3) is a L-valued intuitionistic L-fuzzy convex subgeneralised lattice of type 3 (IF convex subgl of type 3).*

Theorem 4.4. *In a genralised lattice, every L-valued intuitionistic L-fuzzy filter of type 3 (IF filter of type 3) is a L-valued intuitionistic L-fuzzy convex subgeneralised lattice of type 3 (IF convex subgl of type 3).*

Theorem 4.5. *Let $\{A_i\}$ be any family of IF convex subgls of type 3 of P. Then $\bigcap A_i$ is an IF convex subgl of type 3 of P.*

Theorem 4.6. *Let A be an IF ideal of type 3 and B be an IF filter of type 3 of P. Then $A \cap B$ is an IF convex subgl of type 3 of P.*

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