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Dynamics of Tsallis Holographic Dark Energy Interaction in a Bianchi Type-V Universe with a Constant Deceleration Parameter

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Abstract

In this study, we explore a universe characterized by Anisotropic and Homogeneous Bianchi Type-V geometry, filled with both Dark Matter (DM) and Dark Energy (DE) within the framework of General Theory of Relativity. We introduce an interaction between DM and DE, incorporating Tsallis generalized entropy. The field equations are solved using the constant deceleration parameter proposed by Berman (1983). The Equation of State (EoS) parameter of Tsallis Holographic Dark Energy (THDE) elucidates the current accelerated expansion of the universe, exhibiting Phantom-like and approaching ΛCDM. Additionally, we calculate the EoS parameter, Anisotropy Parameter, Deceleration Parameter, and Total Energy Density parameter. Our results align well with observational data.

Keywords: General Theory of Relativity; Tsallis Holographic Dark Energy; Dark Matter; Constant Deceleration Parameter; Bianchi Type-V Universe.

1. Introduction

Observational data from various sources consistently indicates that our universe is experiencing accelerated expansion. The driving force behind this perplexing phenomenon is attributed to Dark Energy [DE] [1-6]. Numerous endeavors have been undertaken to investigate DE, yet its nature remains elusive. Various theories of DE have been postulated, including quintessence, phantom, quintom, and agegraphic models. These theories aim to elucidate the observed accelerating expansion of the universe [7-17]. One intriguing approach to unraveling the nature of DE is through Holographic Dark Energy (HDE), a concept initially proposed by Hooft [18], Holographic Dark Energy (HDE) is grounded in the holographic principle, which originates from the thermodynamics of black holes. HDE model stands as another potential candidate emerging from the holographic principle, initially proposed to provide an explanation for the thermodynamics of black hole physics.

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A novel extension of the HDE model has been introduced by Tsallis and Cirto [19], termed the Tsallis Holographic Dark Energy (THDE) model. This model incorporates Tsallis generalized entropy within the framework of the holographic principle to characterize the properties of DE in the universe. Many researchers have been studied the THDE model such as [20-31]. Recently S. Gupta et al., investigated the Tsallis holographic dark energy scenario in viscous f(Q) gravity with tachyon field [32], C. R. Mahanta et al., conducted a study on Tsallis Holographic Dark Energy in Bianchi Type I Universe in the Framework of $f(R)$ Theory of Gravity [33], Vijaya Santhi et al., studied Tsallis holographic dark energy in Bianchi type II, VIII and IX universes [34]. Motivated by the aforementioned research, our investigation focuses on a spatially homogeneous and anisotropic Bianchi Type-V Universe within the framework of Einstein's theory of relativity. We introduce an interaction between Dark Matter (DM) and Dark Energy (DE). The solutions to the field equations are derived by incorporating specific laws governing the Hubble parameter, resulting in a constant value of the deceleration parameter as proposed by Berman in 1983 [35]. The study delves into a thorough examination of the geometrical and physical aspects of the model.

2. Metric and Field Equation

The Bianchi Type-V spacetime is characterized by:

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2mx}dy^2 - C^2e^{-2mx}dz^2. \tag{1}$$

The cosmic scale factor is represented by A, B and C and $m \neq 0$ is an arbitrary constant in the given context. The equations describing Einstein's field in the natural limit $(8\pi G = c = 1)$ are expressed as follows.

$$R_{ij} - \frac{1}{2}Rg_{ij} = -(T_{ij} + \bar{T}_{ij}), \tag{2}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar and $g_{ij}$is the metric tensor. T_{ij} and $\bar{T}_{ij}$ are the energy momentum tensors of matter and THDE, respectively. The expressions for the energy-momentum tensors are provided as follows.

$$T_{ij} = \rho_m u_i u_j, \tag{3}$$

and

$$\bar{T}_{ij} = (\rho_T + p_T)\, u_i u_j - g_{ij} p_T, \tag{4}$$

where ρ_m, ρ_T and p_T are the energy density of matter, energy density of THDE and pressure of the THDE respectively. The THDE density with Hubble horizon as the IR cutoff is [20].

$$\rho_T = FH^{-2\delta+4}, \tag{5}$$

where $F$is an unknown parameter, H is the Hubble parameter and $\delta$is a free parameter. In comoving coordinate systems, the Einstein field equations (2) for the metric (1), utilizing Equations (3) - (4), can be expressed as:

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\left(\frac{m}{A}\right)^2=-p_T \tag{6}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{C}}{AC}-\left(\frac{m}{A}\right)^2=-p_T \tag{7}$$

$$\frac{\ddot{B}}{B}+\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{B}}{AB}-\left(\frac{m}{A}\right)^2=-p_T \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-3\left(\frac{m}{A}\right)^2=\rho_m+\rho_T \tag{9}$$

$$\frac{\dot{B}}{B}+\frac{\dot{C}}{C}=\frac{2\dot{A}}{A} \tag{10}$$

where an overhead $dot(.)$ represents derivative with respect to time t. Upon integrating Equation (10), we obtain:

$$A^2=\lambda BC, \tag{11}$$

where λ is an integration constant. Without loss of generality we can take $\lambda = 1$. The Directional Hubble Parameter in the direction of x, y and z, denoted respectively, and the average Hubble parameter are defined as:

$$H_1=\frac{\dot{A}}{A},\ \ H_2=\frac{\dot{B}}{B},\ \ H_3=\frac{\dot{C}}{C}, \tag{12}$$

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(H_1+H_2+H_3\right) \tag{13}$$

The special volume, denoted as V and the Average Scale factor, denoted as a are defined as:

$$V=a^3=ABC, \quad a=(ABC)^{\frac{1}{3}} \tag{14}$$

The deceleration parameter, denoted as $q(t)$ is defined by:

$$q=-\frac{a\ddot{a}}{\dot{a}^2} \tag{15}$$

The mean anisotropy parameter of expansion, denoted as A_m, the expansion scalar θ, and the shear scalar σ^2, are defined for the metric as:

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{16}$$

$$\theta=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}=3H \tag{17}$$

$$\sigma^2=\frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2\right]-\frac{\theta^2}{6} \tag{18}$$

By utilizing Equations (3) - (4) and (14), the cosmic scale factor can be expressed as:

$$A = V^{\frac{1}{3}} \tag{19}$$

$$B = DV^{\frac{1}{3}} \exp\left(X \int \frac{dt}{V}\right), \tag{20}$$

$$C = D^{-1}V^{\frac{1}{3}} \exp\left(-X \int \frac{dt}{V}\right) \tag{21}$$

where X, and D are constants of integration. The energy conservation law $T_{j}^{ij} = 0$, yields the continuity equation as

$$\dot{\rho}_m + \dot{\rho}_T + 3H\left(\rho_m + \rho_T + p_T\right) = 0. \tag{22}$$

In the interacting model, where DE interacts with DM through an interaction term Q, the continuity equation transforms to:

$$\dot{\rho}_m + 3H\rho_m = Q \tag{23}$$

$$\dot{\rho}_T + 3H\left(\rho_T + p_T\right) = Q \tag{24}$$

In this study, we opt for the interaction term $Q = 3\gamma H\rho_m$, where γrepresents coupling parameter between DM and DE, setting $\gamma = 0$ results in the non-interacting model. By employing Equation (5) in (24), the Equation of State (EoS) parameter of THDE can be derived as:

$$\omega_T = \frac{p_T}{\rho_T} = -1 + \frac{(2\delta - 4)\dot{H}}{3H^2} - \gamma\frac{\rho_m}{\rho_T}. \tag{25}$$

3. Cosmological Solutions of the Model

To derive exact solutions for the field equations (6) - (10) and the cosmic scale factor A, B and C, we assume a specific law of variation for the Hubble parameter. This assumption leads to a constant value for the deceleration parameter [35]. According to this law, the variation of the mean Hubble parameter is expressed as:

$$H = ka^{-n}, \tag{26}$$

where $k > 0$ and $n \geq 0$. Here, we derive two cosmological models: I) A Model for $n = 0$ and II) A Model for $n \neq 0$.

I) A Model for $n = 0$ (Exponential Volumetric Expansion Model)

For$n = 0$, Equation (26) provides the volume scale factor as

$$V = c_1 e^{3kt}, \tag{27}$$

where $c_1 > 0$ is a constant of integration. By employing Equation (27) in Equations (19) - (21), we derive the exact values of the scale factors as:

$$A = (c_1)^{\frac{1}{3}} e^{kt} \tag{28}$$

$$B = D\,(c_1)^{\frac{1}{3}} \exp\left(kt - \frac{X}{3c_1 k} e^{-3kt}\right) \tag{29}$$

$$C = D^{-1}\,(c_1)^{\frac{1}{3}} \exp\left(kt + \frac{X}{3c_1 k} e^{-3kt}\right) \tag{30}$$

The mean Hubble parameter H, deceleration parameter q, and mean anisotropy parameter of expansion A_m for the model are respectively given as:

$$H = k \tag{31}$$

$$q = -1 \tag{32}$$

$$A_m = \frac{2X^2 e^{-6kt}}{3k^2 c_1^2} \tag{33}$$

The expansion scalar θ and the shear scalar σ^2 are obtained, respectively, as:

$$\theta = 3H = 3k \tag{34}$$

$$\sigma^2 = \frac{X^2 e^{-6kt}}{c_1^2} \tag{35}$$

By substituting Equation (31) into Equation (5), we obtain the energy density of THDE as:

$$\rho_T = F\,(K)^{-2\delta+4}. \tag{36}$$

By substituting Equation (36) into Equation (9), we obtain the energy density of DM as:

$$\rho_m = 3k^2 - \frac{X^2 e^{-6kt}}{c_1^2} - \frac{3m^2 e^{-2kt}}{(c_1)^{2/3}} - F\,(k)^{4-2\delta}. \tag{37}$$

The matter density parameter (Ω_m) and THDE density parameter $(\Omega_T)$are obtained as:

$$\Omega_m = \frac{\rho_m}{3H^2} = 1 - \frac{X^2 e^{-6kt}}{3k^2 c_1^2} - \frac{m^2 e^{-2kt}}{k^2\,(c_1)^{2/3}} - \frac{F\,(k)^{2(1-\delta)}}{3}, \tag{38}$$

$$\Omega_T = \frac{\rho_T}{3H^2} = \frac{F\,(k)^{2(1-\delta)}}{3} \tag{39}$$

The overall density parameter, obtained by utilizing Equations (38) and (39), is:

$$\Omega = \Omega_m + \Omega_T = 1 - \frac{X^2 e^{-6kt}}{3k^2 c_1^2} - \frac{m^2 e^{-2kt}}{k^2\,(c_1)^{2/3}}. \tag{40}$$

By substituting Equations (31), (36), and (37) into Equation (25), the Equation of State (EoS) parameter of THDE is given by:

$$\omega_T = -1 - \gamma \left[\frac{3k^{2(\delta-1)}}{F} - \frac{X^2 e^{-6kt} k^{2(\delta-2)}}{c_1^2 F} - \frac{3m^2 k^{2(\delta-2)}}{(c_1)^{2/3} F} - 1 \right]. \tag{41}$$

II) A Model for $n \neq 0$ (Power-law Volumetric Expansion Model)

For $n \neq 0$, Equation (26) yields the volume scale factor as:

$$V = (nkt + c_2)^{\frac{3}{n}}, \tag{42}$$

where $c_2 > 0$ is a constant of integration. Using Equation (42) into Equation (19) - (21), we obtain the exact values of the scale factors as:

$$A = (nkt + c_2)^{\frac{1}{n}} \tag{43}$$

$$B = D\,(nkt + c_2)^{\frac{1}{n}} \exp\left(\frac{X}{k(n-3)} (nkt + c_3)^{\frac{n-3}{n}} \right) \tag{44}$$

$$C = D^{-1}\,(nkt + c_2)^{\frac{1}{n}} \exp\left(\frac{-X}{k(n-3)} (nkt + c_2)^{\frac{n-3}{n}} \right) \tag{45}$$

The mean Hubble parameter H, deceleration parameter q, and mean anisotropy parameter of expansion A_m for the model are respectively given as:

$$H = k\,(nkt + c_2)^{-1} \tag{46}$$

$$q = n - 1 \tag{47}$$

$$A_m = \frac{2X^2 (nkt + c_2)^{\frac{2(n-3)}{n}}}{3k^2} \tag{48}$$

The expansion scalar θ and the shear scalar σ^2 are obtained as follows:

$$\theta = 3H = 3k(nkt + c_2)^{-1} \tag{49}$$

$$\sigma^2 = X^2\,(nkt + c_2)^{\frac{-6}{n}} \tag{50}$$

By substituting Equation (46) into Equation (5), we obtain the energy density of THDE as:

$$\rho_T = \frac{F\,(nkt + c_2)^{2\delta-4}}{k^{2\delta-4}}. \tag{51}$$

By substituting Equation (43) - (45) into Equation (9), we obtain the energy density of DM as:

$$\rho_m = 3k^2 (nkt + c_2)^{-2} - X^2 (nkt + c_2)^{-6/n} - 3m^2 (nkt + c_2)^{-2/n} - \frac{F(nkt + c_2)^{2\delta-4}}{k^{2\delta-4}}. \tag{52}$$

The matter density parameter (Ω_m) and the THDE density parameter (Ω_T) are given by:

$$\Omega_m = \frac{\rho_m}{3H^2} = 1 - \frac{X^2}{3k^2}(nkt + c_2)^{\frac{2n-6}{n}} - \left(\frac{m}{k}\right)^2 (nkt + c_2)^{\frac{2(n-1)}{n}} - \frac{F(nkt + c_2)^{2\delta-2}}{3k^{2\delta-2}}, \tag{53}$$

$$\Omega_T = \frac{\rho_T}{3H^2} = \frac{F\,(nkt + c_2)^{2\delta-2}}{3k^{2\delta-2}} \tag{54}$$

The overall density parameter, obtained by utilizing Eqs. (53) and (54), is:

$$\Omega = \Omega_m + \Omega_T = 1 - \frac{X^2(nkt + c_2)^{\frac{2n-6}{n}}}{3k^2} + \frac{m^2\,(nkt + c_2)^{\frac{2n-2}{n}}}{k^2}. \tag{55}$$

By substituting Equations (46), (51), and (52) into Equation (25), the Equation of State (EoS) parameter of THDE is given as:

$$\omega_T = -1 - \frac{(2\delta - 4)n}{3} - \gamma \left[\frac{3k^2(nkt + c_2)^{-2} - X^2(nkt + c_2)^{-6/n} - 3m^2(nkt + c_2)^{-2/n}}{Fk^{4-2\delta}\,(nkt + c_2)^{2\delta-4}} - 1\right] \tag{56}$$

4. Concluding Remarks

In this study, we have explored an Anisotropic and Homogeneous Bianchi Type-V universe incorporating THDE. The key features of the model are outlined below:

(i). The anisotropy parameter, as expressed in Equations (33) and (48), converges to zero as cosmic time t becomes significantly large. This behavior aligns with observational data.

(ii). The deceleration parameter for the model $n = 0$ is negative, indicating that the universe is undergoing acceleration. Additionally, for the model $n \neq 0$, the deceleration parameter is positive for $n > 1$, and for values of $0 < n < 1$, the deceleration parameter is negative. This implies that the acceleration and deceleration of the model depend on the specific value of n.

(iii). The overall density parameter Ω in both models approaches 1 as cosmic time t becomes significantly large.

(iv). Our specific choice of a constant deceleration parameter results in a transition from a decelerating phase to an accelerating phase.

(v). The EoS parameter, $\omega_T < -1$ for $t = 0$ and $\omega_T \approx -1$, for significantly large cosmic time t. This behavior is consistent with observational data.

(vi). The special volume V is finite for $t = 0$, expanding to ∞ as cosmic time $t = \infty$ in both models.

(vii). For the model, $n = 0$ the Hubble parameter H, the expansion scalar θ remain constant throughout the evolution of universe. Additionally, the shear scalar σ^2 is constant for $t = 0$ and vanishes for $t = \infty$.

(viii). For the model $n \neq 0$, the Hubble parameter H, the expansion scalar θ, and the shear scalar σ^2 are constant for $t = 0$ and vanish for $t = \infty$.

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