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# Introduction to Soft Set Theory and its Applications

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**Abstract**

The soft set theory offers a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. The main objectives to introduce the basic notions of the theory of soft sets, to present the first results of the theory, and to discuss some problems of the future. We present an application of soft set theory due to Molodtsov [5] in decision making problems that is based on the rough reduction of parameters to keep the optimal choice objects. We have also defined equality of two soft sets, subset and superset of a soft set, complement of a soft set, null soft set and absolute soft set with examples. We have also defined binary operations like AND, OR, union and intersection of soft sets. De Morgan's laws is verified in a soft set theory.

**Keywords:** Soft set; Uncertain; Fuzzy; Crisp data; Vague set.

## 1. Introduction

Set theory is a branch of mathematical logic that studies collections of well-defined objects as sets. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics. The language of set theory can be used to define nearly all mathematical objects.The modern study of set theory was initiated by Georg Cantor and Richard Dedekind in the 1870s. After the discovery of paradoxes in naïve set theory, such as Russell's paradox, numerous axiom systems were proposed in the early twentieth century, of which the Zermelo-Fraenkel axioms are the best-known. Set theory is commonly employed as a foundational system for mathematics Contemporary research into set theory includes a diverse collection of topics, ranging from the structure of the real number line to the study of the consistency of large cardinals. Mathematical topics typically emerge and evolve through interactions among many researchers. Set theory, however, was founded by a single paper in 1874 by Georg Cantor: "On a Property of the Collection of All Real Algebraic Numbers".

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### 1.1 Set Theory in Mathematical Education

As set theory gained popularity as a foundation for modern mathematics, there has been support for the idea of introducing basic theory, or naive set theory, early in mathematics education. Set theory is used to introduce students to logical operators (NOT, AND, OR), and semantic or rule description (technically intentional definition of sets, (e.g. "months starting with the letter A"). This may be useful when learning computer programming, as sets and boolean logic are basic building blocks of many programming languages. Sets are commonly referred to when teaching about different types of numbers (N, Z, R, ...), and when defining mathematical functions as a relationship between two sets. Many mathematical concepts can be defined precisely using only set theoretic concepts. For example, mathematical structures as diverse as graphs, manifolds, rings, and vector spaces can all be defined as sets satisfying various properties. Equivalence and order relations are ubiquitous in mathematics, and the theory of mathematical relations can be described in set theory. Set theory is also a promising foundational system for much of mathematics.

### 1.2 Literature Survey and Applications

Most of our traditional tools for formal modelling, reasoning, and computing are crisp, deterministic, and precise in character. Many problems in economics, engineering, environment, social science, medical science, etc come across complicated problems involving very crisp data. When there is some uncertainty in these data we cannot always use the classical methods. The important existing mathematical theories for dealing with uncertainties are theory of probability, theory of fuzzy sets [1], theory of intuitionistic fuzzy sets [2, 3], theory of vague sets [4], theory of interval mathematics [3, 5], theory of rough sets [6]. But all these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theories and consequently in the year 1999, Molodtsov [7] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties or vagueness which is free from the above difficulties. Soft set theory has a rich potential for applications in several directions like economic, engineering, medical science, etc. Few of which had been shown by Molodtsov in his pioneer work [7]. Soft sets are called (binary, basic, elementary) neighborhood systems [16] and are a special case of context dependent fuzzy sets, as defined by Thielle [15]. Applications of soft set theory in other disciplines and real life problems are now catching momentum. The first practical application of soft sets in decision making problems by using reduction concept in rough set theory was proposed by Majiet. al., [7], in year 2002. Nasef [17] presented another application of soft sets in a decision making problem for real estate marketing with the help of rough mathematics of Pawlak [11, 12, 13]. They have also written an algorithm to select the optimal choice of an object by using fewer parameter select The optimal object for a decision problem.

# 2. Soft Set Theory

## 2.1 Basic Definitions of Soft set

In this section, we present the notion of soft sets which introduced by Molodstov in [7], and mention some useful definitions [12] on rough mathematics. Let U be an initial universe set and E a set of all possible parameters with aspect to U. Parameters are often attributes, characteristics or properties of the objects in U. Then a soft set over U is defined as follows:

The soft set is a parametrized family of subsets of the universal set U. A pair $(F, E)$ is called a soft set over U if and only if F is mapping of E into the set of subsets of the set U, that is $F : E \to P(U)$. In other words, let U be an initial universe set and let E be a set of parameters. Every set $F(e)$, e belongs to E, from this family may be considered as the set of e-elements of the soft set $(F, E)$, or as the set of e-approximate elements of the soft set.

**Example 2.1.** *Let a soft set $(F, E)$ describes the attractiveness of the houses which Mr. X is going to buy.*

U = is the set of houses under consideration.

E = is the set of parameters.

Each parameter is a word or sentence like

$E = \{$expensive; beautiful; wooden; cheap; in the green surroundings; modern; in good repair, in bad repair$\}$

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. It is worth nothing that sets $F(e)$ may be arbitrary. Some may have non-empty intersection. We consider below the same example in more detail for our next discussion. Suppose that there are six houses in the universe set U given by

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6\} \text{ and } E = \{e_1, e_2, e_3, e_4, e_5\}$$

where

$e_1$ stands for the parameter 'expensive'

$e_2$ stands for the parameter 'beautiful'

$e_3$ stands for the parameter 'wooden'

$e_4$ stands for the parameter 'cheap'

$e_5$ stands for the parameter 'in the green surroundings'.

Suppose that

$$F(e_1) = \{h_2, h_4\}$$

$$F(e_2) = \{h_1, h_3\}$$

$$F(e_3) = \{h_3, h_4, h_5\}$$

$$F(e_4) = \{h_1, h_3, h_5\}$$
$$F(e_5) = \{h_1\}$$

The soft set $(F,E)$ is a parametrized family $\{F(e_i), i = 1,2,3,\ldots,8\}$ of subsets of the set U and gives us a collection of approximate descriptions of an object. Consider the mapping F which is "houses (·)" where dot (·) is to be filled up by a parameter $e \in E$. Therefore, $F(e_l)$ means "houses (expensive)" whose functional-value is the set $\{h_2, h_4\}$. Thus, we can view the soft set $(F,E)$ as a collection of approximations as below:

$(F,E) = \{$expensive houses $= \{h_2, h_4\}$, beautiful houses $= \{h_1, h_3\}$, wooden houses $= \{h_3, h_4, h_5\}$, cheap houses $= \{h_1, h_3, h_5\}$, in the green surroundings $= \{h_1\}\}$, where each approximation has two parts:

(i) a predicate p; and

(ii) an approximate value-set v (or simply to be called value-set v).

For example, for the approximation "expensive houses $\{h_2, h_4\}$" , we have the following:

(i) the predicate name is expensive houses; and

(ii) the approximate value set or value set is $\{h_2, h_4\}$.

Thus, a soft set $(F,E)$ can be viewed as a collection of approximation as below:

$$(F,E) = \{p_1 = v_1, p_2 = v_2, p_3 = v_3, \ldots, p_n = v_n\}$$

For the purpose of storing a soft set in a computer, we could represent a soft set in the form of Table 1, as given below (corresponding to the soft set in the above example).

| U | Expensive | Beautiful | Wooden | Cheap | In the green surroundings |
|---|---|---|---|---|---|
| $h_1$ | 0 | 1 | 0 | 1 | 1 |
| $h_2$ | 1 | 0 | 0 | 0 | 0 |
| $h_3$ | 0 | 1 | 1 | 1 | 0 |
| $h_4$ | 1 | 0 | 1 | 0 | 0 |
| $h_5$ | 0 | 0 | 1 | 1 | 0 |
| $h_6$ | 0 | 0 | 0 | 0 | 0 |

Table 1: Tabular representation of a soft set

**Knowledge:** Every primitive attribute $a \in A$ is a total function $a : U \to V_a$, where $V_a$ is the set of the values of $\alpha$ called the domain of $\alpha$. Acknowledge representation system can be formulated as a pair $S = (U, A)$, where U is a nonempty, finite set called the universe, A is a nonempty, finite set of primitive attributes.

**Indiscernibility Relation (IND):** Every subset $B \subseteq A$ is called an attribute. If B is a single element set, then B is called primitive. Otherwise the attribute is said to be compound. Every subset of attributes

$B \subseteq A$, we associate a binary relation $IND(B)$, called an indiscernibility relation, defined by:

$$IND(B) = \{(x, y) \in U^2 : \text{for every } a \in B, a(x) = a(y)\}.$$

Obviously $IND(B)$ is an equivalence relation and $IND(B) = \{IND(a) : a \in B\}$.

**Dispensable:** Let R be a family of equivalence relation and $A \in R$. We will say that A is dispensable in R if $IND(R) = IND(R - \{A\})$; otherwise A is indispensable in R.

**Reduct (RED):** The family R is independent if each $A \in R$ is indispensable in R; otherwise R is dependent.If R is independent and $P \subseteq R$, then P is also independent $Q \subseteq P$ is a reduct of P if Q is independent and $IND(Q) = IND(P)$.

Intuitively, a reduct of knowledge is its essential part, which suffices to define all basic concepts occurring in the considered knowledge, whereas the core is in a certain sense its most important part. The set of all indispensable relations in P will be called the core of P and will be denoted $CORE(P)$. Clearly $CORE(P) = \cap RED(P)$, where $RED(P)$ is the family of all reducts of P.

**Core (CORE):** The set of all indispensable relations in P is called the CORE of P and $CORE(P) = \cap RED(P)$. The concept of the core has twofold use. First, it can be used as a basis for computation of all reducts, for the core is included in every reduct and its computation is straightforward. Secondly, the core can be interpreted as the set of the most characteristic part of knowledge, which cannot be eliminated when reducing the knowledge.

## 2.2 Some Properties of Soft Set

1. **Dependency:** Let $K = (U, R)$ be a knowledge base and $P, Q \subseteq R$.

   (a) Knowledge Q depends on knowledge P iff $IND(P) \subseteq IND(Q)$.

   (b) Knowledge P and Q are equivalent denoted as $P \equiv Q$, iff $P \Rightarrow Q$ and $Q \Rightarrow P$.

   (c) Knowledge P and Q are independent, denoted as $P \equiv Q$, iff neither $P \Rightarrow Q$ nor $Q \Rightarrow P$ hold. Obviously, $P \equiv Q$, iff $IND(P) = IND(Q)$.

2. Forsoft sets $(F, A)$ and $(G, B)$ over a common universe U,

   (a) $(F, A)$ is a soft subset of $(G, B)$ if $A \subset B$, and $F(e) \subseteq G(e)$, for all $e \in A$. We write $(F, A) \subseteq (G, B)$.

   (b) $(F, A)$ is said to be as of the superset of $(G, B)$, if $(G, B)$ is as of the subset of $(F, A)$.

3. A soft set $(F, A)$ over X is said to be

   (a) Null softest denoted by $\Phi$, if $F(e) = \varphi \;\; \forall \;\; e \in A$.

   (b) Absolute soft set denoted by X, if $F(e) = X \;\; \forall \;\; e \in A$.

4. The class of all value sets of a soft set $(F, E)$ is called value-class of the soft set and is denoted by $C(F, E)$.

5. For two soft sets $(F, A)$ and $(G, B)$ over a common universe U, we say that $(F, A)$ is a soft subset of $(G, B)$ if

   (a) $A \subset B$, and

   (b) $\forall\ \epsilon \in A$, $F(\epsilon)$ and $G(\epsilon)$ are identical approximations then we write $(F, A) \subseteq (G, B)$. $(F, A)$ is said to be a soft super set of $(G, B)$, if $(G, B)$ is a soft subset of $(F, A)$.

6. **Equality of soft sets:** Two soft sets $(F, A)$ and $(G, B)$ over a common universe U are said to be soft equal if $(F, A)$ is a soft subset of $(G, B)$ and $(G, B)$ is a soft subset of $(F, A)$ and it is called as Equality of soft sets.

**Example 2.2.** *Let $A = \{e_1, e_3, e_5\} \subset E$, and $B = \{e_1, e_2, e_3, e_5\} \subset E$. Clearly $A \subset B$. Let $(F, A)$ and $(G, B)$ be two soft sets over the same universe $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ such that,*

$$G(e_1) = \{h_2, h_4\}, G(e_2) = \{h_1, h_3\}, G(e_3) = \{h_3, h_4, h_5\}, G(e_5) = \{h_1\}$$
$$F(e_1) = \{h_2, h_4\}, F(e_3) = \{h_3, h_4, h_5\}, F(e_5) = \{h_1\}$$

*Therefore, $(F, A) \subset (G, B)$.*

**Definition 2.3** (Complement of Soft Set)**.** *The complement of soft set $(F, A)$ is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $(F^c, A)$, $P(U)$ is a mapping given by $F^c(\alpha) = U - F(\alpha)$ for all $\alpha \in A$. Let us call $F^c$ to be the soft complement function of F. Clearly $(F^c)^c$ is the same as $F((F, A)^c)^c = (F, A)$.*

**Example 2.4.** *Consider the same example as 1, then $(F, A)^c = \{$not expensive houses $= \{h_1, h_3, h_5, h_6\}$, not beautiful houses $= \{h_2, h_4, h_5, h_6\}$, not wooden houses $= \{h_1, h_2, h_6\}$, not cheap houses $= \{h_2, h_4, h_6\}$, not in the green surroundings houses $= \{h_2, h_3, h_4, h_5, h_6\}$.*

**Definition 2.5** (Null Soft Set)**.** *A soft set $(F, A)$ over U is said to be a NULL soft set denoted by $\Phi$, if for all $e \in A$, $F(\epsilon) = \Phi$ (null-set).*

**Example 2.6.** *Let U be the set of wooden houses and A be the parameter set. Let there be five houses in the universe U is given by $U = \{h_1, h_2, h_3, h_4, h_5\}$ and $A = \{brick, muddy, steel, stone\}$. The soft set $(F, A)$ describes the "construction of the houses", The soft set $(F, A)$ is defined as F(stone) means the stone built houses, the word in brackets means house is built by that. The soft set $(F, A)$ is the collection of approximation as below: $(F, A) = \{$brick built houses $= \Phi$, muddy houses $= \Phi$, steel built houses $= \Phi$, stone built houses $= \Phi\}$. Thus $\{F, A\}$ is Null soft set.*

| U | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ |
|---|---|---|---|---|---|---|---|
| $u_1$ | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| $u_2$ | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| $u_3$ | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| $u_4$ | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| $u_5$ | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| $u_6$ | 0 | 1 | 0 | 1 | 1 | 0 | 0 |

Table 2: Soft Set (F, A)

## 3. Some Important Applications

Let $A = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ be a set of parameters, where $e_1$: stands for costly; $e_2$: stands for cheaper; $e_3$: stands for beautiful; $e_4$: stands for brand-company; $e_5$: stands for brand company). Given U = $\{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a universal set consisting of a set of six Televisions under consideration. A soft set $(F, A)$ describes the "attractiveness of Television" that a customer is going to purchase. Suppose

$$(F, e_1) = \{u_1, u_4, u_5\};$$
$$(F, e_2) = \{u_2, u_6\};$$
$$(F, e_3) = \{u_1, u_3, u_5\};$$
$$(F, e_4) = \{u_1, u_4, u_6\};$$
$$(F, e_5) = \{u_1, u_3, u_6\};$$
$$(F, e_6) = \{u_2, u_5\};$$
$$(F, e_7) = \{u_2, u_3\}.$$

In order to store soft set in the computer easily, the numbers 0-1 are used as two-dimensional table as a set to represent soft set (example, Table 2).

**Definition 3.1** (Two Valued Soft Set). *Given $S = (F, A)$ is a soft set over U, $(U, A, V, Ge)$ is a 2 - valued information system induced by S, to any $x \in U$, $e \in A$, then*

$$Ge = \begin{cases} 1, & \text{if } x \in F(e); \\ 0, & \text{if } x \notin F(e). \end{cases}$$

*Given $S = (F, A)$ is a soft set over U, $(U, A, V, g)$ is a 2- valued information system induced by S, to any $B \subseteq A$, binary relations, induced by S is defined as:*

$$R_B = \{(x, y) \in U \times U : g(x, e) = g(y, 0)(\forall \ e \in B)\}.$$

### 3.1 Application in a Decision Making Problem

Molodtsov [9] presented some applications of the soft set theory in several directions namely: study of the smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability, theory of measurement, etc. Recently in 2012 Maji [8] presented some results as an application of neutrosophicsoftset in a decision making problem. In this section, we present another application of softset theory in a decision making problem for real estate marketing with the help of rough approach [12].

**Example 3.2.** ***Case Study:*** *Let $U = \{c_1, c_2, c_3, c_4, c_5, c_6\}$ be a set of six chalet, $E$ = {expensive; beautiful; wooden; cheap; in the green surroundings; modern; in good repair; in bad repair } be a set of parameters. Consider the soft set $(F, E)$ which describes the "attractiveness of the chalets", given by $(F, E)$ = expensive chalet $= \Phi$, beautiful chalet $= \{c_1, c_2, c_3, c_4, c_5, c_6\}$, wooden chalet $= \{c_1, c_2, c_6\}$, modern chalet $= \{c_1, c_2, c_6\}$, chalet in bad repair $= \{c_2, c_4, c_5\}$, cheap chalet $= \{c_1, c_2, c_3, c_4, c_5, c_6\}$, chalet in good repair $= \{c_1, c_3, c_6\}$, Chalet in green surrounding $= \{c_1, c_2, c_3, c_4, c_6\}\}$. Suppose that, Mr.X is interested in buying a chalet on the basis of his choice parameters beautiful, wooden, cheap, in green surroundings, in good repair, etc., which constitute the subset $A = \{e_1$ = beautiful; $e_2$ = wooden; $e_3$ = cheap; $e_4$ = in green surroundings; $e_5$ = in good repair$\}$ of the set E. That means, out of available chalet in U, he has to select that chalet which qualifies with all (or with maximum number) of parameters of the soft set A. Suppose that, another customer Mr. Y wants to buy a chalet on the basis of the soft set this choice parameters $B \subset E$, where B = {expensive; beautiful; in the green surroundings; in good repair}. Also, Mr. Z wants to buy a chalet on the basis of another set of parameters $C \subset E$. The problem is to select the chalet which is most suitable with the choice parameters of Mr.X. The chalet which is most suitable for Mr.X need not be most suitable for Mr.Y or Mr.Z as the selection is dependent upon the set of choice parameters of each buyer.*

To solve the problem, we do some theoretical characterizations of the soft set theory of Molodtsov, which we present below.

**Tabular Representation of a Soft Set (F,A):** We present an almost analogue representation in the form of a binary table. For this, consider the soft set $(F, A)$ above on the basis of the set A of choice parameters of Mr.X. Then, consider the soft set $(F, A)$. $(F, A) = \{(e_1, U), (e_2, \{c_1, c_2, c_6\}), (e_3, U), (e_4, \{c_1, c_2, c_3, c_4, c_6\}), (e_5, \{c_1, c_3, c_6\})\}$. We can represent this softest $(F, A)$ in a tabular form as shown below. This style of representation will be useful for storing a soft set in a computer memory. If $c_i \in F(e_j)$, then $c_{ij} = 1$, otherwise $c_{ij} = 0$, where $c_{ij}$ are the entries in Table 3. Thus, a softest cannot be viewed as acknowledge representation system, where the set of attributes is replaced by a set of parameters.

**Reduct-Table of a Soft Set:** Consider the soft set $(F, E)$. Clearly, for any $A \subset E$, the set $(F, A)$ is a soft subset of $(F, E)$. We will now define a reduct-soft set of the soft set $(F, A)$.

Consider the tabular representation of the soft set $(F, A)$. If B is a reduction of A, then the soft set $(F, B)$ is called the reduct-soft-set of the soft set $(F, A)$. Intuitively, a reduct-soft-set $(F, B)$ of the soft set $(F, A)$

| U \ A | Beautiful | Wooden | Cheap | In green surroundings | In good repair |
|---|---|---|---|---|---|
| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
| $c_1$ | 1 | 1 | 1 | 1 | 1 |
| $c_2$ | 1 | 1 | 1 | 1 | 0 |
| $c_3$ | 1 | 0 | 1 | 1 | 1 |
| $c_4$ | 1 | 0 | 1 | 1 | 0 |
| $c_5$ | 1 | 0 | 1 | 0 | 0 |
| $c_6$ | 1 | 1 | 1 | 1 | 1 |

in the essential part, which suffices to describe all basic approximate descriptions of the soft set $(F, A)$. The core soft set of $(F, A)$ in the soft set $(F, A)$, where C is the CORE (A) (i. e., $CORE(A) = \cap RED(A)$).

## 4. Conclusion

The soft set theory plays an important role as a mathematical tool for dealing with problem involving vague and uncertain data. We have studied Molodtsov [9], who has explained several possible application sofsoft set theory in many diverse fields. Also we have gone through Maji [8], who presented some results as an application of neutrosophic softest indecision making problem. We have also studied decision making problem explained by El-Sayed [10]. And algorithm of soft set theory ca be developed for further studies. In this project we give new application of soft set theory in a decision making problem for real Estate marketing by the rough technique of Pawlak [12].

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