International Journal of
Mathematics And its Applications
ISSN: 2347-1557

Int. J. Math. And Appl., **12(2)**(2024), 43–50
Available Online: http://ijmaa.in

L-valued Intuitionistic L-fuzzy Generalised Lattice Ordered Groups of the Type 3

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Abstract

A generalised lattice ordered group (gl-group) is a system in which the underlying set is a generalised lattice as well as a group. This article deals with the concept of L-valued intuitionistic L-fuzzy gl-subgroup of the type 3 (IFgl-subgroup of type 3) of a gl-group. Introduced the concept L-valued intuitionistic L-fuzzy gl-subgroup of the type 3 (IFgl-subgroup of type 3) of a gl-group and characterized by the level subsets and discussed some equivalent conditions. Finally proved that intersection of any family of L-valued intuitionistic L-fuzzy gl-subgroups of the type 3 (IFgl-subgroups of type 3) of a gl-group is again an IFgl-subgroup of type 3.

Keywords: group; poset; lattice; l-group; fuzzy set; fuzzy lattice; fuzzy group.
2020 Mathematics Subject Classification: 06-XX.

1. Introduction

The theory related to the concepts fuzzy set (L-fuzzy set, intuitionistic L-fuzzy set), fuzzy group (L-fuzzy group, intuitionistic L-fuzzy group) and fuzzy lattice (L-fuzzy lattice, intuitionistic L-fuzzy lattice) are known from [3–6,18–20] and [21,22]. The theory of lattice ordered groups (l-groups) is well known from the books [13,14] and the concept of fuzzy lattice ordered group introduced and developed by Saibaba in [23]. Mellacheruvu Krishna Murty and U. Madana Swamy (Professors of Andhra University) [7] introduced the concept of generalised lattice and the theory of generalised lattices developed by the author P. R. Kishore in [8,9] that can play an intermediate role between the theories of lattices and posets. The concepts and the corresponding theory of fuzzy generalised lattices [10,11], generalised lattice ordered groups (gl-groups) [12,15,16], fuzzy generalised lattice ordered groups (fuzzy gl-groups) [17] introduced and developed by the author P.R.Kishore. In [24] Gerstenkorn and Tepavcevic introduced the concept L-valued intuitionistic L-fuzzy set of type 3. Later concept of L-valued intuitionistic L-fuzzy generalised lattice of the type 3 (IFgl of type 3) introduced by the author

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P.R.Kishore in [25]. This article deals with the concept of L-valued intuitionistic L-fuzzy gl-subgroup of the type 3 (IFgl-subgroup of type 3) of a gl-group. Section 2 contains some preliminaries from the references. In Section 3, introduced the concept of L-valued intuitionistic L-fuzzy gl-subgroup of the type 3 (IFgl-subgroup of type 3) of a gl-group and characterized that by its level subsets. Finally proved that the intersection of any family of L-valued intuitionistic L-fuzzy gl-subgroups of the type 3 (IFgl-subgroups of type 3) of a gl-group is again an IFgl-subgroup of type 3.

2. Preliminaries

This section contains some preliminaries from the references those are useful in the next sections.

Definition 2.1 ([18]). *Let X be a non-empty set. A collection of objects in the set form* $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ *is called an intuitionistic fuzzy set of X if (i)* $\mu_A : X \to [0,1]$ *is a fuzzy set in X called degree of membership function on X, (ii)* $\nu_A : X \to [0,1]$ *is a fuzzy set in X called degree of non-membership function on X and (iii) for each* $x \in X$*, we have* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Note 2.2. *If* ν_A *is complement of* μ_A *(that is* $\nu_A(x) = 1 - \mu_A(x)$ *for all* $x \in X$*), then the intuitionistic fuzzy set A will be fuzzy set in X.*

Definition 2.3 ([22]). *Let* $(L, \wedge, \vee)$ *be a lattice and* $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in L\}$ *be an intuitionistic fuzzy set of L. Then A is called an intuitionistic fuzzy sublattice of L if the following conditions satisfied: for all* $x, y \in L$*; (i)* $\mu_A(x \vee y) \geq \min\{\mu_A(x), \mu_A(y)\}$ *(ii)* $\mu_A(x \wedge y) \geq \min\{\mu_A(x), \mu_A(y)\}$ *(iii)* $\nu_A(x \vee y) \leq max\{\nu_A(x), \nu_A(y)\}$ *and (iv)* $\nu_A(x \wedge y) \leq max\{\nu_A(x), \nu_A(y)\}$.

Definition 2.4 ([20]). *Let* $(G, \cdot, ^{-1}, e)$ *be a group and* $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in L\}$ *be an intuitionistic fuzzy set of G. Then A is called an intuitionistic fuzzy subgroup of G if the following conditions satisfied: for all* $x, y \in G$*; (i)* $\mu_A(x \cdot y) \geq \min\{\mu_A(x), \mu_A(y)\}$ *(ii)* $\mu_A(x^{-1}) = \mu_A(x)$ *(iii)* $\nu_A(x \cdot y) \leq max\{\mu_A(x), \mu_A(y)\}$ *and (iv)* $\nu_A(x^{-1}) = \nu_A(x)$.

Definition 2.5 ([20]). *Let X be a set and* $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ *be an intuitionistic fuzzy set of X. Let* $\alpha, \beta \in [0,1]$ *with* $\alpha + \beta \leq 1$*. Then the* $(\alpha, \beta)-$*cut of A defined by the set* $C_{\alpha,\beta}(A) = \{x \in X \mid \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$.

The definitions of generalised lattice, subgeneralised lattice, homomorphism and product of generalised lattices are known from [8,9].

Definition 2.6 ([12]). *A system* $(G, \cdot, \leq)$ *is called a generalised lattice ordered group (gl-group) if (i)* $(G, \leq)$ *is a generalised lattice, (ii)* $(G, \cdot)$ *is a group and (iii) every group translation* $x \to a \cdot x \cdot b$ *on G is isotone. That is* $x \leq y \Rightarrow a \cdot x \cdot b \leq a \cdot y \cdot b$ *for all* $a, b \in G$.

Definition 2.7 ([16]). *A subgroup S of G is said to be a gl-subgroup of G if S is a subgeneralised lattice of G.*

Definition 2.8 ([16]). *Let G, H be gl-groups. A group homomorphism $f : G \to H$ is said to be a gl-homomorphism if f is a homomorphism of generalised lattices.*

In [21] Sharma observed that If A is an intuitionistic fuzzy set of a set X, then we have $C_{\alpha,\beta}(A) \subseteq C_{\delta,\theta}(A)$ if $\alpha \geq \delta$ and $\beta \leq \theta$.

Definition 2.9 ([24]). *Let L be a complete lattice with least element 0_L and greatest element 1_L. Let $[0,1]$ be the interval in real line. Let $h : L \to [0,1]$ be a lattice homomorphism, that is $h(\alpha \wedge \beta) = \min\{h(\alpha), h(\beta)\}$ and $h(\alpha \vee \beta) = \max\{h(\alpha), h(\beta)\}$. Let X be a non-empty set. A collection of objects in the set form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ is called a lattice valued intuitionistic fuzzy (L-valued intuitionistic fuzzy) set of type-3 of X if (i) $\mu_A : X \to L$ is a $L-$fuzzy set in X called degree of membership function on X, (ii) $\nu_A : X \to L$ is a $L-$fuzzy set in X called degree of non-membership function on X and (iii) for each $x \in X$, we have $0 \leq h(\mu_A(x)) + h(\nu_A(x)) \leq 1$.*

Definition 2.10 ([25]). *Let L be a complete lattice with least element 0_L and greatest element 1_L. Let $[0,1]$ be the interval in real line. Let $h : L \to [0,1]$ be a lattice homomorphism, that is $h(\alpha \wedge \beta) = \min\{h(\alpha), h(\beta)\}$ and $h(\alpha \vee \beta) = \max\{h(\alpha), h(\beta)\}$. Let P be a generalised lattice. Then a collection of objects in the set form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in P\}$ is called a L-valued intuitionistic L-fuzzy set of the type-3 (IFset of type 3) of P if (i) $\mu_A : P \to L$ is a $L-$fuzzy set in P called degree of membership function on P, (ii) $\nu_A : P \to L$ is a $L-$fuzzy set in P called degree of non-membership function on P and (iii) for each $x \in P$, we have $0 \leq h(\mu_A(x)) + h(\nu_A(x)) \leq 1$.*

Definition 2.11 ([25]). *Let P be a generalised lattice and $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in P\}$ be an L-valued Intuitionistic L-fuzzy set of the type 3 (IFset of type 3) of P. Then A is called an L-valued intuitionistic L-fuzzy subgeneralised lattice of type 3 (IFsubgl of type 3) of P if the following conditions satisfied: for any finite subset X of P; (i) $\mu_A(s) \geq \bigwedge_{x \in X} \mu_A(x)$ for all $s \in mu(X)$ (ii) $\mu_A(t) \geq \bigwedge_{x \in X} \mu_A(x)$ for all $t \in ML(X)$ (iii) $\nu_A(s) \leq \bigvee_{x \in X} \nu_A(x)$ for all $s \in mu(X)$ and (iv) $\nu_A(t) \leq \bigvee_{x \in X} \nu_A(x)$ for all $t \in ML(X)$.*

Definition 2.12. *Let L be a complete lattice with least element 0_L and greatest element 1_L. Let $[0,1]$ be the interval in real line. Let $h : L \to [0,1]$ be a lattice homomorphism, that is $h(\alpha \wedge \beta) = \min\{h(\alpha), h(\beta)\}$ and $h(\alpha \vee \beta) = \max\{h(\alpha), h(\beta)\}$. Let G be a group. Then a collection of objects in the set form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in G\}$ is called a L-valued intuitionistic L-fuzzy set of the type-3 (IFset of type 3) of G if (i) $\mu_A : G \to L$ is a $L-$fuzzy set in G called degree of membership function on G, (ii) $\nu_A : G \to L$ is a $L-$fuzzy set in G called degree of non-membership function on G and (iii) for each $x \in G$, we have $0 \leq h(\mu_A(x)) + h(\nu_A(x)) \leq 1$.*

Definition 2.13. *Let G be a group and $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in G\}$ be an L-valued intuitionistic L-fuzzy set of the type 3 (IFset of type 3) of G. Then A is called a L-valued intuitionistic L-fuzzy subgroup of type 3 (IFsubgroup of type 3) of G if the following conditions satisfied: for all $x, y \in G$; (i) $\mu_A(x \cdot y) \geq \mu_A(x) \wedge \mu_A(y)$ (ii) $\mu_A(x^{-1}) = \mu_A(x)$ (iii) $\nu_A(x \cdot y) \leq \mu_A(x) \vee \mu_A(y)$ and (iv) $\nu_A(x^{-1}) = \nu_A(x)$.*

Definition 2.14 ([25]). *Let P be a generalised lattice and A, B be L-valued intuitionistic L-fuzzy sets of the type 3 (IFsets of type 3) of P. Then define $A \cap B = \{(x, (\mu_A \cap \mu_B)(x), (\nu_A \cap \nu_B)(x)) \mid x \in P\}$ where $(\mu_A \cap \mu_B)(x) = \mu_A(x) \wedge \mu_B(x)$ and $(\nu_A \cap \nu_B)(x) = \nu_A(x) \wedge \nu_B(x)$.*

In [25] it was proved that intersection of any family of IFsubgls of type 3 of a generalised lattice is again an IFsubgl of type 3. By Sharma [20], we can observe that intersection of any family of IFsubgroups of a group is again an IFsubgroup.

3. L-valued Intuitionistic L-fuzzy gl-subgroups of the Type 3 of a gl-group

In this section introduced the concept L-valued intuitionistic L-fuzzy gl-subgroup of the type 3 (IFgl-subgroup of type 3) of a gl-group, discussed its properties and charecterized by its $(\alpha, \beta)-$level subsets. Proved that the set of all L-valued intuitionistic L-fuzzy gl-subgroups of the type 3 (IFgl-groups of type 3) of a gl-group, forms a complete lattice. Throughout this section G denotes for a generalised lattice ordered group (gl-group).

Definition 3.1. *Let L be a complete lattice with least element 0_L and greatest element 1_L. Let $[0,1]$ be the interval in real line. Let $h : L \to [0,1]$ be a lattice homomorphism, that is $h(\alpha \wedge \beta) = \min\{h(\alpha), h(\beta)\}$ and $h(\alpha \vee \beta) = \max\{h(\alpha), h(\beta)\}$. Then a collection of objects in the set form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in G\}$ is called a L-valued intuitionistic L-fuzzy set of the type-3 (IFset of type 3) of G if (i) $\mu_A : G \to L$ is a $L-$fuzzy set in G called degree of membership function on G, (ii) $\nu_A : G \to L$ is a $L-$fuzzy set in G called degree of non-membership function on G and (iii) for each $x \in G$, we have $0 \leq h(\mu_A(x)) + h(\nu_A(x)) \leq 1$.*

Definition 3.2. *Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in G\}$ be an L-valued intuitionistic L-fuzzy set of the type 3 (IFset of type 3) G. Then A is called a L-valued intuitionistic L-fuzzy gl-subgroup of the type 3 (IFgl-subgroup of type 3) of G if (i) A is a L-valued intuitionistic L-fuzzy subgroup of the type 3 (IF subgroup of type 3) of G and (ii) A is a L-valued intuitionistic L-fuzzy subgeneralised lattice of the type 3 (IF subgl of type 3) of G.*

Theorem 3.3. *Let A be a L-valued intuitionistic L-fuzzy gl-subgroup of the type 3 (IFgl-subgroup of type 3) of G. Then $\mu_A(e) \geq \mu_A(x)$ and $\nu_A(e) \leq \nu_A(x)$ for all $x \in G$.*

Proof. Let $x \in G$. Consider $\mu_A(e) = \mu_A(x \cdot x^{-1}) \geq \mu_A(x) \wedge \mu_A(x^{-1})$ (by definitions 3.2 and 2.12) $= \mu_A(x) \wedge \mu_A(x) = \mu_A(x)$. Consider $\nu_A(e) = \nu_A(x \cdot x^{-1}) \leq \nu_A(x) \vee \nu_A(x^{-1})$ (by definitions 3.2 and 2.12) $= \nu_A(x) \vee \nu_A(x) = \nu_A(x)$. Therefore $\mu(e) \geq \mu(x)$ and $\nu(e) \leq \nu(x)$ for all $x \in G$. □

Theorem 3.4. *An IFset of type 3, A of G is IFgl-subgroup of type 3 of G if and only if (i) $\mu_A(x \cdot y^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$ for all $x, y \in G$ (ii) $\nu_A(x \cdot y^{-1}) \leq \nu_A(x) \vee \nu_A(y)$ for all $x, y \in G$ (iii) $\mu_A(s) \wedge \mu_A(t) \geq \bigwedge_{x \in X} \mu_A(x)$ for all $s \in mu(X), t \in ML(X)$ and for any finite subset X of G and (iv) $\nu_A(s) \vee \nu_A(t) \leq \bigvee_{x \in X} \nu_A(x)$ for all $s \in mu(X), t \in ML(X)$ and for any finite subset X of G.*

Proof. Suppose A is an IFgl-subgroup of type 3 of G. (i) Let $x, y \in G$. Consider $\mu_A(x \cdot y^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1})$ (by Definitions 3.2 and 2.12) $= \mu_A(x) \wedge \mu_A(y)$. Therefore $\mu_A(x \cdot y^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$ for all $x, y \in G$. (ii) Let $x, y \in G$. Consider $\nu_A(x \cdot y^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1})$ (by Definitions 3.2 and 2.12) $= \nu_A(x) \vee \nu_A(y)$. Therefore $\nu_A(x \cdot y^{-1}) \leq \nu_A(x) \vee \nu_A(y)$ for all $x, y \in G$. (iii) Let X be a finite subset of G and $s \in mu(X), t \in ML(X)$. By the Definitions 3.2 and 2.10, we have $\mu_A(s) \geq \bigwedge_{x \in X} \mu_A(x)$ and $\mu_A(t) \geq \bigwedge_{x \in X} \mu_A(x)$. Therefore $\mu_A(s) \wedge \mu_A(t) \geq \bigwedge_{x \in X} \mu_A(x)$. (iv) Let X be a finite subset of G and $s \in mu(X), t \in ML(X)$. By the Definitions 3.2 and 2.10, we have $\nu_A(s) \leq \bigvee_{x \in X} \nu_A(x)$ and $\nu_A(t) \leq \bigvee_{x \in X} \nu_A(x)$. Therefore $\nu_A(s) \vee \nu_A(t) \leq \bigvee_{x \in X} \nu_A(x)$.

Conversely suppose the conditions (i), (ii), (iii) and (iv). To show that A is an IFgl-subgroup of type 3 of G : First to show that A is IF subgroup of type 3 of G : Let $x \in G$. Consider $\mu_A(e) = \mu_A(x \cdot x^{-1}) \geq \mu_A(x) \wedge \mu_A(x) = \mu_A(x)$. Consider $\nu_A(e) = \nu_A(x \cdot x^{-1}) \leq \nu_A(x) \vee \nu_A(x) = \nu_A(x)$. Consider $\mu_A(x^{-1}) = \mu_A(e \cdot x^{-1}) \geq \mu_A(e) \wedge \mu_A(x) = \mu_A(x)$. Consider $\nu_A(x^{-1}) = \nu_A(e \cdot x^{-1}) \leq \nu_A(e) \vee \nu_A(x) = \nu_A(x)$. Therefore $\mu_A(e) \geq \mu_A(x)$, $\nu_A(e) \leq \nu_A(x)$ and by Theorem 3.3, $\mu_A(x^{-1}) = \mu_A(x)$, $\nu_A(x^{-1}) = \nu_A(x)$ for all $x \in G$. Let $x, y \in G$. Consider $\mu_A(x \cdot y) = \mu(x \cdot (y^{-1})^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1}) = \mu_A(x) \wedge \mu_A(y)$. Consider $\nu_A(x \cdot y) = \nu(x \cdot (y^{-1})^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1}) = \nu_A(x) \vee \nu_A(y)$. Therefore by Definition 2.12, A is IF subgroup of type 3 of G. Now to show that A is an IFsubgl of type 3 of P : Let X be a finite subset of G. Let $s \in mu(X), t \in ML(X)$. Then $\mu_A(s), \mu_A(t) \geq \mu_A(s) \wedge \mu_A(t) \geq \bigwedge_{x \in X} \mu_A(x)$ and $\nu_A(s), \nu_A(t) \leq \nu_A(s) \vee \nu_A(t) \leq \bigvee_{x \in X} \nu_A(x)$. Therefore by Definition 2.10, A is IFsubgl of type 3 of P. Therefore by Definition 3.2, A is IFgl-subgroup of type 3 of G. □

Theorem 3.5. *An IFset A of G, is an IFgl-subgroup of type 3 of G if and only if $C_{\alpha,\beta}(A)$ is a gl-subgroup of G for all $\alpha \in \mu_A(G) \cup \{l \in L \mid \mu_A(e) \geq l\}$ and $\beta \in \nu_A(G) \cup \{l \in L \mid \nu_A(e) \leq l\}$ with $h(\alpha) + h(\beta) \leq 1$.*

Proof. Suppose A is an IFgl-subgroup of type 3 of G. Let $\alpha \in \mu_A(G) \cup \{l \in L \mid \mu_A(e) \geq l\}$ and $\beta \in \nu_A(G) \cup \{l \in L \mid \nu_A(e) \leq l\}$ with $h(\alpha) + h(\beta) \leq 1$. To show that $C_{\alpha,\beta}(A)$ is a gl-subgroup of G : To show that $C_{\alpha,\beta}(A)$ is a subgroup of G : Let $x, y \in C_{\alpha,\beta}(A)$. Then $\mu_A(x) \geq \alpha, \nu_A(x) \leq \beta$ and $\mu_A(y) \geq \alpha, \nu_A(y) \leq \beta$. Consider $\mu_A(x \cdot y^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1}) = \mu_A(x) \wedge \mu_A(y) \geq \alpha \wedge \alpha = \alpha$. Consider $\nu_A(x \cdot y^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1}) = \nu_A(x) \vee \nu_A(y) \leq \beta \vee \beta = \beta$. Therefore $x \cdot y^{-1} \in C_{\alpha,\beta}(A)$. Therefore $C_{\alpha,\beta}(A)$ is a subgroup of G. To show that $C_{\alpha,\beta}(A)$ is a subgl of G : Let X be a finite subset of $C_{\alpha,\beta}(A)$ and $s \in mu(X), t \in ML(X)$. Then $\mu_A(s) \geq \bigwedge\limits_{x \in X} \mu_A(x) \geq \alpha$, $\nu_A(s) \leq \bigvee\limits_{x \in X} \nu_A(x) \leq \beta$, $\mu_A(t) \geq \bigwedge\limits_{x \in X} \mu_A(x) \geq \alpha$ and $\nu_A(t) \leq \bigvee\limits_{x \in X} \nu_A(x) \leq \beta$. This implies $s, t \in C_{\alpha,\beta}(A)$. That is $mu(X), ML(X) \subseteq C_{\alpha,\beta}(A)$. Therefore $C_{\alpha,\beta}(A)$ is a subgl of G. Therefore $C_{\alpha,\beta}(A)$ is a gl-subgroup of G.

Conversely suppose the condition. To show that A is IFgl-subgroup of type 3 of G : Let $x, y \in G, \alpha = \mu_A(x) \wedge \mu_A(y)$ and $\beta = \nu_A(x) \vee \nu_A(y)$. Clearly $\alpha \in \mu_A(G) \cup \{l \in L \mid \mu_A(e) \geq l\}$ and $\beta \in \nu_A(G) \cup \{l \in L \mid \nu_A(e) \leq l\}$. Consider

$$\begin{aligned}
h(\alpha) + h(\beta) &= h(\mu_A(x) \wedge \mu_A(y)) + h(\nu_A(x) \vee \nu_A(y)) \\
&= \min\{h(\mu_A(x)), h(\mu_A(y))\} + \max\{h(\nu_A(x)), h(\nu_A(y))\} \\
&\leq \min\{1 - h(\nu_A(x)), 1 - h(\nu_A(y))\} + \max\{h(\nu_A(x)), h(\nu_A(y))\} \\
&= 1 - \max\{h(\nu_A(x)), h(\nu_A(y))\} + \max\{h(\nu_A(x)), h(\nu_A(y))\}
\end{aligned}$$

$$= 1$$

Then $x, y \in C_{\alpha,\beta}(A)$ and by hypothesis $C_{\alpha,\beta}(A)$ is a subgroup of G. This implies $x \cdot y^{-1} \in C_{\alpha,\beta}(A)$. Then $\mu_A(x \cdot y^{-1}) \geq \alpha = \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x \cdot y^{-1}) \leq \beta = \nu_A(x) \vee \nu_A(y)$. Let X be a finite subset of G, $\gamma = \bigwedge\limits_{x\in X} \mu_A(x)$ and $\delta = \bigvee\limits_{x\in X} \nu_A(x)$. Clearly $\gamma \in \mu_A(G) \cup \{l \in L \mid \mu_A(e) \geq l\}$, $\delta \in \nu_A(G) \cup \{l \in L \mid \nu_A(e) \leq l\}$. Consider

$$\begin{aligned} h(\gamma) + h(\delta) &= h(\bigwedge_{x\in X} \mu_A(x)) + h(\bigvee_{x\in X} \nu_A(x)) \\ &= \min_{x\in X}\{h(\mu_A(x))\} + \max_{x\in X}\{h(\nu_A(x))\} \\ &\leq \min_{x\in X}\{1 - h(\nu_A(x))\} + \max_{x\in X}\{h(\nu_A(x))\} \\ &= 1 - \max_{x\in X}\{h(\nu_A(x))\} + \max_{x\in X}\{h(\nu_A(x))\} \\ &= 1 \end{aligned}$$

Then $X \subseteq C_{\gamma,\delta}(A)$ and by hypothesis $C_{\gamma,\delta}(A)$ is a subgl of G. This implies $mu(X), ML(X) \subseteq C_{\gamma,\delta}(A)$. Then for any $s \in mu(X), t \in ML(X)$ we have $\mu_A(s) \geq \gamma, \nu_A(s) \leq \delta, \mu_A(t) \geq \gamma$ and $\nu_A(t) \leq \delta$. This implies $\mu_A(s) \wedge \mu_A(t) \geq \gamma = \bigwedge_{x\in X} \mu_A(x)$ and $\nu_A(s) \vee \nu_A(t) \leq \delta = \bigvee_{x\in X} \nu_A(x)$. Therefore by Theorem 3.4, A is IFgl-subgroup of type 3 of G. □

Theorem 3.6. *The intersection of any family of IFgl-subgroups type 3 of G is again an IFgl-subgroup of type 3 of G.*

Definition 3.7. *Let A be an IFset of type 3 of G. Then the smallest IFgl-subgroup of type 3 of G containing A is called IFgl-subgroup of type 3 of G generated by A.*

Recall the definitions of intuitionistic fuzzy empty set and intuitionistic fuzzy whole set defined by Tae Chon Ahn [22].

Definition 3.8. *Let P be a generalised lattice. Define $\phi = \{(x, \mu_\phi(x) = 0_L, \nu_\phi(x) = 1_L) \mid x \in P\}$ and $P = \{(x, \mu_\phi(x) = 1_L, \nu_\phi(x) = 0_L) \mid x \in P\}$. Then ϕ, P are L-valued intuitionistic L-fuzzy sets of the type 3 (IFsets of type 3) of P. Here ϕ is called empty IFset of type 3 and P called whole IFset of type 3.*

Definition 3.9. *Define $\phi = \{(x, \mu_\phi(x) = 0_L, \nu_\phi(x) = 1_L) \mid x \in G\}$ and $G = \{(x, \mu_\phi(x) = 1_L, \nu_\phi(x) = 0_L) \mid x \in G\}$. Then ϕ, G are L-valued intuitionistic L-fuzzy sets of the type 3 (IFsets of type 3) of G. Here ϕ is called empty IFset of type 3 and G called whole IFset of type 3.*

Acknowledgement

The correponding author Parimi Radhakrishna Kishore would like to thank:

- Department of Mathematics, Andhra University, Visakhapatnam, Andhra Pradesh (state), PIN Code: 530003, INDIA

- Department of Mathematics, Dr.B.R.Ambedkar University, Etcherla, Srikakulam, Andhra Pradesh (state), PIN Code: 532410, INDIA

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