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Effects of Viscous Dissipation on Slip Velocity Boundary Layer Flow and Mixed Convection Heat Transfer

T. N. Vasanthakumari[1,*]

[1] *Department of Mathematics, Government First Grade College, Tumakuru, Karnataka, India*

Abstract

In the present study, we investigate the slip velocity and viscous dissipation effects on the boundary layer flow and mixed convective heat transfer over a stationary and rigid surface. The mainstream flow is considered to be varying linearly from the leading boundary layer. The governing boundary layer equations are reduced to a set of ordinary differential equations, using appropriate similarity transformation, which are then solved numerically using Keller-box method. The various results such as velocity and temperature profiles and wall shear stress and temperature gradients are obtained and presented. Our results show that both slip velocity and Eckert number are to influence the boundary layer thickness, velocity and temperature profiles. Particularly, the slip velocity effects are to flatten the boundary layer. The various other physical mechanisms behind these effects are discussed in detail.

Keywords: Boundary layer flows; Mixed convection; Eckert number; Slip velocity; Keller-box method.

1. Introduction

The boundary-layer flow over a surface with incompressible, viscous and Newtonian fluid has been studied in the present work because of ever increasing industrial application and important several technological processes such as cooling of an infinite metallic plate in a cooling bath, the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation process.

The boundary-layer flow is limited to Newtonian fluid with traditional no-slip flow boundary condition over various geometry and effects. The no-slip boundary conditions are generally applied at a liquid-solid interface at which the velocity of the fluid tries to equate to the velocity of the solid boundary. Nevertheless, the fluid with micro or nano scale dimensions have different flow behavior from the traditional fluid flow, that is, the fluid slips on the solid boundary. For such fluid flow it belongs to slip flow region, it also still obey the Navier-Stokes equation with slip boundary

*Corresponding author (tnvasantha123@gmail.com)

conditions. It is advisable to have a general boundary condition that accounts the fluid slip at the boundary, and it is proportional to the tangential stress. In order to reduce the frictional drag, many strategies such as blowing of the fluid, flow additives, polymers, surfactants, plasma fields, have been introduced in the past ([1–4]). The slip velocity is also noticed in the flow of non-Newtonian fluids. Vedantam and Parthasarathy [3] have studied the effects of slip velocity on the Blasius boundary layer flow over flat plate by accounting the slip boundary condition in three different ways, i.e., the slip length, or the slip velocity, or by assuming the slip length to be a function of shear rate. It is shown that in all three cases have the boundary-layer thickness and the displacement thickness are found to decrease upon increase compared to no-slip condition. Aziz [5] has extended the study of Vedantam and Parthasarathy [3] by including heat transfer which varies along x-direction, and has shown that increasing slip velocity on the boundary reduces the surface temperature. Khader and Megahed [6] have investigated the numerical solution of a Newtonian fluid flow over a rigid and nonlinear stretching sheet with slip velocity and variable thickness and shown that the boundary layer velocity is found to decrease near the boundary when slip velocity is increased, but it increases at larger distances downstream.

The thermal boundary layers are found to occur near a solid surface where the temperature of the fluid is expected to change from the temperature of the surface to that of temperature of the mean flow of fluid away from the surface. This generally involves natural or forced convection (Schlichting and Gersten [7]). The former convective boundary layer occurs in the vicinity of a region adjacent to a surface where buoyancy forces drives the fluid motion due to temperature differences between the surface and the fluid., whereas latter convection arises in the boundary layer due to external forces. Mixed convection thermal boundary layers occur when fluid flow near a solid surface is essentially influenced by both buoyancy driven and externally driven forces simultaneously. This type of combination of both convective heat transfers generally appear in heat exchangers, electronics cooling devices, geophysical and environmental heat transfer, solar energy systems to name a few. Because of these ample number of applications numerous studies have taken place in the literature. Chen [8] has studied the mixed convective boundary layers flow on a vertical and power-law stretching sheet numerically using finite difference method. It is shown that effect of mixed convection parameter is to enhance the heat transfer rate. The influence of temperature-dependent viscosity on mixed convection boundary layer flow and associated heat transfer on a continuously moving vertical surface is studied in which the inverse linear function of temperature for fluid viscosity is assumed and shown that there are significant changes have been noticed in local heat transfer and wall shear stress coefficient for various physical parameters (Ali [9]). Ishak [10] have studied the steady mixed convection laminar boundary layer flow in the vicinity of the two-dimensional stagnation-point flow due to a linear stretching vertical sheet in its own plane numerically using finite difference based Keller-box method. It is found that for assisting (opposing) flow, both wall shear stress and temperature gradient are found to increase (decrease) while both are found to increase for increasing Prandtl number. Hayat

[11] have investigated the two-dimensional mixed convection stagnation point boundary layer subjected to applied magnetic field and porous medium accounting vertical stretching plate and thermal radiation. Using the homotopy analysis method, it is shown that the skin-friction coefficient and the temperature gradient are found to decrease gradually for increased Prandtl number in both assisting and opposing flows. Malvandi and Ganji [12] have investigated the mixed convective heat transfer of nanofluid inside a vertical microchannel by adopting Buongiorno's model accounting the Brownian motion and thermophoresis diffusivities. Authors have shown that nanoparticles are shown to move from the heated walls to the core of channel region only construct a non-uniform nanoparticles distribution. It is also shown that increasing bulk mean volume fraction of nanoparticles, slip parameter and mixed convective parameter, there is an enhanced the heat transfer rate. Bilal and Ashbar [13] have explored the influence of mixed convective heat transfer and stratified stretching sheet boundary layer flow of Eyring-Powell fluid taking an account of heat generation/absorption. Numerical solutions have shown that an increment in thermal stratification parameter results in a drastic fall in velocity as well as temperature of fluid while an opposite trend is observed for the heat generation parameter. Roy and Akter [14] have studied the unsteady boundary layer flow and mixed convection heat transfer of an incompressible hybrid nanofluid and shown using perturbation technique that mixed convection parameter is increased the wall shear stress and Nusselt number are also increased.

All the above mentioned studies continued their discussions by assuming slip effect on surface. The purpose of the present study is to investigate slip velocity effect and viscous dissipation on stagnant surface of Newtonian fluid with mixed convective heat transfer. To the best of our knowledge no study has been effectively taken to study the effect of slip velocity and Eckert number on the flow and heat transfer. The model under consideration is derived from the momentum and thermal boundary equations which are then numerically. Here, we introduce a technique known as the Keller-box method based on implicit finite difference to the search for the physical solutions.

We aim in this paper to present a numerical study of mixed convective dissipative boundary-layer flow arising due to slip velocity of Newtonian fluid on stagnant surface. The boundary-layer model is formulated in section 2. The slip effect model is modified into self-similar set of equations. The transformed equations are solved numerically using the Keller-box method which is given in 3. The flow and heat transfer are analyzed by computed numerical results by graphically in 4 in detail. Final section 5 concludes the important findings of the present study.

2. Mathematical Formulation

We consider a steady two-dimensional laminar flow and mixed convective heat transfer of a viscous and incompressible fluid over a impermeable stationary surface. The origin is located over a stationary surface over which the mainstream fluid flows downstream. The surface has the temperature T_w and mainstream temperature T_∞ and it is assumed as $T_w \gg T_\infty$. The rectangular coordinates x is measured as coordinates parallel to the flow and y as the normal to it towards the main stream. The governing equations of the model are the conservation of mass, momentum and energy equations

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\rho(\vec{q} \cdot \nabla)\vec{q} = -\nabla p + \mu \nabla^2 \vec{q} - \rho \vec{g} \tag{2}$$

$$(\vec{q} \cdot \nabla)T = \alpha_1 \nabla^2 T + \Phi \tag{3}$$

where $\vec{q}$ is an intrinsic velocity, ρ is density, p is the pressure, μ is the fluid viscosity, $\vec{g}$ is the gravitational acceleration, T is the temperature, $\alpha_1 = \frac{\kappa}{\rho c_p}$ is thermal diffusivity with c_p being the specific heat at constant pressure and κ is thermal conductivity, and Φ is the viscous dissipation term. It is considered that the temperature of surface and that of fluid is identical to each other which is the case of local thermal equilibrium. Further, it is assumed that the Reynolds number of the mainstream flow is large such that the momentum and thermal boundary layers form near the impermeable surface. In the momentum boundary layer, the viscous effects are considered to be dominant near the surface while away from it these effects are considered to be negligible. Note that the wedge surface has a linear slip-velocity.

By applying usual boundary-layer approximations applicable for the Newtonian fluids the steady two dimensional boundary-layer equations take the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - g_x \tag{5}$$

$$0 = \frac{\partial p}{\partial y} \tag{6}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \Phi \tag{7}$$

where u, v are velocity components, $\nu (= \frac{\mu}{\rho})$ is the kinematics viscosity, g_x is the magnitude of gravity force in the negative x direction. The viscous dissipation term is defined as $\Phi = \frac{\nu}{c_p}(\frac{\partial u}{\partial y})^2$. Further, the first term on right side of (5) can be evaluated using Bernoulli's theorem with $u(x, y) = U(x)$ at the edge of boundary layer as the pressure variation along the normal direction is constant

$$U\frac{dU}{dx} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\rho_\infty}{\rho} g_x \tag{8}$$

where $U(x)$ is a free stream velocity outside the boundary layer, ρ_∞ is the density in the main stream. We assume that the body force $g_x(\rho_\infty - \rho)$ for the mixed convective heat transfer can be written as

$$g_x(\rho_\infty - \rho) = g_x T_c \rho (T - T_\infty) \tag{9}$$

where T_c is the coefficient of thermal expansion. Thus, equation (5) may be rewritten in view of (8)-(9) as

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{(\rho_\infty-\rho)}{\rho}g_x. \tag{10}$$

We define the following boundary conditions which are relevant to the present model

$$\text{at} \qquad y=0: \quad u=N\mu\frac{\partial u}{\partial y}, \;\; v=0, \;\; T=T_w \tag{11}$$

$$\text{as} \qquad y\to\infty: \quad u=U, \;\; T=T_\infty \tag{12}$$

where N is the slip velocity parameter. The boundary conditions define that the velocity u and temperature T decay in an asymptotic manner onto the mainstream velocity U and temperature T_∞ far away from the wedge surface. It is assumed that the surface moves with velocity $U_w(x)$ in the same and opposite to that of the mainstream velocity $U(x)$. We assume that both $U(x)$ is expected to vary in a linear manner from the leading boundary layer edge and the temperature on the surface is assumed as

$$U(x)=U_\infty x, \quad T_w=T_\infty+Ax \tag{13}$$

where $U_\infty>0$ is reference velocity and and A is positive constant.
The continuity equation (4) is satisfied identically by the usual assumption of the stream function $\psi(x,y)$ i.e., $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$ which reduces boundary layer equations to one. Also, two independent variables x and y may be reduced to one using similarity variables, the mathematical analysis of the problem is simplified by introducing dimensionless co-ordinates

$$\psi=(\nu xU)^{\frac{1}{2}}F(\eta), \;\; \frac{T-T_\infty}{T_w-T_\infty}=\theta(\eta), \;\; \eta=\left(\frac{U}{\nu x}\right)^{\frac{1}{2}}y \tag{14}$$

where η is a new similarity transformation. Using the similarity transformations (14) in the momentum and energy equation (10) and (7), we get

$$F'''+FF''+1-F'^2+\alpha\theta=0 \tag{15}$$

$$\frac{1}{Pr}\theta''+F\theta'-F'\theta+EcF''^2=0 \tag{16}$$

and transformed boundary conditions (11)-(12) are

$$F(0)=0, \;\; F'(0)=KF''(0), \;\; \theta(0)=1 \tag{17}$$

$$F'(\infty)=1, \;\; \theta(\infty)=0 \tag{18}$$

where $F=F(\eta)$, $\theta=\theta(\eta)$ are the non-dimensional stream functions and primes denote differentiation with respect to η, $\alpha=\frac{Gr}{Re_x^2}$ is mixed convection parameter, $Gr=\frac{g_xT_c(T_w-T_\infty)L^3}{\nu^2}$ is the Grashoff number and $Re_x = \frac{UL}{\nu}$ is the local Reynolds number, $Pr = \frac{\nu}{\alpha}$ is Prandtl number, $Ec = \frac{U^2}{c_p(T_w - T_\infty)}$ is the Eckert number, $K = N\mu\sqrt{\frac{U_\infty}{\nu}}$ is the velocity slip parameter. The slip velocity describes the relative importance between the magnitude of the slip velocity at the fluid-surface interface and mainstream velocity of the fluid away from the surface. Note that the cases corresponding to $\alpha < 0$ and $\alpha > 0$ are termed as decelerated (or adverse) and accelerated (or assisted) flow, while $\alpha = 0$ is the case for forced convective heat transfer. When $Pr < 1$ and $Pr > 1$, the thickness of the thermal boundary layer is larger and smaller compared to the thickness of the boundary layer whereas $Pr = 1$ is the case for identical boundary layers. The equation (15) and (16) with boundary conditions (17) and (18) are non-linear coupled ordinary differential equation within a domain $[0, \infty)$ and are solved numerically since both equations do not amenable to any analytical solutions.

3. Numerical procedure

The system (15) and (16) is coupled and highly nonlinear in nature which cannot be solved analytically with the boundary conditions (17) and (18), therefore, we solve numerically using Keller box method. This method is used in most of the boundary layer analyses because it is based on a finite difference scheme and computationally efficient [15–17]. Hence, we use the backward finite-difference scheme twinned with the Newton's linearization technique. Keller and Cebeci have made a detailed review and explained on this method who have applied it to solve momentum boundary layer equations. Further Kudenatti *et al.* have applied the method on the non-linear boundary-layer equations pertaining to three-dimensional boundary layer flow and heat transfer and shown that the physical solutions are captured effectively using Keller-box method. The detailed explanation of the method is given in chapter 1. In this section, we discuss the approach of the method to the problem briefly. On introducing some additional unknown functions the total fifth-order differential equation is converted into a system of five first-order equations. Thus, to describe this method the system of differential equations (15), (16), and associated with boundary conditions (17) and (18) are written in terms of first order differential equations i.e.,

$$F' = H, \quad H' = S, \quad S' + FS + 1 - H^2 + \alpha\theta = 0 \tag{19}$$

$$\theta' = T, \quad \frac{1}{Pr}T' + EcS^2 - H\theta + FT = 0 \tag{20}$$

and the respective boundary conditions are given by

$$F(0) = 0, \;\; H(0) = \lambda + KS(0), \;\; H(\infty) = 1, \;\; \theta(0) = 1, \;\; \theta(\infty) = 0 \tag{21}$$

using finite difference approximations with backward difference, we get

$$F_j - F_{j-1} = d(H_j + H_{j-1}) \tag{22a}$$

$$H_j - H_{j-1} = d(S_j + S_{j-1}) \tag{22b}$$

$$\theta_j - \theta_{j-1} = d(T_j + T_{j-1}) \tag{22c}$$

$$S_j - S_{j-1} + \frac{d}{2}(F_j + F_{j-1})(S_j + S_{j-1}) - \frac{d}{2}(H_j + H_{j-1})^2 + \alpha d(\theta_j + \theta_{j-1}) = 0 \tag{22d}$$

$$\frac{1}{Pr}(T_j - T_{j-1}) + \frac{d}{2}Ec(S_j + S_{j-1})^2 - \frac{d}{2}(H_j + H_{j-1})(\theta_j + \theta_{j-1}) + \frac{d}{2}(F_j + F_{j-1})(T_j + T_{j-1}) = 0 \tag{22e}$$

for $j = 1, 2, \cdots, M-1$, where M is number of grid points in boundary layer domain and $d = \frac{h_j}{2}$, h_j is grid-size in η-direction. The above system (22) essentially generates non-linear algebraic equations which are difficult to solve them on their nonlinear structure. We thus linearize them by using Newton's linearization technique

$$[a]^{(k+1)} = [a]^{(k)} + [\delta a]^{(k)} \tag{23}$$

where $\delta a = [\delta F, \delta H, \delta S, \delta\theta, \delta T]$ is a vector of unknown quantities to be determined. Substituting (23) and (22) and neglecting the second and higher order terms gives the system of linear algebraic equations in $[\delta a]$. The vector form of linearized algebraic equations can be represented in matrix form

$$\mathbf{AD} = \mathbf{R} \tag{24}$$

where the matrix A is the block-tridiagonal system with each element is a sub-matrix of order 5×5, $\mathbf{D}$ is the a vector containing the unknowns and R generated from the known coefficients. The factorization method is used to solve the tri-diagonal structure. The solution $\mathbf{D}$ obtained by solving (24) need to be updated using (23) at each iteration until convergence occurs. We have set the error tolerance to 10^{-6} for all the simulations and also verified the error tolerance up to 10^{-8}. Since, there were no difference in both of the solutions, we performed the solution process with 10^{-6}. Using our Keller box code, the various results shall be discussed in the some detail below.

4. Results and Discussion

In this paper, we have studied the velocity and temperature distribution in the steady two-dimensional mixed convection boundary-layer flow and effects of different values of mixed convection parameter α, Prandtl number Pr, Eckert number Ec, velocity ratio parameter λ and the momentum slip parameter K. We have reduced the fully coupled non-linear ordinary differential equations (15)-(16) with the relevant boundary conditions (17)-(18) using the finite difference based Keller-box method. The accuracy and robustness of the Keller-box scheme have been checked and confirmed repeatedly for various parameters. The investigation is carried out for emerging parameters for the dimensionless velocity $F(\eta)$, temperature $\theta(\eta)$, skin-friction $F''(0)$ and temperature gradient $\theta'(0)$. Further, we discuss on the velocity and temperature profiles along with the wall shear stress and rate of heat transfer for different physical parameters.

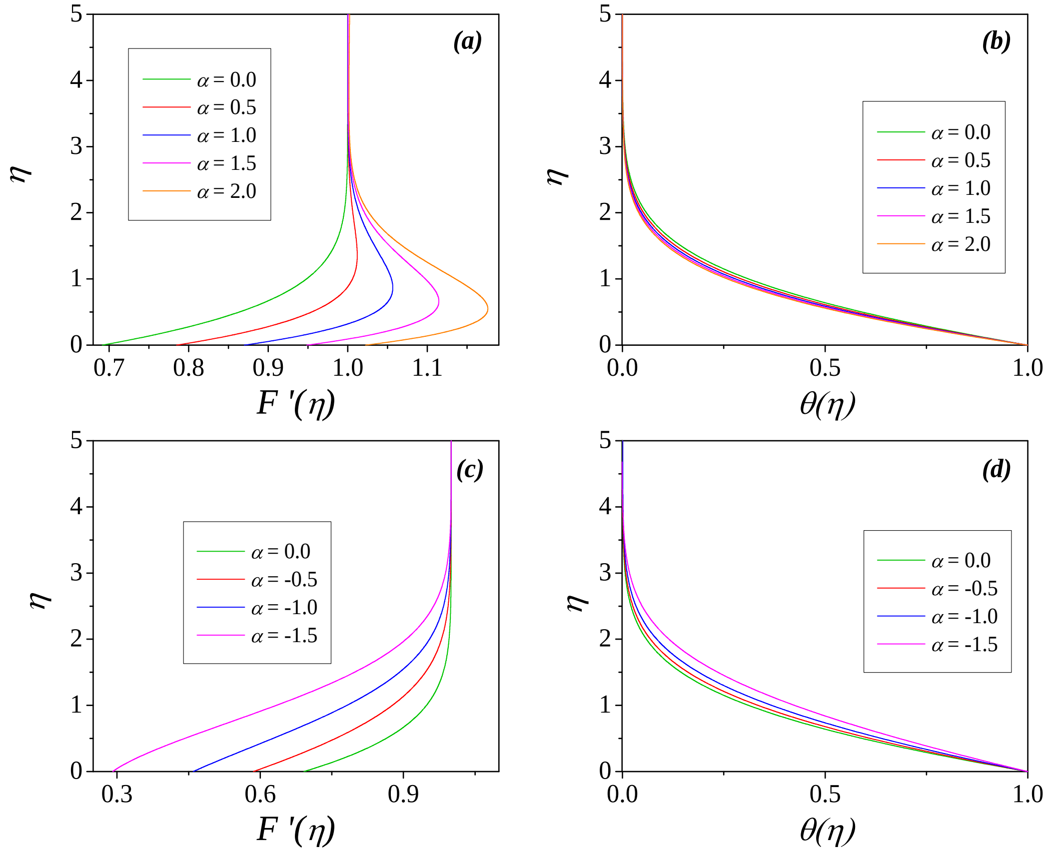

Figure 1: The velocity and thermal graphs for different values of Γ mixed convection parameter.

We first discuss the velocity and temperature profiles for various assisted flow parameter $\alpha \geq 0$ keeping other parameters fixed. Figure 1 infers that the fluid velocity $F'(\eta)$ increases for increasing values of mixed convection parameter α. For increasing α in the positive direction results in increasing fluid velocity. This is because when $\alpha > 0$ induces a favorable pressure gradient that enhances the fluid flow in the boundary layer. For large values of α as shown in figure 1a the velocity curves are seen to produce overshoots near the surface but eventually tend to their asymptotic boundary condition. In figure 1b, there is a very little difference when α is increased, but cannot be neglected as there is a difference in fluid flow involved with the thermal boundary layer. On the other hand, the velocity and temperature curves for negative mixed convection have a typical characteristic of thickening the boundary layer thickness compared to forced convection heat transfer $(\alpha = 0)$. These effects are shown to be in contrast to the results shown in figures 1a-1b. There is a evident heat transfer rate for $\alpha < 0$ at which there is enhanced thermal boundary layer thickness and hence enhanced rate of heat transfer.

We now study the effect of Eckert number that signifies the role of viscous dissipation in the heat transport. To ensure this, we choose the larger values of Ec for which the viscous dissipation in the boundary layer domain significantly which results in an increased internal energy. Thus, the effects of Eckert number on the heat transfer associated with stagnation boundary layer flow are displayed in figures 2a and 2b. The fluid velocity $F'(\eta)$ and the fluid temperature $\theta(\eta)$ increases for increasing values of Eckert number Ec as shown in the figures 2a and 2b respectively. The momentum and thermal boundary layer thickness decreases for increasing Ec due to increase in fluid temperature inside the fluid. Although, the values Ec is larger, there is no evident difference in both profiles and there is a marginal difference in the boundary layer thickness. This impacts temperature gradients on the surface and hence heat transfer rate.

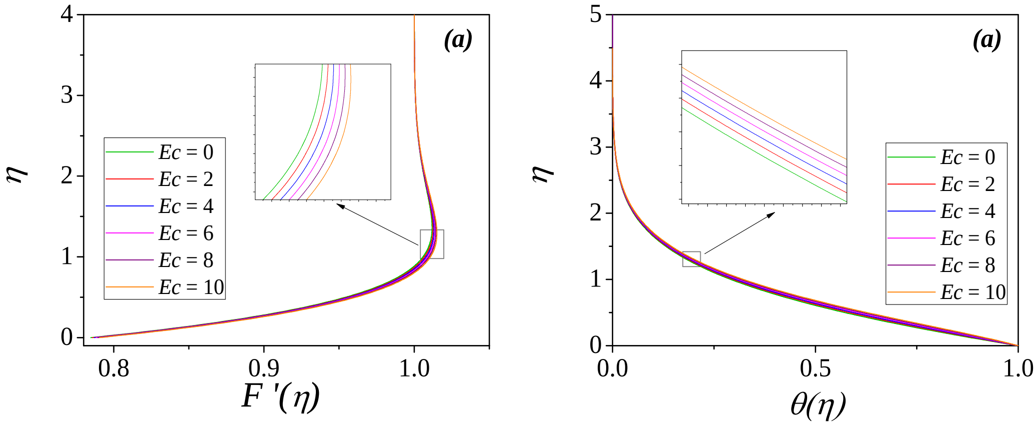

Figure 2: The velocity and thermal graphs for different values of Ec viscous dissipation parameter.

In the case of forced convection i.e, $\alpha = 0$, there will be no contribution of Ec number on the temperature profile since the momentum and energy equations are no more coupled.

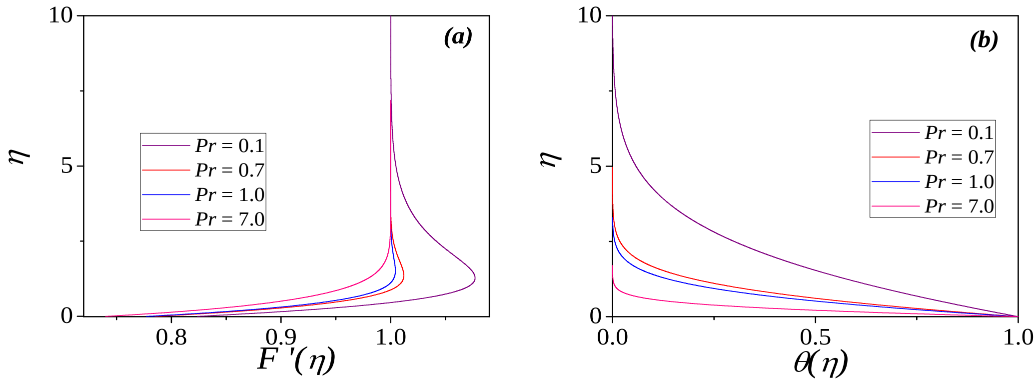

Figure 3: The velocity and thermal graphs for different values of Pr Prandtl number.

Figure 3 demonstrates the effects of the Prandtl number to the velocity and temperature distribution. As already indicated above in section 2, the Prandtl number describes the relative difference in thickness of momentum and thermal boundary layers, and thus affects the heat transfer significantly. In the present study, the range for Pr is chosen from 0.1 to 7 which covers all possible fluids that arise in the applications. It is interesting to notice that for increasing Pr number the velocity and temperature profiles respectively increases and decreases. It is actually an increase in fluid velocity and decrease in the temperature. For $Pr \geq 1$, the momentum velocity is shown to produce the overshoot near the surface, as indicated above in figure 1a. The temperature profiles tend to produce thicker thermal boundary layer thickness for smaller Pr values which results in low convective heat transfer rate.

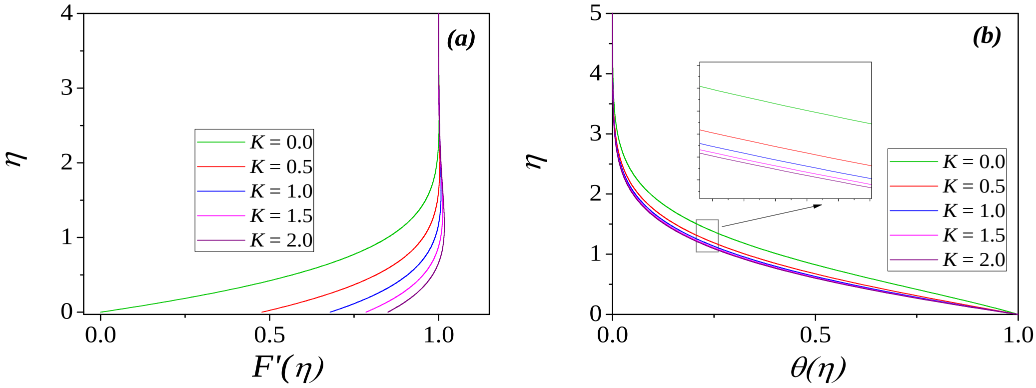

Figure 4: The velocity and thermal graphs for different values of K slip velocity coefficients.

We now discuss the role of slip velocity parameter on the boundary layer flow which is generic of this study. In figures 4, the variation of the velocity slip parameter K for both the velocity and thermal profiles. Note that, when $K = 0$, boundary layer with no-slip condition is achieved. For $K \neq 0$, the velocity is found to originate at different starting points because of the boundary conditions given in (17). These results are shown in figures 4a and 4b for various values of K. For higher values of the slip velocity parameter K as shown in figure 4a, the thickness of the boundary layer near to the surface is greatly affected as seen clearly in the figure, and the slip velocity at the interface of fluid and surface is more pronounced as compared to the fluid velocity away from the surface. This essentially results in a thinner boundary layer because the fluid near the surface is found to move faster for which the viscous effects are generally dominant. Also, there is an evident difference between velocity on the surface and velocity on the boundary layer edge and this difference is found decreasing for higher values of K for which velocity becomes flat which is the result of increased velocity due to slip velocity. Since, the velocity in the boundary layer is influenced by the slip velocity, accordingly thermal boundary layer will also be affected. For higher values of K as shown in figure 4b, there is an enhanced rate of heat transfer due to the thinner thermal boundary layer and altered velocity contribution.

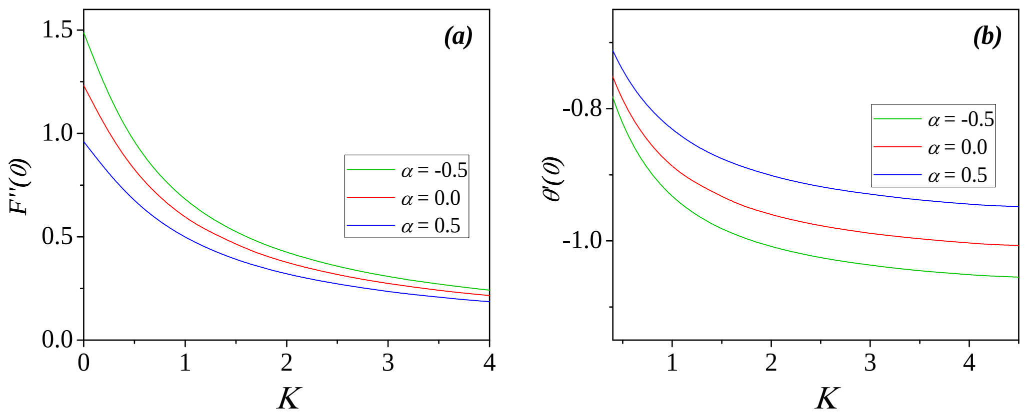

Figure 5: The behaviour of skin-friction distribution and thermal gradient against the slip velocity for various values of mixed convection parameter Γ.

In order to understand the broader structure of the nature of slip velocity, we discuss various results in terms of skin-friction (wall shear stress) $F''(0)$ and the temperature gradient $-\theta'(0)$ for mixed convection parameter α and Prandtl number Pr. The effects of mixed convection parameter α on surface shear stress $F''(0)$ and local Nusselt number $\theta'(0)$ versus velocity slip condition K are presented in figure 5a and figure 5b respectively. In consistency with the earlier graphs described for velocity and temperature with increase in α, wall shear stress is reduced from no-slip velocity condition i.e, the flow is accelerated along the surface which is the impact of the momentum transfer between the fluid and the surface. With an increasing K, the Nusselt number is also decreasing and profiles are generally monotonic decay for decreases in mixed convection parameter. Maximum Nusselt number arises at the slip velocity is negligible small and minimized with greater slip velocity. The skin friction co-efficient local Nusselt number are maximized for the no slip condition and increase or decrease value of T. Further, both the skin-friction and temperature gradient is found decreasing gradually for increasing K in that they an opposite phenomena for mixed convection parameter.

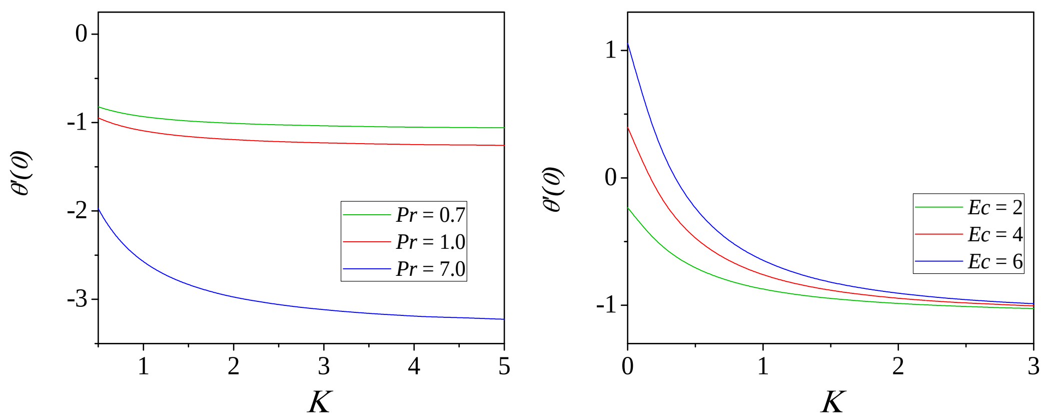

Figure 6: The behaviour of Nusselt number against the distribution of the slip velocity for various values of Ec Eckert number.

Similar but not identical results are found for increasing Prandtl Pr (figure 6a) and Eckert numbers (figure 6a). Since the Prandtl and Eckert numbers are associated with heat transport equation, only temperature gradient is discussed. Figures 6a and 6b show the variation of Nusselt number $\theta'(0)$ on the wall with velocity slip parameter for different Prandtl number Pr and Eckert number Ec. The results for $Pr < 1$ have different characteristics than those of $Pr > 1$. The former results show a steady decrease for $K > 2$ while for latter results, there is a nonlinear decay for small K. The values for $\theta'(0)$ shown in figure 6b for small K decrease while for $K > 2.5$, the values seem to overlap regardless of Eckert number. This means that the temperature gradients seem to be more uniform along the flow direction at higher Ec and K. In both the cases, when k is small the temperature is large where as for K large the local heat transfer rate increases.

5. Conclusion

The present paper studies the two-dimensional boundary layer flow of a viscous fluid and associated mixed convective heat transfer, emphasizing the effects of slip velocity and viscous dissipation on the thermal boundary layers. The governing nonlinear equations have been obtained using the similarity transformations in the momentum and thermal boundary layer equations, and the finite difference based Keller-box method is used for their simulations. We have found the following important results:

1. the mixed convection parameter is increased, the velocity profiles are to become flat close to the boundary layer edge. The heat transfer rate is enhanced with thicker boundary layer thickness.
2. the temperature profiles have thicker thermal boundary layer thickness for smaller Prandtl numbers. This results in low convective heat transfer rate.
3. the slip velocity influences momentum boundary layer to be flat due to increased velocity near the surface. Since, changes in the the shear stress distribution at the boundary, the slip velocity tends to reduce the thickness of the thermal boundary layer and to enhance convective heat transfer.
4. at large Eckert numbers, internal heating due to viscous dissipation is large which leads higher temperatures near the boundary, affecting the thickness of the thermal boundary layer and hence heat transfer rate.

Thus, the slip velocity, Eckert and Prandtl numbers each are found to play a crucial role in determining the various characteristics of momentum and thermal boundary layers. These physical quantities are seen to influence boundary layer thickness and heat transfer rate in the stagnation boundary layers.

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