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Elliptic Delta and Modified Elliptic Delta Indices of Certain Networks

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Abstract

In this study, we introduce the elliptic delta and modified elliptic delta indices and their corresponding exponentials of a graph. Furthermore, we compute these newly definied elliptic delta indices and their corresponding exponentials for certain networks of chemical importance like silicate networks, honeycomb networks, oxide networks and hexagonal networks.

Keywords: elliptic delta index; modified elliptic delta index; silicate networks; honeycomb networks; oxide networks and hexagonal networks.
2020 Mathematics Subject Classification: 05C07, 05C09, 05C92.

1. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set E(G). The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. Let $\delta(G)$ denote the minimum degree among the vertices of G. We refer [1] for undefined notations and terminologies. A graph index is a numerical parameter mathematically derived from the graph structure. Several graph indices have been considered in Theoretical Chemistry and many graph indices were defined by using vertex degree concept [2]. The Zagreb, Banhatti, Revan, Gourava, delta indices are the most degree based graph indices in Chemical Graph Theory, see [3-17]. Graph indices have their applications in various disciplines in Science and Technology [18, 19]. The elliptic Sombor index [20] of a graph G is defined as

$$ESO\,(G) = \sum_{uv \in E(G)} (d_G(u) + d_G(v))\sqrt{d_G(u)^2 + d_G(v)^2}.$$

Recently, some elliptic indices were studied in [21-25]. The δ vertex degree was defined by Kulli in [26] as

$$\delta_u = d_u - \delta\,(G) + 1.$$

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Motivated by the elliptic Sombor index, we define the elliptic delta index of a graph G as

$$E\delta(G) = \sum_{uv \in E(G)} (\delta_u + \delta_v) \sqrt{\delta_u^2 + \delta_v^2}.$$

Considering the elliptic delta index, we introduce the elliptic delta exponential of a graph G and defined it as

$$E\delta(G, x) = \sum_{uv \in E(G)} x^{(\delta_u + \delta_v) \sqrt{\delta_u^2 + \delta_v^2}}.$$

We define the modified elliptic delta index of a graph G as

$$^{m}E\delta(G) = \sum_{uv \in E(G)} \frac{1}{(\delta_u + \delta_v) \sqrt{\delta_u^2 + \delta_v^2}}.$$

Considering the modified elliptic delta index, we introduce the modified elliptic delta exponential of a graph G and defined it as

$$^{m}E\delta(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{(\delta_u + \delta_v) \sqrt{\delta_u^2 + \delta_v^2}}}.$$

Recently, some delta indices were studied in [27-36]. In this work, we determine the elliptic delta and modified elliptic delta indices and their exponentials for certain families of networks.

2. Results for Silicate Networks

Silicate networks are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is denoted by SL_n. A 2-D silicate network is presented in Figure 1.

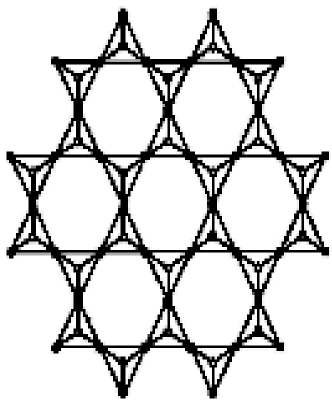

Figure 1: A 2-D silicate network

Let G be the graph of a silicate network SL_n. We obtain that G has $15n^2 + 3n$ vertices and $36n^2$ edges. In G, there are three types of edges as follows:

$$E_1 = \{uv \in E(G) | d_G(u) = d_G(v) = 3\}, \quad |E_1| = 6n.$$

$$E_2 = \{uv \in E(G) | d_G(u) = 3, d_G(v) = 6\}, \quad |E_2| = 18n^2 + 6n.$$

$$E_3 = \{uv \in E(G) | d_G(u) = d_G(v) = 6\}, \qquad |E_3| = 18n^2 - 12n.$$

We have $\delta(G) = 3$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 2$. Hence there are 3 types of δ-edges as given in Table 1.

$\delta_u, \delta_v \setminus uv \in E(G)$	(1, 1)	(1, 4)	(4, 4)
Number of edges	6n	$18n^2 + 6n$	$18n^2 - 12n$

Table 1: δ-edge partition of SL_n

Theorem 2.1. *Let G be the graph of a silicate networks SL_n. Then*

$$E\delta\,(G) = \left(90\sqrt{17} + 576\sqrt{2}\right) n^2 + \left(30\sqrt{17} - 372\sqrt{2}\right) n.$$

Proof. From definition and by using Table 1, we deduce

$$\begin{aligned} E\delta\,(G) &= \sum_{uv \in E(G)} (\delta_u + \delta_v)\sqrt{\delta_u^2 + \delta_v^2} \\ &= 6n\,(1+1)\sqrt{1^2+1^2} + \left(18n^2 + 6n\right)(1+4)\sqrt{1^2+4^2} + + \left(18n^2 - 12n\right)(4+4)\sqrt{4^2+4^2} \\ &= \left(90\sqrt{17} + 576\sqrt{2}\right) n^2 + \left(30\sqrt{17} - 372\sqrt{2}\right) n. \end{aligned}$$

□

Theorem 2.2. *Let G be the graph of a silicate networks SL_n. Then*

$$E\delta\,(G,x) = 6nx^{2\sqrt{2}} + \left(18n^2 + 6n\right) x^{5\sqrt{17}} + \left(18n^2 - 12n\right) x^{32\sqrt{2}}.$$

Proof. From definition and by using Table 1, we derive

$$\begin{aligned} E\delta\,(G,x) &= \sum_{uv \in E(G)} x^{(\delta_u+\delta_v)\sqrt{\delta_u^2+\delta_v^2}} \\ &= 6nx^{(1+1)\sqrt{1^2+1^2}} + \left(18n^2 + 6n\right) x^{(1+4)\sqrt{1^2+4^2}} + \left(18n^2 - 12n\right) x^{(4+4)\sqrt{4^2+4^2}} \\ &= 6nx^{2\sqrt{2}} + \left(18n^2 + 6n\right) x^{5\sqrt{17}} + \left(18n^2 - 12n\right) x^{32\sqrt{2}}. \end{aligned}$$

□

Theorem 2.3. *Let G be the graph of a silicate networks SL_n. Then*

$$^{m}E\delta\,(G) = \frac{6n}{2\sqrt{2}} + \frac{18n^2 + 6n}{5\sqrt{17}} + \frac{18n^2 - 12n}{32\sqrt{2}}.$$

Proof. From definition and by using Table 1, we obtain

$$^{m}E\delta\,(G) = \sum_{uv \in E(G)} \frac{1}{(\delta_u + \delta_v)\sqrt{\delta_u^2 + \delta_v^2}}$$

$$= \frac{6n}{(1+1)\sqrt{1^2+1^2}} + \frac{18n^2+6n}{(1+4)\sqrt{1^2+4^2}} + \frac{18n^2-12n}{(4+4)\sqrt{4^2+4^2}}$$
$$= \frac{6n}{2\sqrt{2}} + \frac{18n^2+6n}{5\sqrt{17}} + \frac{18n^2-12n}{32\sqrt{2}}.$$

□

Theorem 2.4. *Let G be the graph of a silicate networks SL_n. Then*

$$^{m}E\delta(G,x) = 6nx^{\frac{1}{2\sqrt{2}}} + \left(18n^2+6n\right)x^{x^{\frac{1}{5\sqrt{17}}}} + + \left(18n^2-12n\right)x^{x^{\frac{1}{32\sqrt{2}}}}.$$

Proof. From definition and by using Table 1, we deduce

$$^{m}E\delta(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{(\delta_u+\delta_v)\sqrt{\delta_u^2+\delta_v^2}}}$$
$$= 6nx^{\frac{1}{(1+1)\sqrt{1^2+1^2}}} + \left(18n^2+6n\right)x^{x^{\frac{1}{(1+4)\sqrt{1^2+4^2}}}} + + \left(18n^2-12n\right)x^{x^{\frac{1}{(4+4)\sqrt{4^2+4^2}}}}$$
$$= 6nx^{\frac{1}{2\sqrt{2}}} + \left(18n^2+6n\right)x^{x^{\frac{1}{5\sqrt{17}}}} + + \left(18n^2-12n\right)x^{x^{\frac{1}{32\sqrt{2}}}}.$$

□

3. Results for Honeycomb Networks

If we recursively use hexagonal tiling in particular pattern, honeycomb networks are formed. These networks are very useful in Chemistry and also in Computer Graphics. A honeycomb network of dimension n is denoted by HC_n. A honeycomb network of dimension four is shown in Figure 2.

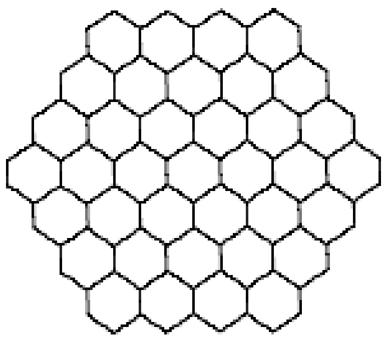

Figure 2: Honeycomb network of dimension four

Let G be the graph of a honeycomb network HC_n. We obtain that G has $6n^2$ vertices and $9n^2 - 3n$ edges. In G, there are three types of edges as follows:

$$E_1 = \{uv \in E(G) | d_G(u) = d_G(v) = 2\}, \quad |E_1| = 6.$$
$$E_2 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 3\}, \quad |E_2| = 12n - 12.$$
$$E_3 = \{uv \in E(G) | d_G(u) = d_G(v) = 3\}, \quad |E_3| = 9n^2 - 15n + 6.$$

We have $\delta(G) = 2$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 1$. Hence there are 3 types of δ-edges as given in Table 2.

$\delta_u, \delta_v \setminus uv \in E(G)$	(1, 1)	(1, 2)	(2, 2)
Number of edges	6	$12n - 12$	$9n^2 - 15n + 6$

Table 2: δ-edge partition of HC_n

Theorem 3.1. *Let G be the graph of a honeycomb network HC_n. Then*

$$E\delta\left(G\right) = 72\sqrt{2}n^2 + \left(36\sqrt{5} - 120\sqrt{2}\right) n + 60\sqrt{2} - 36\sqrt{5}.$$

Proof. From definition and by using Table 2, we deduce

$$\begin{aligned} E\delta\left(G\right) &= \sum_{uv\in E(G)} \left(\delta_u + \delta_v\right) \sqrt{\delta_u^2 + \delta_v^2} \\ &= 6\left(1+1\right)\sqrt{1^2+1^2} + \left(12n-12\right)\left(1+2\right)\sqrt{1^2+2^2} + \left(9n^2-15n+6\right)\left(2+2\right)\sqrt{2^2+2^2} \\ &= 72\sqrt{2}n^2 + \left(36\sqrt{5} - 120\sqrt{2}\right) n + 60\sqrt{2} - 36\sqrt{5}. \end{aligned}$$

□

Theorem 3.2. *Let G be the graph of a honeycomb network HC_n. Then*

$$E\delta\left(G, x\right) = 6x^{2\sqrt{2}} + \left(12n - 12\right) x^{3\sqrt{5}} + \left(9n^2 - 15n + 6\right) x^{8\sqrt{2}}.$$

Proof. From definition and by using Table 2, we derive

$$\begin{aligned} E\delta\left(G, x\right) &= \sum_{uv\in E(G)} x^{(\delta_u+\delta_v)\sqrt{\delta_u^2+\delta_v^2}} \\ &= 6x^{(1+1)\sqrt{1^2+1^2}} + \left(12n-12\right) x^{(1+2)\sqrt{1^2+2^2}} + \left(9n^2-15n+6\right) x^{(2+2)\sqrt{2^2+2^2}} \\ &= 6x^{2\sqrt{2}} + \left(12n - 12\right) x^{3\sqrt{5}} + \left(9n^2 - 15n + 6\right) x^{8\sqrt{2}}. \end{aligned}$$

□

Theorem 3.3. *Let G be the graph of a honeycomb network HC_n. Then*

$$^{m}E\delta\left(G\right) = \frac{6}{2\sqrt{2}} + \frac{12n-12}{3\sqrt{5}} + \frac{9n^2-15n+6}{8\sqrt{2}}.$$

Proof. From definition and by using Table 2, we obtain

$$\begin{aligned} ^{m}E\delta\left(G\right) &= \sum_{uv\in E(G)} \frac{1}{\left(\delta_u+\delta_v\right)\sqrt{\delta_u^2+\delta_v^2}} \\ &= \frac{6}{\left(1+1\right)\sqrt{1^2+1^2}} + \frac{12n-12}{\left(1+2\right)\sqrt{1^2+2^2}} + \frac{9n^2-15n+6}{\left(2+2\right)\sqrt{2^2+2^2}} \\ &= \frac{6}{2\sqrt{2}} + \frac{12n-12}{3\sqrt{5}} + \frac{9n^2-15n+6}{8\sqrt{2}}. \end{aligned}$$

□

Theorem 3.4. *Let G be the graph of a honeycomb network HC_n. Then*

$$ {}^{m}E\delta\left(G,x\right) = 6nx^{\frac{1}{2\sqrt{2}}} + \left(18n^2+6n\right) x^{x^{\frac{1}{5\sqrt{17}}}} + + \left(18n^2-12n\right) x^{x^{\frac{1}{32\sqrt{2}}}}. $$

Proof. From definition and by using Table 2, we deduce

$$ \begin{aligned} {}^{m}E\delta\left(G,x\right) &= \sum_{uv\in E(G)} x^{\frac{1}{(\delta_u+\delta_v)\sqrt{\delta_u^2+\delta_v^2}}} \\ &= 6x^{\frac{1}{(1+1)\sqrt{1^2+1^2}}} + (12n-12)\,x^{\frac{1}{(1+2)\sqrt{1^2+2^2}}} + \left(9n^2-15n+6\right) x^{\frac{1}{(2+2)\sqrt{2^2+2^2}}} \\ &= 6x^{\frac{1}{2\sqrt{2}}} + (12n-12)\,x^{\frac{1}{3\sqrt{5}}} + \left(9n^2-15n+6\right) x^{\frac{1}{8\sqrt{2}}}. \end{aligned} $$

□

4. Results for Oxide Networks

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by of OX_n. An oxide network of dimension five is shown in Figure 3.

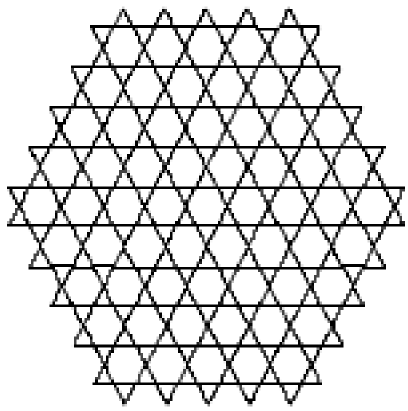

Figure 3: Oxide network of dimension 5

Let G be the graph of an oxide network OX_n. We find that G has $9n^2+3n$ vertices and $18n^2$ edges. In G, there are two types of edges based on degrees of end vertices of each edge as follows:

$$ E_1 = \{uv \in E(G) | d_G(u) = 2, d_G(v) = 4\}, \quad |E_1| = 12n. $$

$$ E_2 = \{uv \in E(G) | d_G(u) = d_G(v) = 4\}, \quad |E_2| = 18n^2 - 12n. $$

We have $\delta(G) = 2$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 1$. Thus there are two types of δ-vertices as given in Table 4.

$\delta_u, \delta_v \setminus uv \in E(G)$	(1, 3)	(3, 3)
Number of edges	6n	$9n^2-3n$

Table 3: δ-vertex partition of OX_n

Theorem 4.1. *Let G be the graph of an oxide network OX_n. Then*

$$E\delta(G) = 162\sqrt{2}n^2 + \left(24\sqrt{10} - 54\sqrt{2}\right) n.$$

Proof. From definition and by using Table 3, we deduce

$$\begin{aligned} E\delta(G) &= \sum_{uv \in E(G)} (\delta_u + \delta_v) \sqrt{\delta_u^2 + \delta_v^2} \\ &= 6n(1+3)\sqrt{1^2+3^2} + \left(9n^2 - 3n\right)(3+3)\sqrt{3^2+3^2} \\ &= 162\sqrt{2}n^2 + \left(24\sqrt{10} - 54\sqrt{2}\right) n. \end{aligned}$$

□

Theorem 4.2. *Let G be the graph of an oxide network OX_n. Then*

$$E\delta(G, x) = 6nx^{4\sqrt{10}} + \left(9n^2 - 3n\right) x^{18\sqrt{2}}.$$

Proof. From definition and by using Table 3, we derive

$$\begin{aligned} E\delta(G, x) &= \sum_{uv \in E(G)} x^{(\delta_u + \delta_v)\sqrt{\delta_u^2 + \delta_v^2}} \\ &= 6nx^{(1+3)\sqrt{1^2+3^2}} + \left(9n^2 - 3n\right) x^{(3+3)\sqrt{3^2+3^2}} \\ &= 6nx^{4\sqrt{10}} + \left(9n^2 - 3n\right) x^{18\sqrt{2}}. \end{aligned}$$

□

Theorem 4.3. *Let G be the graph of an oxide network OX_n. Then*

$${}^{m}E\delta(G) = \frac{6n}{4\sqrt{10}} + \frac{9n^2 - 3n}{18\sqrt{2}}.$$

Proof. From definition and by using Table 3, we obtain

$$\begin{aligned} {}^{m}E\delta(G) &= \sum_{uv \in E(G)} \frac{1}{(\delta_u + \delta_v)\sqrt{\delta_u^2 + \delta_v^2}} \\ &= \frac{6n}{(1+3)\sqrt{1^2+3^2}} + \frac{9n^2 - 3n}{(3+3)\sqrt{3^2+3^2}} \\ &= \frac{6n}{4\sqrt{10}} + \frac{9n^2 - 3n}{18\sqrt{2}}. \end{aligned}$$

□

Theorem 4.4. *Let G be the graph of an oxide network OX_n. Then*

$${}^{m}E\delta(G, x) = 6nx^{\frac{1}{4\sqrt{10}}} + \left(9n^2 - 3n\right) x^{\frac{1}{18\sqrt{2}}}.$$

Proof. From definition and by using Table 3, we deduce

$$
\begin{aligned}
{}^{m}E\delta\left(G,x\right) &= \sum_{uv\in E(G)} x^{\frac{1}{(\delta_u+\delta_v)\sqrt{\delta_u^2+\delta_v^2}}} \\
&= 6nx^{\frac{1}{(1+3)\sqrt{1^2+3^2}}} + \left(9n^2-3n\right)x^{\frac{1}{(3+3)\sqrt{3^2+3^2}}} \\
&= 6nx^{\frac{1}{4\sqrt{10}}} + \left(9n^2-3n\right)x^{\frac{1}{18\sqrt{2}}}.
\end{aligned}
$$

□

5. Results for Hexagonal Networks

It is known that there exist three regular plane tilings with composition of some kind of regular polygons such as triangular, hexagonal and square. Triangular tiling is used in the construction of hexagonal networks. This network is denoted by HX_n. A hexagonal network of dimension six is shown in Figure 4.

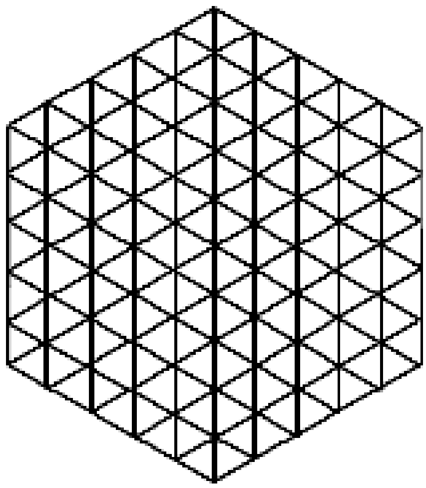

Figure 4: Hexagonal network of dimension six

Let G be the graph of a hexagonal network HX_n. We obtain that G has $3n^2-3n+1$ vertices and $9n^2-15n+6$ edges. In G, there are five types of edges based on degrees of end vertices of each edge as follows:

$$
\begin{aligned}
E_1 &= \{uv \in E(G) | d_G(u)=3, d_G(v)=4\}, && |E_1| = 12. \\
E_2 &= \{uv \in E(G) | d_G(u)=3, d_G(v)=6\}, && |E_2| = 6. \\
E_3 &= \{uv \in E(G) | d_G(u)=d_G(v)=4\}, && |E_3| = 6n-18. \\
E_4 &= \{uv \in E(G) | d_G(u)=4, d_G(v)=6\}, && |E_4| = 12n-24. \\
E_5 &= \{uv \in E(G) | d_G(u)=d_G(v)=6\}, && |E_5| = 9n^2-33n+30.
\end{aligned}
$$

Thus $\delta(G)=3$ and hence $\delta_u = d_G(u)-\delta(G)+1 = d_G(u)-2$. There are five types of δ-edges as given in Table 4.

$\delta_u, \delta_v \setminus uv \in E(G)$	(1, 2)	(1, 4)	(2, 2)	(2, 4)	(4, 4)
Number of edges	12	6	$6n-18$	$12n-24$	$9n^2-33n+30$

Table 4: δ-edge partition of HX_n

Theorem 5.1. *Let G be the graph of a hexagonal network HX_n. Then*

$$E\delta(G) = 288\sqrt{2}n^2 + \left(144\sqrt{5} - 1008\sqrt{2}\right)n + 30\sqrt{17} - 252\sqrt{5} + 816\sqrt{2}.$$

Proof. From definition and by using Table 3, we deduce

$$\begin{aligned}
E\delta(G) &= \sum_{uv\in E(G)} (\delta_u+\delta_v)\sqrt{\delta_u^2+\delta_v^2} \\
&= 12(1+2)\sqrt{1^2+2^2} + 6(1+4)\sqrt{1^2+4^2} + (6n-18)(2+2)\sqrt{2^2+2^2} \\
&+ (12n-24)(2+4)\sqrt{2^2+4^2} + \left(9n^2-33n+30\right)(4+4)\sqrt{4^2+4^2} \\
&= 288\sqrt{2}n^2 + \left(144\sqrt{5} - 1008\sqrt{2}\right)n + 30\sqrt{17} - 252\sqrt{5} + 816\sqrt{2}.
\end{aligned}$$

□

Theorem 5.2. *Let G be the graph of a hexagonal network HX_n. Then*

$$E\delta(G,x) = 12x^{3\sqrt{5}} + 6x^{5\sqrt{17}} + (6n-18)x^{8\sqrt{2}} + (12n-24)x^{12\sqrt{5}} + \left(9n^2-33n+30\right)x^{32\sqrt{2}}.$$

Proof. From definition and by using Table 4, we derive

$$\begin{aligned}
E\delta(G,x) &= \sum_{uv\in E(G)} x^{(\delta_u+\delta_v)\sqrt{\delta_u^2+\delta_v^2}} \\
&= 12x^{(1+2)\sqrt{1^2+2^2}} + 6x^{(1+4)\sqrt{1^2+4^2}} + (6n-18)x^{(2+2)\sqrt{2^2+2^2}} \\
&+ (12n-24)x^{(2+4)\sqrt{2^2+4^2}} + \left(9n^2-33n+30\right)x^{(4+4)\sqrt{4^2+4^2}} \\
&= 12x^{3\sqrt{5}} + 6x^{5\sqrt{17}} + (6n-18)x^{8\sqrt{2}} + (12n-24)x^{12\sqrt{5}} + \left(9n^2-33n+30\right)x^{32\sqrt{2}}.
\end{aligned}$$

□

Theorem 5.3. *Let G be the graph of a hexagonal network HX_n. Then*

$$^mE\delta(G) = \frac{12}{3\sqrt{5}} + \frac{6}{5\sqrt{17}} + \frac{6n-18}{8\sqrt{2}} + \frac{12n-24}{12\sqrt{5}} + \frac{9n^2-33n+30}{32\sqrt{2}}.$$

Proof. From definition and by using Table 4, we obtain

$$\begin{aligned}
^mE\delta(G) &= \sum_{uv\in E(G)} \frac{1}{(\delta_u+\delta_v)\sqrt{\delta_u^2+\delta_v^2}} \\
&= \frac{12}{(1+2)\sqrt{1^2+2^2}} + \frac{6}{(1+4)\sqrt{1^2+4^2}} + \frac{6n-18}{(2+2)\sqrt{2^2+2^2}} \\
&+ \frac{12n-24}{(2+4)\sqrt{2^2+4^2}} + \frac{9n^2-33n+30}{(4+4)\sqrt{4^2+4^2}}
\end{aligned}$$

$$= \frac{12}{3\sqrt{5}} + \frac{6}{5\sqrt{17}} + \frac{6n-18}{8\sqrt{2}} + \frac{12n-24}{12\sqrt{5}} + \frac{9n^2-33n+30}{32\sqrt{2}}.$$

□

Theorem 5.4. *Let G be the graph of a hexagonal network HX_n. Then*

$$^{m}E\delta\left(G,x\right) = 12x^{\frac{1}{3\sqrt{5}}} + 6x^{\frac{1}{5\sqrt{17}}} + (6n-18)\,x^{\frac{1}{8\sqrt{2}}} + (12n-24)\,x^{\frac{1}{12\sqrt{5}}} + \left(9n^2-33n+30\right)x^{\frac{1}{32\sqrt{2}}}.$$

Proof. From definition and by using Table 4, we deduce

$$\begin{aligned} ^{m}E\delta\left(G,x\right) &= \sum_{uv\in E(G)} x^{\frac{1}{(\delta_u+\delta_v)\sqrt{\delta_u^2+\delta_v^2}}} \\ &= 12x^{\frac{1}{(1+2)\sqrt{1^2+2^2}}} + 6x^{\frac{1}{(1+4)\sqrt{1^2+4^2}}} + (6n-18)\,x^{\frac{1}{(2+2)\sqrt{2^2+2^2}}} \\ &+ (12n-24)\,x^{\frac{1}{(2+4)\sqrt{2^2+4^2}}} + \left(9n^2-33n+30\right)x^{\frac{1}{(4+4)\sqrt{4^2+4^2}}} \\ &= 12x^{\frac{1}{3\sqrt{5}}} + 6x^{\frac{1}{5\sqrt{17}}} + (6n-18)\,x^{\frac{1}{8\sqrt{2}}} + (12n-24)\,x^{\frac{1}{12\sqrt{5}}} + \left(9n^2-33n+30\right)x^{\frac{1}{32\sqrt{2}}}. \end{aligned}$$

□

6. Conclusion

In this work, we have determined the elliptic delta and modified elliptic delta indices and their corresponding exponentials for certain families of networks.

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