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Numerical Study of Mixed Convective Heat Transfer Over a Moving Wedge with Slip Velocity Effects

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Abstract: We study the combined effects of pressure gradient, the slip velocity and viscous dissipation on the laminar and mixed convective boundary layer flow associated with heat transfer over a moving wedge. Both mainstream flow and wedge velocity are taken to be as a power of distance along the wedge wall. The modeled boundary layer equations are reduced upon using the suitable similarity transformations to a set of ordinary differential equations. These governing equations are solved numerically using shooting method. The physical quantities such as the momentum velocity, temperature profiles, skin-friction coefficient and Nusselt number are determined and discussed. The numerical simulations show that both pressure gradient, slip velocity and Eckert number are significantly affect the boundary layer thickness, velocity and temperature profiles. These model parameters are evidently enhance the rate of heat transfer. Interesting physical mechanisms are discussed in detail.

MSC: 76D10, 80A21.

Keywords: Mixed convection, Boundary layer flows, Slip velocity, Eckert number, Shooting method.

1. Introduction

The boundary-layer flow over a moving wedge has attracted many researchers due to ample number of applications in industry and technology such as long thread rolling between a wind-up and feed roll, metal spinning, wire drawing, polymer sheets and filaments, and also helps understanding aerodynamical problems such as drag, skin-frictions. such as cooling of an infinite metallic plate in a cooling bath, the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation process. In most of these applications, it is required that the mechanical property of the end product would entirely depend on the stretching/moving rate. The boundary layer flow problems would give satisfactory results for these applications which are based on the boundary layer solutions. Numerous research investigations are available in the literature on solutions of boundary layer flow problems [1–5].

The thermal boundary layers are often found in the vicinity of the surface where the fluid temperature varies from the temperature of the stationary/moving wedge to that of temperature of the mainstream flow. This phenomena is usually found to occur through conduction or convection, and the latter involves either natural or forced convection. In a natural convection boundary layer problems, the buoyancy forces are expected to make the fluid motion because of the predominant temperature difference between the fluid flow and the wedge [6, 7]. However, the forced convection generally occurs due to external forces. An excellent review on these problems can be found in Schlichting and Gersten [8] and Hussein *et al.* [9]. On the other side, the mixed convection heat transfer is seen to occur when flow of the fluid near a wedge is influenced by

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mixture of natural and forced convection which is found geophysical and environmental heat transfer, electronics cooling devices, heat exchangers, solar energy systems, etc. Because of these ample number of applications numerous studies have taken place in the literature. The mixed convective boundary layers flow due to the power-law stretching sheet is investigated numerically using finite difference method, and shown that there is a substantial heat transfer for nonlinear stretching sheet since the thickness of the thermal boundary layer is thin (Chen [10]). Chamkha *et al.* [11] have explored a steady dissipative layer, which is known as Marangoni mixed convection boundary layer, would form in surface driven flows along the interface of two immiscible fluids. Harris *et al.* [12] have studied a steady laminar stagnation boundary layer flow on an impermeable vertical surface which has slip velocity embedded in a porous medium, and shown that there are at least two solutions for the boundary layer flow for all mixed convection parameters, stability analysis is used to predict which of the two solutions is practically possible. Rahman *et al.* [13] have examined mixed convection boundary layer flow on a vertical surface while taking appropriate forms of mainstream flow are considered in order to obtain the governing equations. It is shown that there are at least dual solutions for each set of values. Ghalambaz *et al.*[14] have studied the mixed convection boundary layer flow and heat transfer of Al2O3-Cu/water hybrid nanofluid over a vertical plate and shown numerically that the suitable composition of nanoparticles can significantly affect the boundary layer flow and heat transfer.

It is worth mentioning that the boundary layer flows are usually utilized the no-slip velocity boundary conditions in most of the applications in which are found at a solid-fluid interface. However, the fluid velocity will certainly be different at least at pore level from that of classical fluid flow, i.e., the fluid slips on the solid boundary. To reduce the frictional drag drastically or at least moderately, strategies such as flow additives, blowing of the fluid, plasma fields, polymers, surfactants, are made use in the past ([15–18]. The effects of slip velocity on the flat boundary layer flow with the slip boundary conditions that have either the slip length, or the slip velocity or the slip length as a function of shear rate, and the correspond results show that in these have the boundary layer thickness thinner for no-slip condition (Vedantam and Parthasarathy [17]). This study was later extended with heat transfer which varies downstream, and shown that inclusion of slip velocity in the study reduces the surface temperature (Aziz [7]). Khader and Megahed [19] have illustrated numerically that the fluid flow over a impermeable and with inordinate stretching of the sheet which has the slip velocity, and have shown that that the momentum in the boundary layer decreasing near the boundary for increased slip-velocity.

In the present study, we study the combined influences of pressure gradient and slip velocity on the boundary layer flow and associated heat transfer over a moving wedge. The mixed convection and viscous dissipation effects are also taken into study. No such investigation on the present model is available in the literature. We use the numerical shooting technique for the solution of the governing equation and to analyse the simulations.

The present paper is organized in a following manner. The conservation mass, momentum and energy equations which reduce to the momentum and thermal boundary-layer model are given in section §2. This section also extends to include the stream function and similarity transformations. Section §3 devotes to use numerical scheme to solve the governing equations. In section §4, the various results are simulated and analysed in terms of figures and tables. Section §5 presents the details of gradients of velocity and temperature on the wall. Final section (§6) summarizes findings of the present investigation.

2. Mathematical Formulation

Two-dimensional laminar flow of a viscous and incompressible fluid over a moving wedge by incorporating the effects of slip velocity and viscous dissipation is investigated. It is also assumed that the slip velocity which refers to the relative velocity between a moving fluid and a wedge boundary is considered. Note that the wedge surface has a linear slip-velocity. The Cartesian coordinates are used in which the x axis is taken along the direction of the flow and y is normal to the flow. The wedge surface is assumed to have the temperature T_w and mainstream temperature T_∞ with a condition that $T_w \gg T_\infty$. The governing equations of the model are the conservation of mass, momentum and energy equations

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\rho(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} - \rho \mathbf{g} \tag{2}$$

$$(\mathbf{u} \cdot \nabla)T = \alpha_1 \nabla^2 T + V_d \tag{3}$$

where $\mathbf{u}$ is an intrinsic velocity of the fluid, ρ is the fluid density, p is the pressure, μ is the fluid viscosity, $\mathbf{g}$ is the gravitational acceleration, T is the fluid temperature, $\alpha_1 = \frac{\kappa}{\rho c_p}$ is thermal diffusivity with c_p being the specific heat at constant pressure and κ is thermal conductivity, and V_d is the viscous dissipation in the model. It is considered that the Reynolds number of the fluid is asymptotically large so that both the momentum and thermal boundary layers form near the moving wedge surface. In that the momentum boundary layer in which the viscous effects are predominant in the vicinity of the moving wedge surface while away from it their effects gradually decrease to zero.

Using the usual boundary-layer approximations that are applicable to the above mentioned model, the system (1)-(3) takes the following form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - g \tag{5}$$

$$0 = \frac{\partial p}{\partial y} \tag{6}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + V_d. \tag{7}$$

Here, the velocity components $u,\ v$ are taken along x and y directions, $\nu(= \frac{\mu}{\rho})$ is the kinematics viscosity of the fluid, g is the magnitude of gravity force in the negative x direction. The viscous dissipation V_d is defined as $V_d = \frac{\nu}{c_p}(\frac{\partial u}{\partial y})^2$. Note that the velocity in the boundary layer is expected to approach the mainstream flow at the edge of the boundary layer that is $u(x, y) = U(x)$. Accordingly, the pressure variation normal to the wedge is treated as constant and whence obtain

$$U\frac{dU}{dx} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\rho_\infty}{\rho}g \tag{8}$$

where $U(x)$ is a mainstream velocity away from the boundary layer, ρ_∞ is the density in the main stream. We further approximate that the body force $g(\rho_\infty - \rho)$ for the present study on mixed convective heat transfer can be taken as

$$g_x(\rho_\infty - \rho) = g_x T_c \rho(T - T_\infty) \tag{9}$$

where T_c is the thermal expansion coefficient. It is approximated that the mainstream flow has a variation in power of distance from the leading edge i.e. $U(x) = U_\infty x^m$ where U_∞ is constant and positive always and m is related to imposed pressure gradient. Thus, equation (5) may be rewritten in view of (8)-(9) as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty^2 x^{2m-1} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{(\rho_\infty - \rho)}{\rho}g. \tag{10}$$

To render the physical solution for the above mode, we define the relevant boundary conditions in the following manner

$$u = U_w + \mu S\frac{\partial u}{\partial y}, \quad v = 0, \quad T = T_w, \qquad \text{at } y = 0, \tag{11}$$

$$u = U, \quad T = T_\infty, \quad \text{as } y \to \infty \tag{12}$$

where U_w is the velocity of the wedge along/opposite to the mainstream flow which is approximated as a power of distance downstream i.e. $U_w(x) = U_0 x^m$ with U_0 is either positive or negative depending on whether the wedge is moving either along or opposite to the mainstream flow. The parameter S is the slip velocity which is relative velocity between the wedge and fluid. The boundary conditions characterize the flow velocity u and temperature T to decay in the asymptotic manner onto the mainstream velocity U and temperature T_∞. We assume that the temperature varies from wedge temperature to the mainstream in the manner $T_w = T_\infty + Ax^N$ where A is positive constant and N is the wall exponent parameter.
In order to obtain the physically meaningful solutions, we taken the velocity components u and v in the following manner

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{13}$$

which are found to satisfy the continuity equation, where $\psi(x, y)$ is the stream function. We define the new similarity variables in the following manner

$$\psi = \left(\frac{2\nu x U}{m+1}\right)^{\frac{1}{2}} \phi(\eta), \quad \frac{T - T_\infty}{T_w - T_\infty} = \theta(\eta), \quad \eta = \left(\frac{(m+1)U}{2\nu x}\right)^{\frac{1}{2}} y \tag{14}$$

where $\phi(\eta)$, θ and η are a new set of similarity transformations. Using the similarity transformations (14) in the momentum and energy equation (10) and (7), we get

$$\phi'''(\eta) + \phi(\eta)\phi''(\eta) + \beta(1 - \phi'^2(\eta)) + \alpha\theta(\eta) = 0 \tag{15a}$$

$$\theta''(\eta) + Pr\phi(\eta)\theta'(\eta) - (2 - \beta)PrN\phi'(\eta)\theta(\eta) + PrEc\phi''^2(\eta) = 0 \tag{15b}$$

and transformed boundary conditions (11)-(12) take the form

$$\phi(0) = 0, \quad \phi'(0) = \lambda + K\phi''(0), \quad \theta(0) = 1 \tag{16a}$$

$$\phi'(\infty) = 1, \quad \theta(\infty) = 0 \tag{16b}$$

where $\phi = \phi(\eta)$ and $\theta = \theta(\eta)$ are the non-dimensional stream functions and primes denote differentiation with respect to η. When $\alpha = 0$ and $K = 0$, the most celebrated Falkner-Skan boundary layer flow over a moving wedge is recovered ([3, 8]). The various parameters in the above system are given below:

1. The parameter $\beta = 2m/(m+1)$ is the pressure gradient in that $\beta < 0(> 0)$ is a decelerated (accelerated) pressure gradient and $\beta = 0$ gives the flow over a flat plate.

2. The variable $\alpha = \frac{Gr}{Re_x^2}$ is mixed convection parameter, $Gr = \frac{g_x T_c (T_w - T_\infty) L^3}{\nu^2}$ is the Grashoff number and $Re_x = \frac{UL}{\nu}$ is the local Reynolds number. Note that the cases corresponding to $\alpha < 0$ and $\alpha > 0$ are termed as the opposing and assisted flow, while $\alpha = 0$ is the case for forced convective heat transfer.

3. The Prandtl number $Pr = \frac{\nu}{\alpha}$ infers for $Pr < 1$ and $Pr > 1$ as the thickness of the thermal boundary layer is larger and smaller compared to the thickness of the boundary layer whereas $Pr = 1$ is the case for identical boundary layers.

4. λ is defined as the ratio of wedge velocity to the mainstream flow velocity.

5. The slip velocity $K = S\mu\sqrt{\frac{U_\infty}{\nu}}$ describes the relative importance between the magnitude of the slip velocity at the fluid-surface interface and mainstream velocity of the fluid away from the surface.

6. The Eckert number $Ec = \frac{U^2}{c_p(T_w - T_\infty)}$ is defined as the ratio of the kinetic energy of a fluid to its thermal energy. And

7. N is the wall exponent parameter which signifies how the heat transfer coefficient varies with distance from the wedge wall.

System (15) - (16a) is highly non-linear coupled ordinary differential equations defined in the boundary layer domain $[0, \infty)$ and hence solved numerically using the shooting method since any analytical solution is almost impossible.

3. Numerical Procedure

We numerically simulate the physically genuine solutions of the model under consideration using shooting method. The shooting method is the hybrid method which is a combination of Runge-Kutta method and Secant method, and involves the following steps: we first convert the boundary value problems to the initial value problem by introducing some additional unknown functions. That is

$$\phi'(\eta) = \phi_1(\eta), \qquad \phi_1'(\eta) = \phi_2(\eta), \qquad \theta'(\eta) = \theta_1(\eta) \tag{17}$$

$$\phi_2'(\eta) = -\phi(\eta)\phi_2(\eta) - \beta(1 - \phi_1^2(\eta)) \tag{18}$$

$$\theta_1'(\eta) = -\Pr\phi(\eta)\theta_1(\eta) + (2-\beta)PrN\phi_1(\eta)\theta - PrEc\phi_2^2(\eta) \tag{19}$$

along with the converted boundary conditions

$$\phi(0) = 0, \quad \phi_1(0) = \lambda + K\phi_1'(0), \quad \theta(0) = 0, \quad \phi_1(\infty) = 1, \quad \theta(\infty) = 0 \tag{20}$$

We then integrate the above converted system using the Runge-Kutta method from the wedge surface by assuming initial guesses for $\phi_2(0)$ and $\theta_1(0)$ until the edge of boundary layer. The secant method procedure supplies the significant initial guess to the wall-shear stress $\phi_2(0)$ and temperature gradient $\theta_1(0)$. The required solutions for $\phi_2(0)$ and $\theta_1(0)$ on the wedge surface are accelerated using standard secant method. The shooting method is stopped when the difference between two consecutive solutions are identical up to pre-specified error tolerance which is taken 10^{-5} in the present studies. At this convergence, the physically genuine profiles are obtained which are discussed in detail in the following sections.

4. Results and discussion

We have investigated the combined effects of the pressure gradient, slip velocity and viscous dissipation on the two-dimensional boundary layer flow and mixed convection heat transfer over a moving wedge numerically. We have obtained the velocity and temperature distribution in the mixed convection boundary-layer flow and effects of different values of mixed convection parameter α, Prandtl number Pr, Eckert number Ec, velocity ratio parameter λ and the momentum slip parameter K. Using the similarity variables, the thermal and momentum boundary layer equations (7) and (10) have been transformed into a system of ordinary differential equations with appropriate boundary conditions. The accuracy and grid independence test of numerical scheme are checked repeatedly and confirmed for various parameters. The numerical solutions produce physical solutions for emerging parameters for the dimensionless velocity $\phi(\eta)$, temperature $\theta(\eta)$, skin-friction $\phi''(0)$ and temperature gradient $\theta'(0)$.

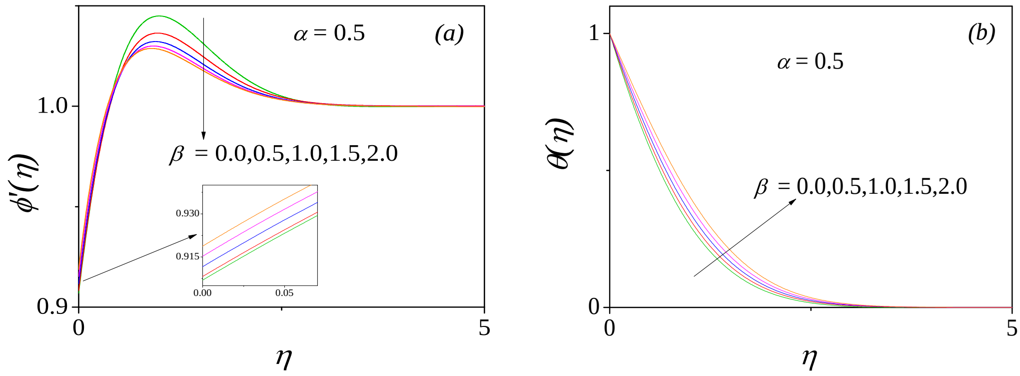

Figure 1. The variation of (a) the velocity and (b) the temperature profiles with η for different values of β with $\alpha = 0.5$, $K = 1.2$, $\lambda = 0.5$, $Ec = 0.5$, $Pr = 0.7$ and $N = 0.4$.

We first discuss the effect of pressure gradient β that plays a major role in shaping the boundary layer around a moving wedge for assisted flow $\alpha > 0$. For increasing values of β, as shown in figure 1a, favorable pressure gradient for which pressure in the direction of flow decreases makes the boundary layer attached to the wedge that promotes benign flow. Also, a favorable pressure gradient parameter, the boundary layer thickness, as expected, is thin since the flow for $\beta > 0$ gradually accelerates along the wedge surface by reducing viscous effects. Interestingly, for $\beta > 0$, all velocity profiles have an overshoot ($\phi'(\eta) > 1$ for some η near the surface) character which is not usually seen in the forced convection heat transfer investigations (Riley and Weidman [3, 5, 20]). On the other hand, the accelerated pressure gradient also has a notable influence on the thermal boundary layer which is expected to vary from the wedge temperature to mainstream temperature wherein a significant variation is seen near the surface. For favorable pressure gradient $\beta > 0$, the thermal boundary layer is found to be thin due to fluid flow accelerates smoothly along the wedge surface which results into enhancing heat transfer rate. Although there is a overshoot in momentum velocity but its effects are not affecting thermal boundary layer.

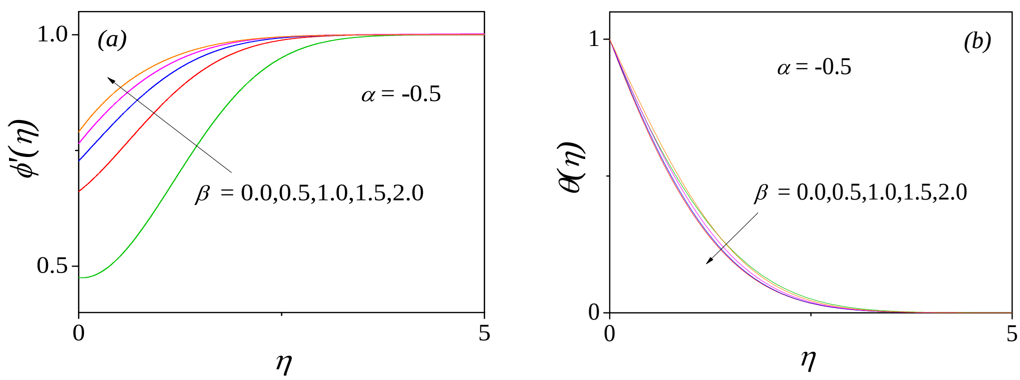

Figure 2. The variation of (a) the velocity and (b) thermal profiles with η for different values of β with $\alpha = -0.5$, $K = 1.2$, $\lambda = 0.5$, $Ec = 0.5$, $Pr = 0.7$ and $N = 0.4$.

For an opposing flow $\alpha < 0$, figures 2a-2b have produced distinctive velocity profiles showing the influence of pressure gradient simultaneously along with slip velocity effects on the boundary layers. Due to the presence of slip velocity, the origin of profiles is not unique hence start at different positions of the wedge but eventually tend to edge boundary condition. There is an evident difference in the velocity for each β varied from $\beta = 0$ case. The temperature profiles for opposing flow have a minimum influence for increasing pressure gradient, as the thermal boundary layer thickness is found thicker. Very similar temperature profiles have been obtained by Turkyilmazoglu [21].

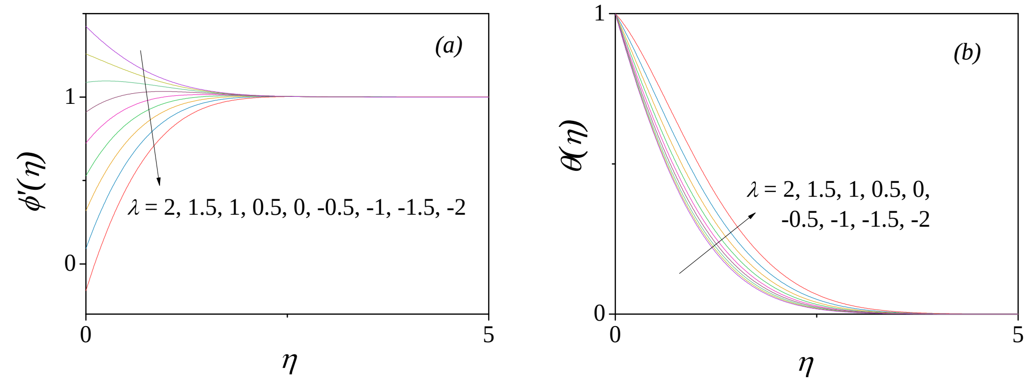

Figure 3. The variation of (a) the velocity and (b) temperature profiles with η for different values of velocity ratio parameter λ with $\alpha = 0.5$, $K = 1.2$, $\beta = 0.8$, $Ec = 0.5$, $Pr = 0.7$ and $N = 0.4$.

We continue to discuss the effects of velocity ratio parameter λ on both boundary layers, and the corresponding results are shown in figures 3a-3b. Note that when $\lambda > 1$ and $\lambda < 1$, the boundary layer characteristics are expected to change in that for the former case, the wedge velocity is faster than the mainstream flow velocity, while the contrary happened for latter case. When $\lambda = 1$, both velocities are equal hence no boundary layer forms over the wedge and thermal boundary layer as well. These results are shown in the figures. Nevertheless, all the velocity profiles eventually decay to the mainstream boundary condition. Similar trend is also shown for temperature profiles in which thermal boundary layer thickness is found to be thicker for increasing λ. The effect of Grashof number (mixed convection parameter) on the boundary layers is studied by emphasizing both assisted and opposing flows. In order to show these effects, figures 4a-4b are shown to produce both velocity and temperature profiles keeping other parameters fixed. For increasing α results in increasing fluid velocity. This is because when $\alpha > 0$ induces a favorable pressure gradient that enhances the fluid flow in the boundary layer. For increasing α as shown in figure 4a, there is variation close proximity to the wedge surface enhancing the velocity of the moving fluid but when the velocity of fluid in terms of slip velocity is seen to cross the value, the the velocity curves have a tendency of producing overshoots ($\phi'(\eta) > 1$ for some range of η near the surface but in the end all the curves approach the end condition $\phi'(\infty) \to 1$. On the other side, figure 4b, there is a gradual variation in temperature curves when α is decreased, affecting heat transfer rate, but cannot be neglected. The velocity and temperature curves for varying Grashof number have a typical characteristic of thickening the boundary layer thickness compared to the case $\alpha = 0$.

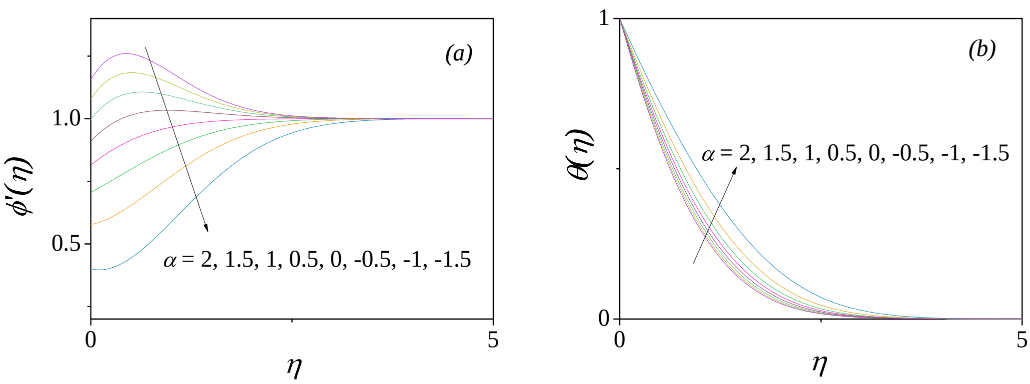

Figure 4. The variation of (a) the velocity and (b) the temperature profiles with η for different values of α with $\beta = 0.8$, $K = 1.2$, $\lambda = 0.5$, $Ec = 0.5$, $Pr = 0.7$ and $N = 0.4$.

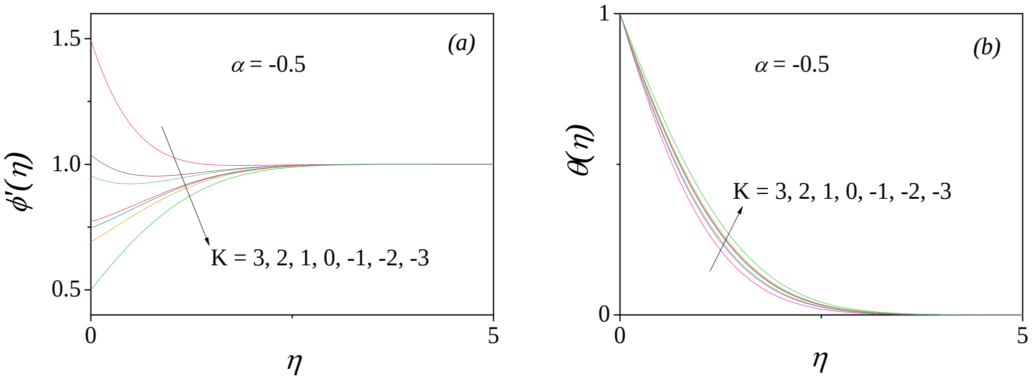

Figure 5. The velocity and thermal graphs for different values of K with $\beta = 0.6$, $\lambda = 0.5$, $\alpha = -0.5$, $Ec = 0.5$, $Pr = 0.7$ and $N = 0.4$.

We discuss the influence of slip velocity boundary condition on the boundary layer flow and associated heat transfer rate. In figures 5a-5b, the slip velocity K in the range from -3 to 3 for both the velocity and thermal profiles are shown at $\alpha = -0.5$. The no-slip condition is recovered when $K = 0$. It is shown that though the velocity curves originate at different starting points because of the boundary conditions given in (16b), they decay to their mainstream conditions. For increasing K, the thickness of the boundary layer near to the surface is evidently altered. This seems to make boundary layer thinner due to the fluid near the moving wedge is found to move faster since the viscous effects are found predominant.

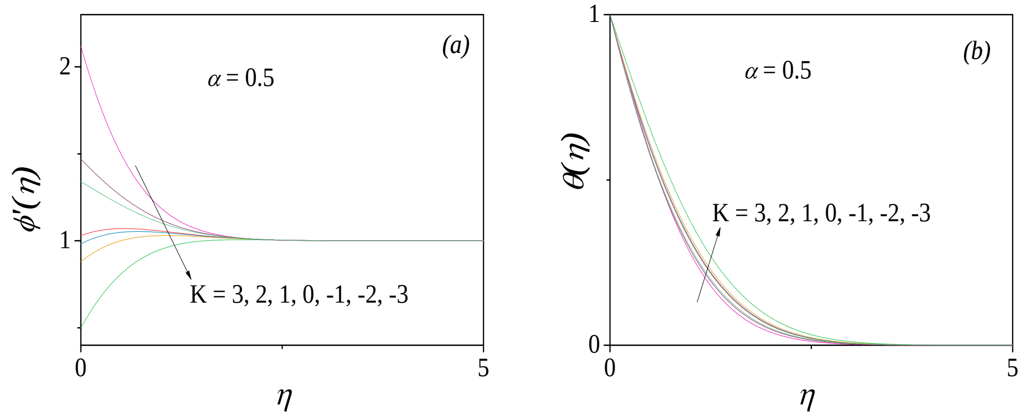

Figure 6. The velocity and thermal graphs for different values of K at $\alpha = 0.5$, with $\beta = 0.6$, $\lambda = 0.5$, $Ec = 0.5$, $Pr = 0.7$ and $N = 0.4$.

Moreover, an evident difference between velocity on the moving surface and the boundary layer edge is found decreasing for increasing K and this is due to the slip velocity. Further, the temperature of the fluid depends on the velocity of the fluid which influenced by the slip velocity, accordingly thermal boundary layer is noticed to be affected. There is an enhanced rate of heat transfer due to the thinner thermal boundary layer and altered velocity contribution for increased K as shown in figure 5b. Similar scenario of the velocity and temperature profiles are seen as shown in figures 6a -6b for an assisted flow when $\alpha = 0.5$ for which the thickness of boundary layer is rather thick, however the physical mechanisms for this chosen α are remain same.

We also discuss the effects of the Prandtl number on the moving wedge with regard to slip velocity on the surface. Note that due to the mixed convection or Grashof number, the Prandtl number affects both momentum and thermal boundary layers as depicted in figures 7a-7b. The Prandtl number exhibits the relative difference in thickness of thermal and momentum boundary layers which affects the heat transfer significantly. The results for $Pr > 1$ have different characteristics than those of $Pr < 1$ for which the momentum boundary layer is smaller than the thermal boundary layer and the contrast is found for $Pr < 1$. Both figures reveal that that for when Prandtl number is decreased, the velocity profiles have different structure in that the thickness of the momentum boundary layer is decreasing which is due to an increase in fluid velocity and decrease in the temperature. Exactly an opposite scenario is shown in figure 7b for increasing Pr. These profiles are seen to exhibit thicker thermal boundary layer thickness for increasing Pr values which results in low convective heat transfer rate.

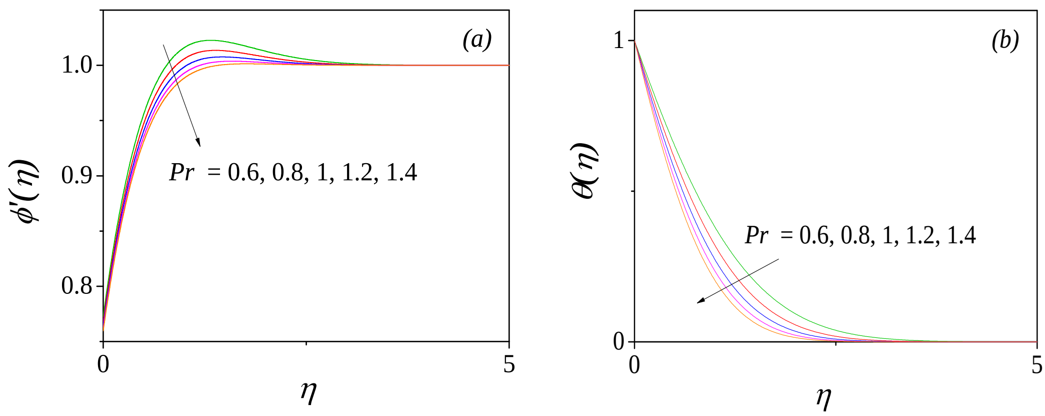

Figure 7. The variation of (a) velocity and the (b) thermal graphs for different values of Pr with $\beta = 0.6$, $\lambda = 0.5$, $\alpha = 0.5$, $Ec = 0.5$, $K = 0.5$ and $N = 0.4$.

We also continue to investigate the effects of the Eckert number which characterizes the role of viscous dissipation in the thermal boundary layer. The Eckert number particularly affects the heat transfer in the boundary layer due to the interchange of energy between the fluid flow and the wedge surface. In order to understand the significance of viscous dissipation which leads to an increased internal energy, we need to choose the values of $Ec \in [0, 4]$. The corresponding results are shown in terms of the velocity and temperature profiles in figures 8a-8b. The velocity of the fluid experiences the overshoot near the moving wedge and after attaining it starts to merge into the mainstream velocity. The momentum velocity $\phi'(\eta)$ and the temperature $\theta(\eta)$ have, as expected, increase for increasing Ec, and both thicknesses are seen to be thin compared to $Ec = 0$ due to faster interaction between the wedge and fluid. Since, the kinetic energy of the fluid is more for larger Ec, the enhanced heat transfer rate is also expected.

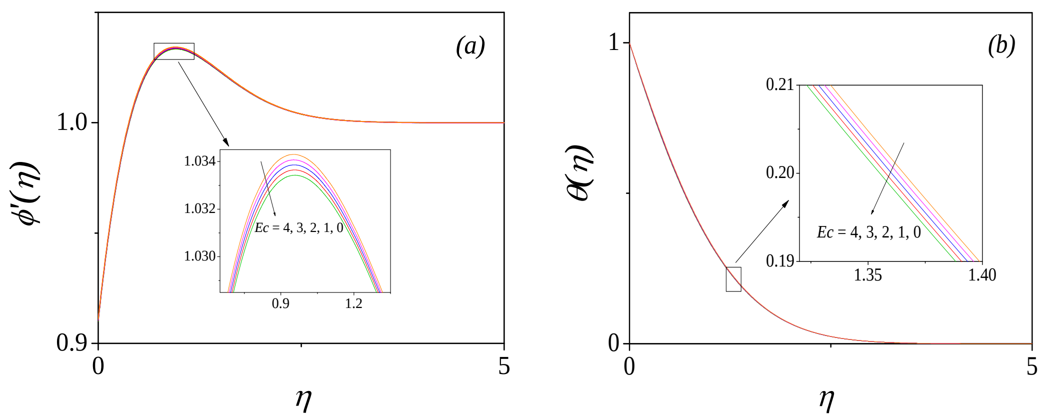

Figure 8. The variation of (a) velocity and (b) thermal graphs for different values of Ec with $\beta = 0.8$, $\lambda = 0.5$, $\alpha = 0.5$, $Pr = 0.7$, $K = 1.2$ and $N = 0.4$.

The velocity profiles, similar to figures 8a-8b, are shown in figures 9a-9b for increased wall exponent parameter. For increasing N values, both thicknesses become thinner leading to benign velocity and enhanced heat transfer. For $N = -1.5$ is rather different in which the velocity and temperature curves have a larger overshoot which is attributed to the presence of slip velocity.

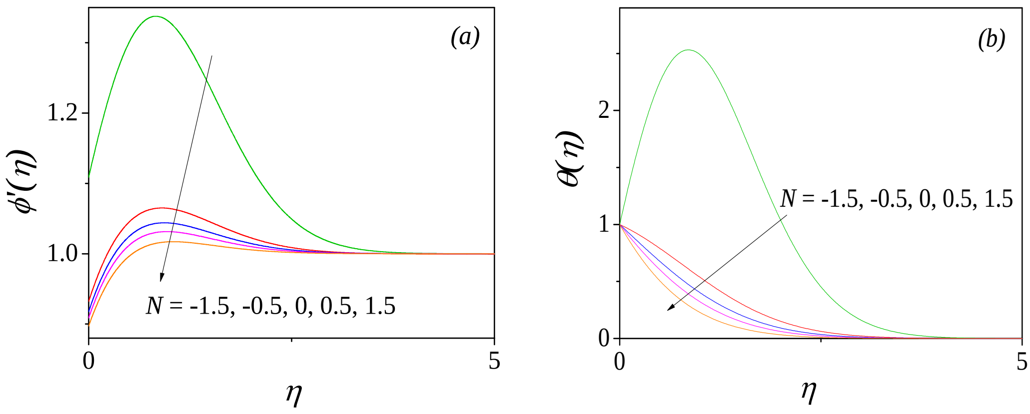

Figure 9. The velocity and thermal graphs for different values of N with $\beta = 0.8$, $\lambda = 0.5$, $\alpha = 0.5$, $Pr = 0.7$, $K = 1.2$ and $Ec = 0.5$.

5. Physical Quantities of Engineering Interest

In what follows below, the broader structure of the nature of slip velocity and pressure gradient can be understood through the wall shear stress or skin-friction-$\phi''(0)$ and temperature gradient $\theta'(0)$ we discuss various results in terms of skin-friction (wall shear stress) $F''(0)$ and the temperature gradient $-\theta'(0)$ which are important for engineering interest and are discussed for pressure gradient and viscous dissipation. Both $\phi''(0)$ and $\theta'(0)$ are determined numerically and presented as a function of slip-velocity parameter.

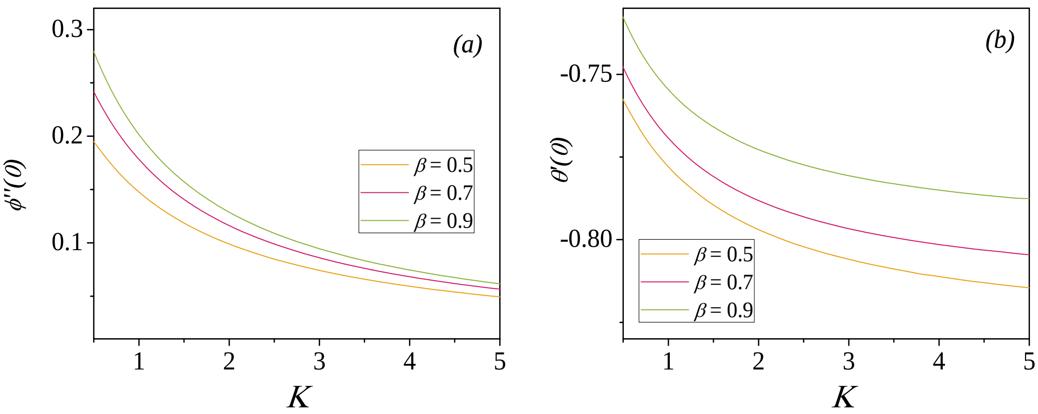

Figure 10. The variation of (a) wall shear stress and (b) temperature gradient for $\alpha = -0.5$, $Ec = 0.5$, $Pr = 0.7$, $N = 0.4$ and $\lambda = 0.5$.

In figures 10a-10b, we demonstrate the gradients of velocity and temperature for various pressure gradient β. Note that for each of these values there are both velocity and temperature variations which have already been discussed in previous figures. For each β considered, both wall shear stress and temperature gradient on the surface are seen to decrease gradually for increasing slip velocity K. In that the case for $\beta = 0.9$ of wall shear stress has a larger variation but decreasing trend is observed, and so is for temperature gradient.

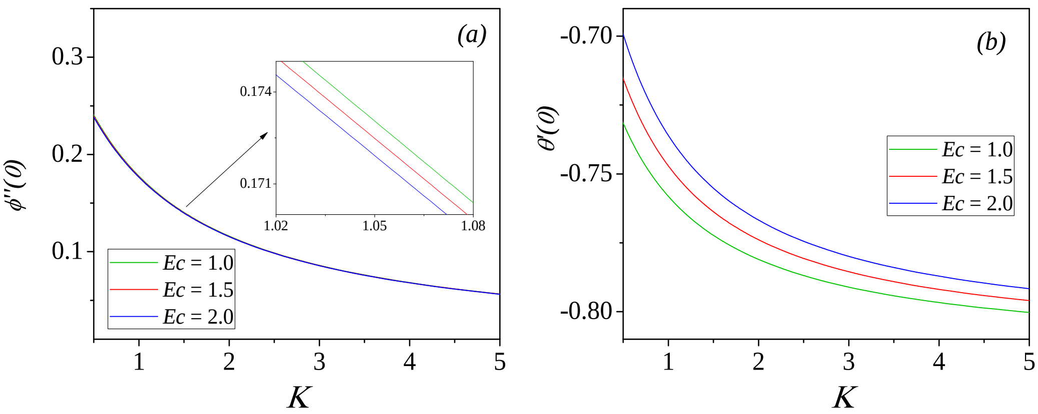

Figure 11. The variation of (a) wall shear stress $\phi''(0)$ and (b) temperature gradient $\theta'(0)$ for $\alpha = -0.5, \beta = 0.7, Pr = 0.7, N = 0.4, \lambda = 0.5$.

The similar but slightly different scenario is expected when plotted for different Eckert number Ec, as shown in figures 11a-11b. In these figures, the viscous dissipation has a minute effect but cannot be neglected for wall shear stress $\phi''(0)$, but the same has an influential effect for temperature gradient $\theta'(0)$, and both have decreasing scenario. When the Prandtl number Pr is taken into account, these variations of $\phi''(0)$ and $\theta'(0)$, as shown in figures 12a-12b, typical trend of decreasing values gradually upon increasing K from zero.

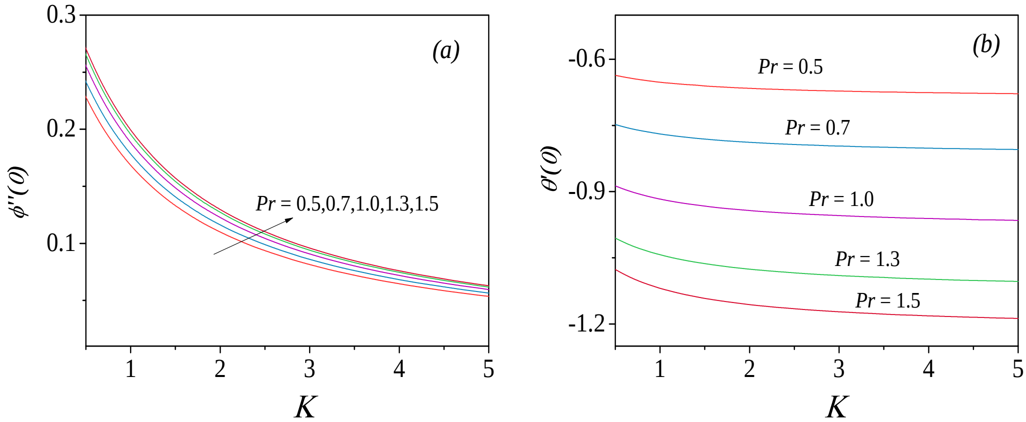

Figure 12. The variation of (a) wall shear stress $\phi''(0)$ and (b) temperature gradient $\theta'(0)$ for $\alpha = -0.5, \beta = 0.7, Ec = 0.5, N = 0.4, \lambda = 0.5$.

6. Conclusion

We have investigated the two-dimensional mixed convection boundary layer flow over a moving wedge which has slip velocity, by taking an account of viscous dissipation. The modeled equations are solved numerically using shooting technique which is a judicial combination of Runge-Kutta and secant methods. Various physical derived quantities in terms of velocity, temperature and their gradients are determined for the system parameters. The numerical simulations show the following important findings:

1. The imposed pressure gradient on both momentum and thermal boundary layer flows make the thicknesses thinning which helps the flow to be adhering to the wedge and enhanced heat transfer rate. For an opposing flow ($\alpha = -0.5$) and for the same β, there is an evident difference in the velocity.

2. For both assisted and opposing flows, boundary layer and associated heat transfer rate are seen to produce benign characteristics in that enhanced rate of heat transfer.

3. The slip velocity influences momentum boundary layer to form from below for $K \leq 0$ while the contrast results are seen for $K > 0$. Since, the wall shear stress distribution at the boundary is found to change, there is reduction in the thickness of the thermal boundary layer and to enhanced rate of heat transfer.

4. Both velocity and temperature profiles have thicker boundary layer thickness for smaller Prandtl numbers, out of which the velocity curves seem to have produced overshoots. The thicker thickness for smaller Pr exhibit low convective heat transfer rate.

5. At large Eckert numbers Ec, the internal heating due to viscous dissipation is large which leads higher temperatures which affect the thickness of the thermal boundary layer and hence heat transfer rate.

6. The wall shear stress $\phi''(0)$ and temperature gradient $\theta'(0)$ have a decreasing effect for increasing K.

Thus, the imposed pressure gradient, the slip velocity, viscous dissipation and wall exponent parameter are found to play a major role in identifying suitable momentum and thermal boundary layers in order to model more efficient and technologically useful.

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