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A New Algorithm to Solve Fuzzy Sequential Non-Linear Programming Problem

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Abstract

In this paper, an algorithm is proposed to solve a Fuzzy sequential non-linear programming problem. This algorithm applies to the problem when the objective function is non-linear and the constraints are linear. Initially, the Fuzzy sequential linear programming problem is converted into a fuzzy linear programming problem using the fuzzy Frank Wolfe algorithm and then it is solved by the Fourier Motzkin Elimination Method. In nonlinear programming problems, the Sequential quadratic programming (SQP) algorithm is seen as quite possibly the most effective method. Since the 1970s, numerous specialists in China and abroad have explored this kind of algorithm and made some allure results. Through their work, the SQP process has drawn in a basic situation in tackling controlled nonlinear streamlining problems. Notwithstanding, the SQP algorithm realistic so far has a horrid limitation, (i.e.) this sort of strategy necessitates that their quadratic programming sub-problems have limited solutions at every emphasis. Considering that the imperatives of the sub-problems are linear guesses of one of the extraordinary problems, attainable arrangements of such sub-problems might be unfilled. In this way, the surpassing case is difficult to be fulfilled.

Keywords: Algorithm; Fuzzy; SQP; Fourier Motzkin Elimination Method.
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1. Introduction

For nonlinear programming problems, the Sequential quadratic programming (SQP) algorithm is seen at there as quite possibly the most productive method. Since the 1970s, numerous analysts in China and abroad have done much research into this kind of algorithm and made some allure results. Through their work, the SQP process has drawn in a basic situation in taking care of controlled nonlinear streamlining problems. In any case, the SQP algorithm reachable so far has a troubling limitation, (i.e.) this sort of strategy necessitates that their quadratic programming sub-problems have limited solution at every emphasis. Taking into account that the requirements of the sub-problems are linear guesses

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of one of the kind problems, practical arrangements of such sub-problems might be unfilled. Thus, the surpassing case is difficult to be fulfilled. Instructions to vanquish this intricacy is a hot issue in the investigation of SQP strategy [1].

2. Fuzzy Set

Under many conditions, accurate information is lacking to show the real circumstances. Human decisions including inclinations are frequently obscure and they can't appraise inclination with precise numerical information. A fresh set ascents/falls suddenly, making its components thoroughly disjoint with different individuals from the universe. Such an arrangement doesn't exist as long as the human thinking process is concerned. While noticing an actual work, we attempt to show it with various types of perception [2]. While demonstrating it, the model might be a straightforward assertion, a figure, a square outline, and so on and the boundaries of the model can never be freshly characterized. The all-out problem space from the littlest to the biggest permissible worth of the variable viable is known as the universe of talk. The trademark capacity of a fresh set allows a worth either 0 or 1 to every person in the all-inclusive set, consequently separating among individuals and non-members of the fresh set viable. This capacity can be summed up to such an extent that the qualities doled out to components of the general set fall inside the predefined go and demonstrate the participation grade of these components in the set. The most usually utilized scope of upsides of enrolment work is in the unit stretch [0, 1]. For this situation, every participation work maps components of a given general set X, which is consistently a fresh set, into a genuine number into [0,1].

Let X be the fuzzy set given by $\widehat{A} = \{\langle x, \mu_{\widehat{A}}(x)\rangle, x \in X, \mu_{\widehat{A}}(x) \in [0,1]\}$ where $\mu_{\widehat{A}}(x) : X \to [0,1]$ is the pair $\langle x, \mu_{\widehat{A}}(x)\rangle$, the first element x belongs to the classical crisp set X the second element $\mu_{\widehat{A}}(x)$ belongs to the interval $[0,1]$ called membership function of $x \in X$. Each fresh set is a fuzzy set yet not on the other hand. The numerical implanting of traditional set hypothesis into fuzzy sets is a characteristic as installing the genuine numbers into the mind boggling plane. In this manner, the possibility of fluffiness is one of enhancement, not of replacement.

An Algorithm to Solve the Fuzzy Sequential Non-Linear Programming Problem

Step 1: Assume an initial point $x^{(0)}$, two convergence parameters ε and δ. Set an iteration Counter $t = 0$.

Step 2: Calculate $\nabla f\left(x^{(t)}\right)$. If $\left\|\nabla f\left(x^{(t)}\right)\right\| \leq \varepsilon$, Terminate; Else go to Step 3.

Step 3: Frame the Fuzzy Linear Programming Problem as

Maximize Minimize $f\left(x^{(t)}\right) + \nabla f\left(x^{(t)}\right)\left(x - x^{(t)}\right)$

Subject to constraints

$$g_j\left(x^t\right) + \nabla g_j\left(x^t\right)\left(x - x^t\right) \geq 0; j = 1, 2, \ldots, J$$

$$g_j\left(x^t\right) + \nabla g_j\left(x^t\right)\left(x - x^t\right) \leq 0; j = 1, 2, \ldots, J$$

$$h_k\left(x^t\right)+\nabla h_k\left(x^t\right)\left(x-x^t\right)=0; k=1,2,\ldots,K; \qquad x^t\leq x_i\leq x^u$$

Step 4: As the fluffy straight programming issue contains the objective work in the goals, change the same sign '=' in under or comparable sign '$\leq$' for increase issues and more noticeable than or identical sign '$\geq$' for minimization in the objective work [3].

Step 5: Eliminate the variable one by one in the order as $(\hat{x}_1,\hat{x}_2,\ldots,\hat{x}_n)$.

(i). Divide each equation by its modulus value of $\hat{x}_1$ coefficient or all the equations.

(ii). Now we have three classes of $\hat{x}_1$ coefficient, i.e., '-1' or '$+1$' or '0' linear equations.

(iii). Adding or taking away any two classes of conditions to dispense with $\hat{x}_1$.

Step 6: Rehash the above interaction until all the 'n' fuzzy factors are dispensed with.

Step 7: In the wake of killing all the 'n' fuzzy factors, we get the $\tilde{Z}$ characteristics and substitute the $\tilde{Z}$ in above, we get the potential gains of fluffy variables in continuous substitution. Then we get y^t to be the optimal solution to the above fuzzy LPP.

Step 8: Find $\widetilde{\alpha}^t$ that minimizes $f\left(x^t\right)+\widetilde{\alpha}\left(\tilde{y}^t-x^t\right)$ in the range $\alpha\in(0,1)$.

Step 9: Calculate $x^{t+1}=x^t+\widetilde{\alpha}^t\left(\tilde{y}^t-x^t\right)$.

Step 10: If $\left\|x^{t+1}-x^t\right\|\leq\delta\left\|x^t\right\|$ and if $\left\|f\left(x^{t+1}\right)-f\left(x^t\right)\right\|\leq\epsilon\left\|f\left(x^t\right)\right\|$. Terminate. Else set $t=t+1$ and go to Step 2.

Objectives of the Study

1. To review on Algorithm to Solve the Fuzzy Sequential Non-Linear Programming Problem.
2. To review on Fuzzy Non-Linear Programming.

3. Numerical Example

Example 3.1. *Contemplate the fluffy successive nonlinear programming issue, with a three-sided fluffy number is,*

Maximize $f(x)=5x_1-x_1^2+8x_2-2x_2^2$

Subject to the constraints $3x_1+2x_2\leq(5,6,7)$

Non-negative restriction $x_1\geq(0,0,0)$; $x_2\geq(0,0,0)$.

Initial Point $t=0$

Max $f\left(x^{(0)}\right)+\nabla f\left(x^{(0)}\right)\left(x-x^{(0)}\right)$

$$\nabla f\left(x^{(t)}\right)=\begin{pmatrix}\frac{\partial f}{\partial x_1}\\ \frac{\partial f}{\partial x_2}\end{pmatrix}_{(x_1,x_2)}$$

$$\nabla f\left(x^{(t)}\right)=\begin{pmatrix}\frac{\partial f}{\partial x_1}\\ \frac{\partial f}{\partial x_2}\end{pmatrix}_{(x_1,x_2)}$$

$$= \begin{pmatrix} 5 - 2x_1 \\ 8 - 4x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$$f\left(x^0\right) = 0$$

$$f\left(x\right) = 0 + \begin{pmatrix} 5 \\ 8 \end{pmatrix} \begin{pmatrix} x_1 - 0 \\ x_2 - 0 \end{pmatrix}$$

$$\text{Max } f\left(x\right) = 0 + \begin{pmatrix} 5x_1 \\ 8x_2 \end{pmatrix}$$

$$\text{Max } f\left(x\right) = 5x_1 + 8x_2$$

Linearized Fuzzy problem is given by

Max $f\left(x\right) = 5x_1 + 8x_2$

Subject to the constraints $3x_1 + 2x_2 \leq (5, 6, 7)$

Non-negative limitations are $x_1 \geq (0, 0, 0)$; $x_2 \geq (0, 0, 0)$.

Recollect the objective work for the imperatives, for the amplification issue, change the same $'='$ in the objective as $'\leq'$ and (for a minimization issue, change $'\geq'$) [4-8].

$$\begin{aligned} \text{Max } \tilde{z} &\leq 5x_1 + 8x_2 \\ 3x_1 + 2x_2 &\leq (5, 6, 7) \\ x_1 \geq (0, 0, 0)\,; &\quad x_2 \geq (0, 0, 0) \end{aligned} \tag{1}$$

Change every one of the imbalances in the framework as $'\leq'$ for maximization (and $\geq$ for minimization)

$$\begin{aligned} -5x_1 - 8x_2 + z &\leq (0, 0, 0) \\ 3x_1 + 2x_2 &\leq (5, 6, 7) \\ -x_1 \leq (0, 0, 0)\,; &\quad -x_2 \leq (0, 0, 0) \end{aligned} \tag{2}$$

To get rid of x_1 segment each coefficient of the system (2) by the coefficient of x_1 we have

$$\begin{aligned} -x_1 - 1.6x_2 + 0.2\tilde{z} &\leq (0, 0, 0) \\ x_1 + 0.66x_2 &\leq (1.66, 2, 2.33) \\ x_1 \leq (0, 0, 0)\,; &\quad -x_2 \leq (0, 0, 0) \end{aligned} \tag{3}$$

Rearranging the equations in (3), to eliminate x_2

$$\begin{aligned} -0.94x_2 + 0.2\tilde{z} &\leq (1.66, 2, 2.33) \\ -1.6x_2 - 0.2\tilde{z} &\leq (0, 0, 0) \\ 0.66x_2 \leq (1.66, 2, 2.33)\,; &\quad -x_2 \leq (0, 0, 0) \end{aligned} \tag{4}$$

Utilizing a similar methodology as the end of x_1, we get

$$\begin{aligned} -x_2 + 0.213\tilde{z} &\leq (1.76, 2.13, 2.47) \\ -x_2 - 0.125\tilde{z} &\leq (0, 0, 0) \\ x_2 \leq (2.52, 3.03, 3.53)\,; &\quad -x_2 \leq (0, 0, 0) \end{aligned} \tag{5}$$

Presently, the set conditions were acquired by wiping out x_2

$$\begin{aligned} 0.088\tilde{z} &\leq (1.76, 2.13, 2.47) \\ 0.125\tilde{z} &\leq (2.52, 3.03, 3.53) \\ 0.125\tilde{z} &\leq (0, 0, 0) \\ 0.213\tilde{z} &\leq (4.27, 5.16, 6.00) \\ 0.213\tilde{z} &\leq (1.76, 2.13, 2.47) \end{aligned} \tag{6}$$

From the above condition (6), we have

$$\begin{aligned} \tilde{z} &\leq (20, 24.20, 28.06) \\ \tilde{z} &\leq (20.16, 24.24, 28.24) \\ \tilde{z} &\leq (0, 0, 0) \\ \tilde{z} &\leq (20.04, 24.22, 28.16) \\ \tilde{z} &\leq (8.26, 10, 11.59) \end{aligned}$$

Now choosing the value for $\tilde{z}$ which fulfils every one of the requirements. In this way, the ideal arrangement is given by $\tilde{z} = (20, 24.20, 28.06)$. Using the obtained $\tilde{z}$ in (5),

$$\begin{aligned} -x_2 + 0.213(20, 24.20, 28.06) &\leq (1.76, 2.13, 2.47) \\ -x_2 + 0.125(20, 24.20, 28.06) &\leq (0, 0, 0) \\ x_2 &\leq (2.52, 3.03, 3.53) \\ -x_2 &\leq (0, 0, 0) \end{aligned}$$

We get,

$$\begin{aligned} -x_2 + (4.26, 5.15, 5.97) &\leq (1.76, 2.13, 2.47) \\ -x_2 + (2.5, 3.02, 3.50) &\leq (0, 0, 0) \\ x_2 &\leq (2.52, 3.03, 3.53) \\ -x_2 &\leq (0, 0, 0) \end{aligned}$$

Find: $\widetilde{\alpha}$

$$f\left(x^{t}\right)+\widetilde{\alpha}\left(\tilde{y}^{t}-x^{t}\right)\Rightarrow f\left\{\begin{array}{c}(0,0,0)\\(0,0,0)\end{array}+\widetilde{\alpha}\left(\left(\begin{array}{c}(0,0,0)\\(2.52,3.03,3.53)\end{array}\right)-\left(\begin{array}{c}(0,0,0)\\(0,0,0)\end{array}\right)\right)\right\}$$

$$\Rightarrow f\left\{\begin{array}{c}(0,0,0)\\(0,0,0)\end{array}+\widetilde{\alpha}\left(\left(\begin{array}{c}(0,0,0)\\(2.52,3.03,3.53)\end{array}\right)\right)\right\}$$

$$\Rightarrow f\left\{\begin{array}{c}(0,0,0)\\(0,0,0)\end{array}+\left(\begin{array}{c}(0,0,0)\,\widetilde{\alpha}\\(2.52,3.03,3.53)\,\widetilde{\alpha}\end{array}\right)\right\}$$

$$\Rightarrow f\left\{(0,0,0)\,\widetilde{\alpha},(2.52,3.03,3.53)\,\widetilde{\alpha}\right\}$$

$$f(x)=5\left[(0,0,0)\,\widetilde{\alpha}\right]-\left[(0,0,0)\,\widetilde{\alpha}\right]^{2}+8\left[(2.52,3.03,3.53)\,\widetilde{\alpha}\right]-2\left[(2.52,3.03,3.53)\,\widetilde{\alpha}\right]^{2}$$

$$=(20.16,24.24,28.24)-(12.7,18.36,24.92)\,\widetilde{\alpha}^{2}$$

$$\frac{\partial f(x)}{\partial\widetilde{\alpha}}=(20.16,24.24,28.24)-2(12.5,18.24,24.5)\widetilde{\alpha}$$

$\widetilde{\alpha}=(0.41,0.66,1.21)$ in the range (0,1).

$$f\left\{(0,0,0)\,\widetilde{\alpha},(2.52,3.03,3.53)\,\widetilde{\alpha}\right\}$$

$f\{(0,0,0),(0.41,0.66,1.21),(2.52,3.03,3.53),(0.41,0.66,1.21)\}$; $f\{(0,0,0),(2.81,2.00,1.45)\}$. Therefore the optimal solution is $x_1=(0,0,0)$; $x_2=(2.81,2.00,1.45)$.

Fuzzy Non-Linear Programming:

Max (or Min) $f(\tilde{x})=\sum_{j=0}^{n}\widetilde{C}_j\tilde{x}^n$

Subject to $\sum_{j=0}^{n}\tilde{a}_{ij}\tilde{x}_j^n\leq(or\geq)\ b_i,\ i=1,2,3,\ldots,m\ \tilde{x}_j\geq0,\ j=1,2,3,\ldots,n$, where $\tilde{C}_j\ (j=1,2,3,\ldots,n)$, $\tilde{a}_{ij}$ and $\tilde{b}_i\ (i=1,2,3,\ldots,m)$ are triangular fuzzy numbers and $\tilde{x}_j\ \ (j=1,2,3,\ldots,n)$ are crisp variables. We utilize the positioning capacity on the problem (1) to get

Max (or Min) $f(\tilde{x})=\sum_{j=0}^{n}Ra(\widetilde{C}_j\tilde{x}^n)$

Subject to $\sum_{j=0}^{n}Ra(\tilde{a}_{ij}\tilde{x}_j^n)\leq(or\geq)\ \ Ra(b_i),\ i=1,2,3,\ldots,m;\ \tilde{x}_j\geq0,\ j=1,2,3,\ldots,n.$

This is equal to:

Max (or Min) $f(\tilde{x})=\sum_{j=0}^{n}Ra(\widetilde{C}_j)\tilde{x}^n$

Subject to $\sum_{j=0}^{n}Ra(\tilde{a}_{ij}\tilde{x}_j^n)\leq(or\geq)\ Ra(b_i),\ i=1,2,3,\ldots,m;\ \tilde{x}_j\geq0,\ j=1,2,3,\ldots,n$, where, $Ra\left(\widetilde{C}_j\right)=C'_j$, $Ra\left(\tilde{a}_{ij}\right)=a'_{ij}$, $Ra\left(\tilde{b}_i\right)=b'_i$ ($i=1,2,3,\ldots,m$ and $j=1,2,3,\ldots,n$). Then we have,

Max (or Min) $\tilde{f}(\tilde{x})=\sum_{i=0}^{n}C'_jx^n$

Subject to $\sum_{j=0}^{n}a'_{ij}\tilde{x}_j^n\leq(or\geq)\,b'_i,\ i=1,2,3,\ldots,m,\ \tilde{x}_j\geq0,\ j=1,2,3,\ldots,n.$ The above connection we reason that the ideal solutions of (1) and (4) are same [9-13].

4. Conclusion

Successive quadratic programming calculation is seen at there as conceivably the most useful technique. Through their work, the cycle has attracted an essential circumstance in handling controlled nonlinear progression issues. In any case, the SQP calculation reachable so far has an alarming limit; i.e. this kind of procedure requires that their quadratic programming sub issues have restricted arrangement at each accentuation. Considering that the requirements of the sub issues are a direct gauge of one of the excellent issues, potential game plans of such sub-issues may be unfilled. Thus, the astounding case is hard to satisfy. Bit by bit directions to defeat this multifaceted design are a hot issue in the examination of the SQP procedure. This paper proposes the ideal arrangement for fluffy non-straight programming issues using a fluffy consecutive quadratic programming procedure. In the first place, we converted the issue into the new model and thereafter the new construction was handled by the methodologies for fluffy consecutive quadratic programming. Mathematical models support the proposed procedure's ampleness [14-16].

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