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# A Multicriteria Decision Making in Medical Diagnosis using Vague Sets

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**Abstract**

A disease may concern a set of symptoms, and the same set of symptoms may appear in different diseases. Also, symptoms may occur in different patients differently. It creates uncertainty, which is to be addressed to make the diagnostic decision more accurate. By categorizing symptoms into criteria, the proposed model aims to address the inherent ambiguity and vagueness associated with medical data. Vague sets, a generalization of fuzzy sets, are employed to capture the imprecise and uncertain nature of symptoms and their relevance to different diseases. The aim of the present study is to contribute to the development of more reliable and efficient medical diagnostic systems. A numerical example is demonstrated to discuss the proposed method and provides a practical and visual tool to assess potential outcomes of the proposed technique.

**Keywords:** vague set; lower and upper membership function; vague point; triangular vague number.

## 1. Introduction

The theory of fuzzy sets proposed by Zadeh [1] has been successfully applied in decision-making problems carrying vagueness and imprecision in medicine, healthcare, economics, etc. Fuzzy sets focus on the membership degree of an element in the defined domain. If the hesitation part is introduced in the study, then membership degree and non-membership degree play significant roles in the subject. Such objects are called intuitionistic fuzzy sets, as defined by Attanassov [2]. Kumar and Pandey [3] improved the switching function between intuitionistic fuzzy set and type-2 fuzzy set and applied it in medical diagnosis. They also presented fuzzy linear programming in setting up waiting time targets for the outpatients in an outpatient department [4]. Dutta and Goala [5] proposed fuzzy decision-making in medical diagnosis using an advanced distance measure on intuitionistic fuzzy sets. Similar to intuitionistic fuzzy sets, the concept of a vague set given by Gau and Buehrer [6] takes into account the favourable and unfavourable evidence separately. These evidence provide a lower and an upper bound, respectively, within which the membership grade may lie. Vague sets are higher-order fuzzy

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sets [7, 8], which are capable of providing more authentic and realistic results than fuzzy sets, and Bustince and Burillo [7] showed that vague sets are intuitionistic fuzzy sets, and a vague set reduces to the fuzzy set if $\mu_x = 1 - v_x$.

Chen and Tan [9], Hong and Choi [10] applied vague set theory to multicriteria fuzzy decision problems. Pandey [11] defined the concept of triangular vague number in a different way than Chen [12]. They extended the concept of signed distance between two vague numbers and used these concepts in decision-making in medical diagnosis. Chen worked on reliability theory by applying vague set theory. Deng [13] proposed a novel distance between vague sets and demonstrated its applications in decision-making. Aung applied multicriteria decision-making to analyze medical waste management systems [14]. In 2020, Alhazaymeh [15] introduced the cubic vague set and presented its application in decision-making. Recently, Kumar gave a concise note on vague set theory [16]. They applied vague set theory in career determination using assumed data.

In this paper, a membership function contributing truth membership and a non-membership function contributing false membership are considered for different types of relations. In the face of uncertainty concerning the observed symptoms of the patients and the relation of symptoms to diseases, it is useful to introduce the multicriteria analysis for different symptoms. This develops a relationship between patients and diseases via symptom characters. This assumption makes the analysis efficient. The approach is thoroughly illustrated using a hypothetical example.

## 2. Basic Concepts of Vague Sets

**Vague set:** A vague set $\tilde{A}$ in the universe of discourse $X$ is characterized by a membership or truth membership function, $\mu_{\tilde{A}} : X \to [0,1]$ and a non-membership or false membership function, $v_{\tilde{A}} : X \to [0,1]$. The grade of membership for any element $x$ in the vague set is bounded by a sub interval $[\mu_{\tilde{A}}(x), 1 - v_{\tilde{A}}(x)]$, where the grade $\mu_{\tilde{A}}(x)$ is called the lower bound of membership grade of $x$ derived from evidence for $x$ and $v_{\tilde{A}}(x)$ is the lower bound of the membership grade on the negation of $x$ derived from the evidence against $x$ and $\mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1$. In the extreme case of equality $\mu_{\tilde{A}}(x) = 1 - v_{\tilde{A}}(x)$, a vague set reduces to a fuzzy set with interval value of the membership grade reducing to a single value $\mu_{\tilde{A}}(x)$. In general, however, $\mu_{\tilde{A}}(x) \leq$ exact membership grade of $x \leq 1 - v_{\tilde{A}}(x)$. Expressions given below can be used to represent a vague set $\tilde{A}$ for a finite or continuous universe of discourse $X$, respectively.

$$\tilde{A} = \sum_{k=1}^{n} \frac{[\mu_{\tilde{A}}(x_k), 1 - v_{\tilde{A}}(x_k)]}{x_k}, \tilde{A} = \int_{x \in X} \frac{[\mu_{\tilde{A}}(x), 1 - v_{\tilde{A}}(x)]}{x}$$

A vague set is represented pictorially [12] as

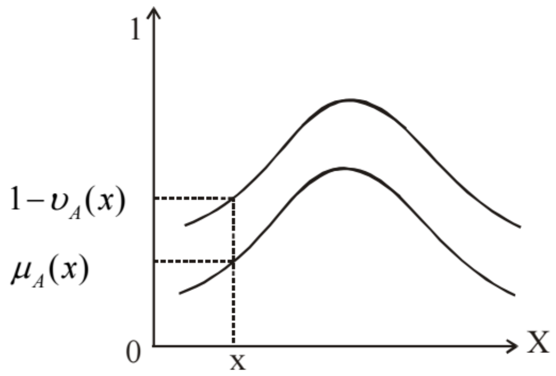


Figure 1: Vague Set

Concept of a vague number [12] is introduced as follows:

**Convex vague set:** Let $\tilde{A}$ be a vague set of the universe of discourse X with $\mu_{\tilde{A}}$ and $v_{\tilde{A}}$ as its membership and non-membership functions, respectively. The vague set is convex iff for every $x_1, x_2$ in $X$

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)\, x_2) \geq \min(\mu_{\tilde{A}}\,(x_1)\,, \mu_{\tilde{A}}\,(x_2))$$
$$1 - v_{\tilde{A}}(\lambda x_1 + (1-\lambda)\, x_2) \geq \min(1 - v_{\tilde{A}}\,(x_1)\,, 1 - v_{\tilde{A}}\,(x_2)), \text{ where } \lambda \in [0,1].$$

**Normal vague set:** A vague set $\tilde{A}$ in the universe of discourse X is called normal if $\exists\ x_i \in X$ such that $1 - v_{\tilde{A}}\,(x_i) = 1$ i.e. $v_{\tilde{A}}\,(x_i) = 0$.

## 3. Multicriteria Decision-making for Medical Diagnosis Using Vague Sets

Let $S = \{S_1, S_2, \ldots, S_n\}$, $P = \{P_1, P_2, \ldots, P_l\}$, and $D = \{D_1, D_2, \ldots, D_m\}$ be the crisp sets of symptoms, patients, and diseases, respectively. Let for each $p \in \{1, 2, \ldots, n\}$, $S_p = \left\{S_{p1}, S_{p2}, \ldots, S_{pn(p)}\right\}$ be the set of characters (criteria) of $S_p$. In vague set theory, we need fuzzy relations between symptom characters and diseases called membership relations and non-membership relations denoted by $\tilde{R}_{pL}$, $\tilde{R}_{pN}$, for each $p = 1, 2, \ldots, n$, respectively. $\tilde{R}_{pL}$, representing the lower bounds and $\tilde{R}_{pN}$, contributing the upper bounds. Let fuzzy relations $\tilde{R}_{pL}$ and $\tilde{R}_{pN}$ between $S_p$ and $D$ be expressed as follows:

$$\tilde{R}_{pL} = \begin{bmatrix} r_{p11} & \cdots & r_{p1m} \\ \vdots & \ddots & \vdots \\ r_{pn(p)1} & \cdots & r_{pn(p)m} \end{bmatrix}, \quad \tilde{R}_{pN} = \begin{bmatrix} c_{p11} & \cdots & c_{p1m} \\ \vdots & \ddots & \vdots \\ c_{pn(p)1} & \cdots & c_{pn(p)m} \end{bmatrix} \tag{1}$$

where $0 \leq r_{pij} \leq 1$, $0 \leq c_{pij} \leq 1$, where $p = \{1, 2, \ldots, n\}$, $i = \{1, 2, \ldots, n(p)\}$; $j = \{1, 2, \ldots, m\}$. Substituting $r'_{pij} = 1 - c_{pij}$ using $\tilde{R}_{pN}$, we get the upper bound of the membership relation (upper membership relation) $\tilde{R}_{pU}$ for each $p$.

$$\tilde{R}_{pU} = \begin{bmatrix} r'_{p11} & \cdots & r'_{p1m} \\ \vdots & \ddots & \vdots \\ r'_{pn(p)1} & \cdots & r'_{pn(p)m} \end{bmatrix} \tag{2}$$

The vague membership relations between patients and symptom characters, that is, $\tilde{A}_{pL}$ and $\tilde{A}_{pU}$ can be obtained by the examining of states of the patients:

$$\tilde{A}_{pL} = \begin{bmatrix} a_{p11} & \cdots & a_{p1n(p)} \\ \vdots & \ddots & \vdots \\ a_{pl1} & \cdots & a_{pln(p)} \end{bmatrix}, \tilde{A}_{pU} = \begin{bmatrix} a'_{p11} & \cdots & a'_{p1n(p)} \\ \vdots & \ddots & \vdots \\ a'_{pl1} & \cdots & a'_{pln(p)} \end{bmatrix} \tag{3}$$

**Determination of weight:** The weight $\tilde{A} = (a_1, a_2, \ldots, a_n)$ for $(S_1, S_2, \ldots, S_n)$ is taken as $a_p = \frac{N_p}{N}$, $p = 1, 2, \ldots, n$, where $N = \sum_{p=1}^{n} N_p$.

**Determination of $r_{pij}$, $r'_{pij}$, $a_{pij}$ and $a'_{pij}$:** Let $N_p$ be the number of past patients for each $p = 1, 2, \ldots, n$; to estimate the membership grade for each $S_{pi}$, $i = 1, 2, \ldots, n(p)$, let the numbers

$N_{pij}$ : Number of cases that have the symptom character $S_{pi}$ in the diagnosis $d_j$;

$N'_{pij}$ : Number of cases having symptom characters $S_{pi}$ but not the diseases $d_j$;

$N_{pi}$ : Number of the total cases having symptom character $S_{pi}$;

$N'_{pi}$ : Number of the total cases having symptom character $S_{pi}$;

$(i = 1, 2, \ldots, n(p);\ j = 1, 2, \ldots, m)$; $\sum_{j=1}^{m} N_{pij} = N_{pi}$, $\sum_{j=1}^{m} N'_{pij} = N'_{pi}$ for each i and p. $r_{pij}$ and $r'_{pij}$ can now be determined by the following expressions: $r_{pij} = \frac{N_{pij}}{N_{pi}}$ and $r'_{pij} = 1 - c_{pij}$, where $c_{pij} = \frac{N'_{pij}}{N'_{pi}}$. We also have $\sum_{j=1}^{m} r_{pij} = 1$, $\sum_{j=1}^{m} c_{pij} = 1$ for each $i \in \{1, 2, \ldots, n(p)\}$. Membership grades $a_{pij}$ and $a'_{pij}$ are determined by physician's experience. We now need a relation between the set of patients and the set of diseases to facilitate the diagnoses for the patients. By using the compositional rule of inference we get $\tilde{T}_{pL}$ and $\tilde{T}_{pU}$ for each $p \in \{1, 2, \ldots, n\}$, given by

$$\tilde{T}_{pL} = \begin{bmatrix} t_{p11} & \cdots & t_{p1m} \\ \vdots & \ddots & \vdots \\ t_{pl1} & \cdots & t_{plm} \end{bmatrix}, \tilde{T}_{pU} = \begin{bmatrix} t'_{p11} & \cdots & t'_{p1m} \\ \vdots & \ddots & \vdots \\ t'_{pl1} & \cdots & t'_{plm} \end{bmatrix} \tag{4}$$

To obtain the elements of $\tilde{T}_{pL}$ and $\tilde{T}_{pU}$ we may use any of the following two composition rules:

$$t_{pij} = \max_h \left[\min\left(a_{pih}, r_{pih}\right)\right], \quad t'_{pij} = \max_h \left[\min\left(a'_{pih}, r'_{pih}\right)\right] \tag{5}$$

$$t_{pij} = \min_h \left[\sum_{h=1}^{n(p)} \left(a_{pih} r_{pih}, 1\right)\right], \quad t'_{pij} = \min_h \left[\sum_{h=1}^{n(p)} \left(a'_{pih} r'_{pih}, 1\right)\right] \tag{6}$$

for $p = 1,2,\ldots,n$, $i = 1,2,\ldots,l$ and $j = 1,2,\ldots,m$. Entries $t_{pij}$ and $t'_{pij}$ in the $i^{th}$ row $\left[t_{pi1}, t_{pi2}, \ldots, t_{pim}\right]$ and $\left[t'_{pi1}, t'_{pi2}, \ldots, t'_{pim}\right]$ of $\tilde{T}_{pL}$ and $\tilde{T}_{pU}$ represent the lower membership and upper membership of the diseases $d_j$ of the $p^{th}$ symptom of the patient $P_i$ for $j = 1,2,\ldots,m$. Fuzzy sets $\tilde{B}_i$ and $\widetilde{B'}_i$ of possible diseases of the patient $P_i$ associated with $\tilde{T}_{pL}$ and $\tilde{T}_{pU}$ are obtained by the compositional rule of inference as

$$\tilde{B}_i = (a_1, a_2, \ldots, a_n)\, o \begin{bmatrix} t_{1i1} & \cdots & t_{1im} \\ \vdots & \ddots & \vdots \\ t_{ni1} & \cdots & t_{nim} \end{bmatrix}$$

$$\widetilde{B'}_i = (a_1, a_2, \ldots, a_n)\, o \begin{bmatrix} t'_{1i1} & \cdots & t'_{1im} \\ \vdots & \ddots & \vdots \\ t'_{ni1} & \cdots & t'_{nim} \end{bmatrix} \tag{7}$$

Hence we get $\tilde{B}_i = [b_{i1}, b_{i2}, \ldots, b_{im}]$ and $\widetilde{B'}_i = [b'_{i1}, b'_{i2}, \ldots, b'_{im}]$ by using two types of compositions rule defined as

$$b_{ij} = \max_p \left[\min\left(a_p, t_{pij}\right)\right], \quad b'_{ij} = \max_h \left[\min\left(a'_{pih}, r'_{pij}\right)\right] \tag{8}$$

$$b_{ij} = \min\left[\sum_{p=1}^{n}\left(a_p t_{pij}, 1\right)\right], \quad b'_{ij} = \min\left[\sum_{p=1}^{n}\left(a'_p t'_{pij}, 1\right)\right] \tag{9}$$

where $i = 1,2,\ldots,l$; $j = 1,2,\ldots,m$, $p \in \{1,2,\ldots,n\}$. It is necessary here to use some aggregation operation to combine $\tilde{B}_i$ and $\widetilde{B'}_i$ to get a single relation between symptoms and diseases. We prefer to use the arithmetic mean on normalized forms of $\tilde{B}_i$ and $\widetilde{B'}_i$. Thus

$$\tilde{B}_i^{\circ *} = \frac{\tilde{B}_i^* + \tilde{B}_i'^*}{2} \tag{10}$$

Where $\tilde{B}_i^*$ and $\tilde{B}_i'^*$ are the normalized forms of $\tilde{B}_i$ and $\widetilde{B'}_i$ respectively. Using the probability distribution rule, each $b_{i1}{}^{\circ *}$ in $\tilde{B}_i^{\circ *}$ gives the expected percentage of the presence of disease $d_j$in the patient $P_i$. Further if $\max_{1 \le h \le m} b_{ih}{}^{\circ *} = b_{ij}{}^{\circ *}$, then patient $P_i's$ diagnosis can be given as $d_j$.

## 4. An Example

The numerical computation is presented to illustrate the calculations based on the present model. Let S, D, P denote sets of symptoms, diseases, and patients, respectively.

$S = \{S_1\ (headache), S_2\ (fever), S_3\ (phlegm)\}$;

$S_1 = \{S_{11}\ (minor), S_{12}\ (median), S_{13}\ (strong)\}$;

$S_2 = \{S_{21}\ (low), S_{22}\ (medium), S_{23}\ (high)\}$;

$S_3 = \{S_{31}\ (light), S_{32}\ (thick)\}$ are the symptom characters associated to corresponding symptoms.

$D = \{D_1\ (cold), D_2\ (pulmonary\ tuberculosis), D_3\ (pertussis)\}$, $P = \{P_1, P_2\}$.

The relations between patients and symptom characters obtained from physician investigation are denoted by $\tilde{A}_{1L}$, $\tilde{A}_{2L}$, $\tilde{A}_{3L}$, respectively. Now, for symptoms like headache, fever, and phlegm, the relations are represented as:

$$\tilde{A}_{1L} = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.0 & 0.3 & 0.7 \end{bmatrix}, \ \tilde{A}_{2L} = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.2 & 0.8 \end{bmatrix} \ and \ \tilde{A}_{3L} = \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix}.$$

Use following non-membership relations between patients and symptom characters based on the physician investigation:

$$\tilde{A}_{1N} = \begin{bmatrix} 0.1 & 0.8 & 0.9 \\ 0.9 & 0.7 & 0.2 \end{bmatrix}, \ \tilde{A}_{2N} = \begin{bmatrix} 0.0 & 0.9 & 1.0 \\ 0.9 & 0.7 & 0.1 \end{bmatrix} \ and \ \tilde{A}_{3N} = \begin{bmatrix} 0.1 & 0.7 \\ 0.8 & 0.1 \end{bmatrix}.$$

From non membership relations, following upper membership relations are obtained:

$$\tilde{A}_{1U} = \begin{bmatrix} 0.9 & 0.2 & 0.1 \\ 0.1 & 0.3 & 0.8 \end{bmatrix}, \ \tilde{A}_{2U} = \begin{bmatrix} 1.0 & 0.1 & 0.0 \\ 0.1 & 0.3 & 0.9 \end{bmatrix} \ and \ \tilde{A}_{3U} = \begin{bmatrix} 0.9 & 0.3 \\ 0.2 & 0.9 \end{bmatrix}.$$

In representing vague sets, it is essential to determine the non-membership relations $\tilde{R}_{1N}, \tilde{R}_{2N}, \tilde{R}_{3N}$ between symptom characters and diseases. It requires the simple statistical analysis. For this purpose we need the values of $N'_{pij}$ that can be determined only if $N_{pij}$ is known for each $p = 1, 2$, and 3. Assume values for $N_{pij}$, $i = 1, 2, 3$; $j = 1, 2, 3$ as follows:

For *p*=1, *i*= 1, 2, 3, *j*=1, 2, and 3:

$$N_{111} = 4, N_{112} = 2, N_{113} = \ 14$$

$$N_{121} = 6, N_{122} = 8, N_{123} = \ 6$$

$$N_{131} = 4, N_{132} = 20, N_{133} = \ 16$$

For *p*=2, *i*= 1, 2, 3, *j*=1, 2,3:

$$N_{211} = 24, N_{212} = 3, N_{213} = \ 3$$

$$N_{221} = 25, N_{222} = 15, N_{223} = \ 10$$

$$N_{231} = 12, N_{232} = 4, N_{233} = \ 4$$

For p=3, *i*= 1, 2, *j*=1, 2, 3:

$$N_{311} = 6, N_{312} = 4, N_{313} = \ 10$$

$$N_{321} = 3, N_{322} = 9, N_{323} = \ 18$$

Using statistical analysis of section-3, relations $\tilde{R}_{1L}, \tilde{R}_{2L}, \tilde{R}_{3L}$ and $\tilde{R}_{1N}, \tilde{R}_{2N}, \tilde{R}_{3N}$ are obtained as:

$$\tilde{R}_{1L} = \begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}, \tilde{R}_{2L} = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.5 & 0.3 & 0.2 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}, \ \tilde{R}_{3L} = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$$

$$\tilde{R}_{1N} = \begin{bmatrix} 0.4 & 0.45 & 0.15 \\ 0.35 & 0.3 & 0.35 \\ 0.45 & 0.25 & 0.3 \end{bmatrix}, \tilde{R}_{2N} = \begin{bmatrix} 0.1 & 0.45 & 0.45 \\ 0.25 & 0.35 & 0.40 \\ 0.2 & 0.40 & 0.40 \end{bmatrix}, , \tilde{R}_{3N} = \begin{bmatrix} 0.35 & 0.4 & 0.25 \\ 0.45 & 0.35 & 0.2 \end{bmatrix}$$

Now the upper membership relations are

$$\tilde{R}_{1U} = \begin{bmatrix} 0.60 & 0.55 & 0.85 \\ 0.65 & 0.70 & 0.65 \\ 0.55 & 0.75 & 0.70 \end{bmatrix}, \tilde{R}_{2U} = \begin{bmatrix} 0.90 & 0.55 & 0.55 \\ 0.75 & 0.65 & 0.60 \\ 0.80 & 0.60 & 0.60 \end{bmatrix}, \tilde{R}_{3U} = \begin{bmatrix} 0.65 & 0.60 & 0.75 \\ 0.55 & 0.65 & 0.80 \end{bmatrix}$$

Using compositional rule of inference given in equation (5) we get

$$\tilde{T}_{1L} = \tilde{A}_{1L} o \tilde{R}_{1L} = \begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.3 & 0.5 & 0.4 \end{bmatrix}, \qquad \tilde{T}_{2L} = \tilde{A}_{2L} o \tilde{R}_{2L} = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.6 & 0.2 & 0.2 \end{bmatrix},$$

$$\tilde{T}_{3L} = \tilde{A}_{3L} o \tilde{R}_{3L} = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}, \qquad \tilde{T}_{1U} = \tilde{A}_{1U} o \tilde{R}_{1U} = \begin{bmatrix} 0.6 & 0.55 & 0.85 \\ 0.55 & 0.75 & 0.7 \end{bmatrix},$$

$$\tilde{T}_{2U} = \tilde{A}_{2U} o \tilde{R}_{2U} = \begin{bmatrix} 0.9 & 0.55 & 0.55 \\ 0.8 & 0.6 & 0.6 \end{bmatrix}, \qquad \tilde{T}_{3U} = \tilde{A}_{3U} o \tilde{R}_{3U} = \begin{bmatrix} 0.65 & 0.6 & 0.75 \\ 0.55 & 0.65 & 0.8 \end{bmatrix}$$

Using the property given in (8) we get

$$\tilde{B}_1 = \begin{bmatrix} 0.3 & 0.2 & 0.5 \end{bmatrix}, \ \tilde{B}_2 = \begin{bmatrix} 0.3 & 0.3 & 0.5 \end{bmatrix} \ \textit{and} \ \widetilde{B'}_1 = \begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix}, \ \widetilde{B'}_2 = \begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Normalizing $\tilde{B}_1$, $\tilde{B}_2$, $\widetilde{B'}_1$, $\widetilde{B'}_2$, we get $\tilde{B}_1^*$, $\tilde{B}_2^*$, $\tilde{B}_1'^*$, $\tilde{B}_2'^*$ as follows:

$$\tilde{B}_1^* = \begin{bmatrix} 0.3 & 0.2 & 0.5 \end{bmatrix}, \qquad \tilde{B}_2^* = \begin{bmatrix} 0.2727 & 0.2727 & 0.4546 \end{bmatrix}$$

$$\tilde{B}_1'^* = \begin{bmatrix} 0.3333 & 0.3333 & 0.3334 \end{bmatrix}, \quad \widetilde{B'}_2 = \begin{bmatrix} 0.3333 & 0.3333 & 0.3334 \end{bmatrix}$$

Now after aggregating we get

$$\tilde{B}_1^{\circ *} = \begin{bmatrix} 0.3167 & 0.2667 & 0.4166 \end{bmatrix} \ \textit{and} \ \tilde{B}_2^{\circ *} = \begin{bmatrix} 0.3030 & 0.3030 & 0.3940 \end{bmatrix} \tag{11}$$

$$\tilde{B}_1^{\circ *} = \begin{bmatrix} 0.3167 & 0.2667 & 0.4166 \end{bmatrix} \ \textit{and} \ \tilde{B}_2^{\circ *} = [\ 0.3030 \ \ 0.3030 \ \ 0.3940\ ]$$

Using (11), the expected percentage of the presence of diseases in the two patients can be given as

$$P_1 : D_1 \; 31.67\ \%, \quad D_2 \; 26.67\ \%, \quad and \quad D_3 \; 41.66\ \%$$
$$P_2 : D_1 \; 30.30\ \%, \quad D_2 \; 30.30\ \%, \quad and \quad D_3 \; 39.40\ \%. \qquad (12)$$

It can be observed from (12) that the diagnosis for the patient $P_1$ is $D_3$ and for $P_2$ it is $D_3$. Using compositional rule of inference given in equation (6), equation (9) and equation (10) we get the following diagnosis:

$$P_1 : D_1 \; 37.91\ \%, \quad D_2 \; 23.16\ \%, \quad D_3 \; 38.93\ \%$$
$$P_2 : D_1 \; 30.69\ \%, \quad D_2 \; 31.92\ \%, \quad D_3 \; 37.39\ \% \qquad (13)$$

It can be observed from (13), the diagnosis for the patient $P_1$ is $D_3$ for $P_2$ it is $D_3$. Observing our results given in (12), it is observed that the conclusion about patient $P_1$ having disease $D_3$ and $P_2$ having disease $D_3$. In the current methodology which is based on the favourable and unfavorable evidence separately, the margins between the highest and second highest percentages are 1-10% and 6-9% for patients $P_1$ and $P_2$ respectively. Since our results are sufficiently credible, these would help the consultant clinician to watch the first line of treatment more closely and gather further evidence on the basis of the response of the patient before finalizing the diagnostic decision between two close diseases

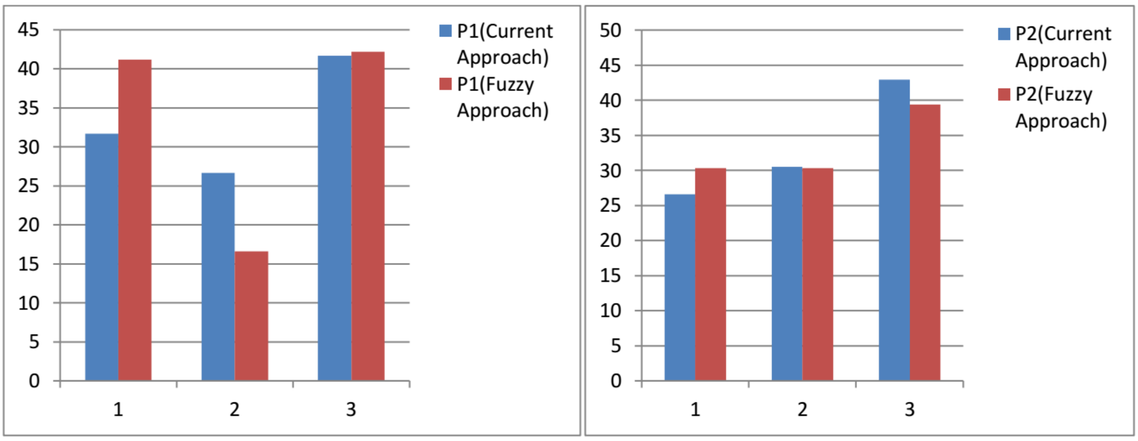


Figure 2: Comparative bar diagrams of the current approach and the traditional fuzzy approach using composition max-min

## 5. Conclusion

A multi-criteria decision-making using vague sets is discussed by classifying symptoms into symptom characters. Relations between patients and symptom characters and symptom characters and diseases are constructed. Decision-making for medical diagnosis based on vague sets is then introduced. To cover up the vagueness in the patient-symptom relationship and the imprecision in observations of the symptom - disease relationship, a good decision model must take into account the favourable and unfavourable evidence separately. It must also be ready to accept some hidden or unobserved uncertainties, e.g. a patient may have some observed symptoms like cough, fever, and chest pain but might not have either observed or given importance to swelling in the axilla (armpit). In such situations, $\mu_x \neq 1 - \nu_x$ and would rather satisfy the inequality $\mu_x + \nu_x < 1$; hence, it may be useful to identify meaningful lower and upper bounds for the membership grades to make the diagnostic decision more credible. Consideration of higher fuzzy sets creates difficulties in the solution process, but use of the computer software of the model removes the difficulties and handles the decision - making with more reality and credibility.

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