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Eccentric Degree Based Topological Indices of Pentagonal triple Chains

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Abstract

In chemistry, pharmacology, medicine and physics molecular graphs have been used to model molecular substances and networks. In graph theory, Pentagonal chains are important tools in network theory and some chemical applications. Recently, topological indices of Pentagonal chain are studied by some researchers. In this paper, we study and calculated eccentric degree based topological indices of pentagonal triple chain.

Keywords: Eccentric vertex degrees; Eccentric degree topological indices; Pentagonal triple chain.
2020 Mathematics Subject Classification: 05C07, 05C30, 05C35.

1. Introduction

Graph theory has many applications for science, technology and social sciences. Graph theory enables suitable toys to researches to model real world problems. Chemical graph theory is considered the intersection of graph theory, chemistry and information science. A topological graph index is a mathematical formula which is invariant for all isomorphic graphs. Topological graph indices have been defined and studied in the last eight decades. Topological indices have been derived from the molecular graphs of chemical compounds and networks. The most important applications of topological graph indices are in Chemical Graph Theory. Any molecule can be modeled by a graph constructed so that each vertex corresponds to an atom and each edge corresponds to a chemical bond between the corresponding atoms. There are many papers on topological graph indices of several molecular structures. Let G be a simple, undirected, connected graph with vertex set $V(G)$ and edge set $E(G)$. Given $v \in V(G)$, the degree of v is denoted by $d(v)$ and is defined as the number of edges incident with v. For the vertices $u, v \in V(G)$, the distance between u and v is denoted by $d(u,v)$ and is defined as the length of the shortest path connecting u and v in G.

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2. Definitions

Definition 2.1 (Eccentricity of a vertex). *Eccentricity of a vertex is denoted by $e(v)$. It is defined as the maximum distance from v to any other vertex. i.e $e(v) = max\{d(v,u); u \in V(G)\}$.*

Definition 2.2 (Eccentric set of a vertex). *Eccentric set $E(v)$ of a vertex v is defined as $E(v)= \{u \in V(G); d(v,u) = e(v)\}$. This set include all vertices that are at the maximum distance $e(v)$ from v [10].*

Definition 2.3 (Eccentric open neighbourhood of a vertex). *The Eccentric open neighbourhood of a vertex u is defined as $N_e(u) = \{v \in V(G); d(u,v) = e(u)\}$. The cardinality of $N_e(u)$ is called degree of vertex u in G and denoted by $deg_e(u)$ or $d_e(u)$.*

In 2023, Bhairaba kumar Majhi et.all calculated some indices of pentagonal triple chains [11]. Motivated by this we have work on Eccentric Degree Based Topological Indices(EDBTI) of Pentagonal triple Chains. A Eccentric Degree Pentagonal triple Chain consisting of k units of Pentagons is denoted by ${}^eC^3_{5,k}$ and illustrated in Figure 1 and 2:

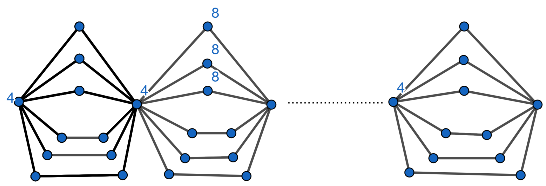

Figure 1:

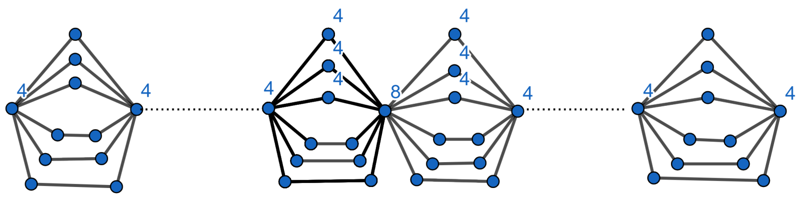

Figure 2:

In following sections, we shall make our proofs by means of this eccentric degree and edge partition table.

$(d_e(u), d_e(v))$	Number of vertices (u, v)
(4,8)	6
(4,4)	15k-6

Table 1: The edge partition table of eccentric $C^3_{5,k}$ is given for the vertices $n = 10k + 1$, if k is odd

$(d_e(u), d_e(v))$	Number of vertices(u, v)
(4,8)	12
(4,4)	15k-12

Table 2: The edge partition table of eccentric $C^3_{5,k}$ is given for the vertices $n = 10k + 1$, if k is even

3. Additive Eccentric Topological Indices of $C_{5,k}^3$

In this section, we calculate some additive EDBTI of the pentagonal triple chain. The first and second eccentric degree zagreb index is defined as

$$^{e}M_1(G) = \sum_{uv \in E(G)} d_e(u) + d_e(v) \text{ and } ^{e}M_2(G) = \sum_{uv \in E(G)} d_e(u)d_e(v)$$

The eccentric inverse sum index is defined by

$$^{e}ISI(G) = \sum_{uv \in E(G)} \frac{d_e(u)d_e(v)}{d_e(u) + d_e(v)}$$

The eccentric Sigma index is an important irregularity measure defined by

$$^{e}\sigma(G) = \sum_{uv \in E(G)} (d_e(u) - d_e(v))^2$$

The eccentric Harmonic index is defined by

$$^{e}H(G) = \sum_{uv \in E(G)} \frac{2}{d_e(u) + d_e(v)}$$

Generalized eccentric Harmonic index is similarly defined by taking arbitrary power α as follows:

$$^{e}H_{\alpha}^{*}(G) = \sum_{uv \in E(G)} \left(\frac{2}{d_e(u) + d_e(v)} \right)^{\alpha}$$

The eccentric Atom bond connectivity index (ABC) is defined by:

$$^{e}ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_e(u) + d_e(v) - 2}{d_e(u)d_e(v)}}$$

The eccentric Geometric-arithmetic index is defined as:

$$^{e}GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_e(u)d_e(v)}}{d_e(u) + d_e(v)}$$

The eccentric Augmented Zagreb index is:

$$^{e}AZ(G) = \sum_{uv \in E(G)} \left(\frac{d_e(u)d_e(v)}{d_e(u) + d_e(v) - 2} \right)^3$$

The eccentric irregularity index is the Albertson index is defined by the sum of absolute values of all the differences between degrees of pairs of vertices forming an edge:

$$^{e}Alb(G) = \sum_{uv \in E(G)} |d_e(u) - d_e(v)|$$

The eccentric Randi´c index is defined as:

$$^{e}R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_e(u)d_e(v)}}$$

The eccentric Reciprocal Randi´c index is defined similarly to eccentric Randi´c index:

$$^{e}RR(G) = \sum_{uv \in E(G)} \sqrt{d_e(u)d_e(v)}$$

And the eccentric Sum connectivity index as:

$$^{e}\chi(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_e(u) + d_e(v)}}$$

Now we present our results on above definitions:

Theorem 3.1. *Let $G = C^3_{5,k}$ be a pentagonal triple chain then Some additive eccentric degree-based topological indices of the graph G with $n = 10k + 1$ vertices and $m = 15k$ edges,where k is odd are as follows:*

(1) $^{e}M_1(G) = 120k + 24$

(2) $^{e}M_2(G) = 240k + 96$

(3) $^{e}ISI(G) = 30k + 4$

(4) $^{e}\sigma(G) = 96$

(5) $^{e}H(G) = \dfrac{15k - 2}{4}$

(6) $^{e}H^*_\alpha(G) = (15k - 6)4^{-\alpha} + 6^{(1-\alpha)}$

(7) $^{e}ABC(G) = \dfrac{6\sqrt{5} + (15k - 6)\sqrt{6}}{4}$

(8) $^{e}GA(G) = 15k - 6 + 4\sqrt{2}$

(9) $^{e}AZ(G) = \dfrac{24576}{125} + \dfrac{(5k - 2)512}{9}$

(10) $^{e}Alb(G) = 24$

(11) $^{e}R(G) = \dfrac{15k - 6 + 3\sqrt{2}}{4}$

(12) $^{e}RR(G) = 4(15k - 6 + 6\sqrt{2})$

(13) ${}^{e}\chi(G) = \sqrt{3} + \frac{15k-6}{2\sqrt{2}}$

Proof. Let $C^3_{5,k}$ be a Pentagonal triple chain of order $n = 10k+1$, size $m = 15k$ and k is odd. Now we calculate the eccentric additive topological indices of $C^3_{5,k}$ by using existing additive topological indices are mentioned above and using the eccentric degrees of Table 1. First We calculate the first additive eccentric Zagreb index of $C^3_{5,k}$. ${}^{e}M_1(G) = 6(4+8) + (15k-6)(4+4) = 120k+24$. Next we find second additive eccentric Zagreb index. ${}^{e}M_2(G) = 6(4*8) + (15k-6)(4*4) = 240k+96$ in a similar manner we calculated all other indices. □

Theorem 3.2. *Let $G = C^3_{5,k}$ be a pentagonal triple chain then Some additive eccentric degree-based topological indices of for the graph G with $n = 10k+1$ vertices and $m = 15k$ edges,where k is even are as follows*

(1) ${}^{e}M_1(G) = 120k+48$

(2) ${}^{e}M_2(G) = 240k+192$

(3) ${}^{e}ISI(G) = 30k+8$

(4) ${}^{e}\sigma(G) = 192$

(5) ${}^{e}H(G) = \frac{15k-4}{4}$

(6) ${}^{e}H^*_\alpha(G) = (15k-12)4^{-\alpha} + 2\times 6^{(1-\alpha)}$

(7) ${}^{e}ABC(G) = \frac{12\sqrt{5}+(15k-12)\sqrt{6}}{4}$

(8) ${}^{e}GA(G) = 15k-12+8\sqrt{2}$

(9) ${}^{e}AZ(G) = \frac{49152}{125} + \frac{(5k-4)512}{9}$

(10) ${}^{e}Alb(G) = 48$

(11) ${}^{e}R(G) = \frac{15k-12+6\sqrt{2}}{4}$

(12) ${}^{e}RR(G) = 4(15k-12+12\sqrt{2})$

(13) ${}^{e}\chi(G) = 2\sqrt{3} + \frac{15k-12}{2\sqrt{2}}$

Proof. Let $C^3_{5,k}$ be a Pentagonal triple chain of order $n = 10k+1$, size $m = 15k$ and k is even. Now we calculate the eccentric additive topological indices of $C^3_{5,k}$ by using existing additive topological indices are mentioned above and using the eccentric degrees of Table 2. ${}^{e}M_1(G) = 12(4+8) + (15k-12)(4+4) = 120k+48$. The second additive eccentric Zagreb index. ${}^{e}M_2(G) = 12(4*8) + (15k-12)(4*4) = 240k+192$ similarly we find out all other indices. □

4. Multiplicative Topological Indices of $C_{5,k}^3$

In this section, we calculate some multiplicative EDBTI of pentagonal triple chain $C_{5,k}^3$. These multiplicative versions of the indices are obtained by replacing the sum sign with a product sign.These Indices are listed below:

$$^{e}PM_1(G) = \prod_{uv \in E(G)} (d_e(u) + d_e(v)) \text{ and } ^{e}PM_2(G) = \prod_{uv \in E(G)} d_e(u)d_e(v)$$

these are called the Multiple eccentric first and second Zagreb indices. Similarly the multiplicative eccentric forgotten index, sometimes named as the multiplicative eccentric third Zagreb index is defined by:

$$^{e}\prod_3(G) = \prod_{uv \in E(G)} (d_e^2(u) + d_e^2(v))$$

The eccentric Geometric-arithmetic multiplicative index is defined as:

$$^{e}GA\prod(G) = \prod_{uv \in E(G)} \frac{2\sqrt{d_e(u)d_e(v)}}{d_e(u) + d_e(v)}$$

The eccentric first and second multiplicative hyper Zagreb indices are:

$$^{e}H\prod_1(G) = \prod_{uv \in E(G)} (d_e(u) + d_e(v))^2 \text{ and } ^{e}H\prod_2(G) = \prod_{uv \in E(G)} (d_e(u)d_e(v))^2$$

General eccentric sum connectivity index is given by

$$^{e}H\prod_\alpha(G) = \prod_{uv \in E(G)} (d_e(u) + d_e(v))^\alpha$$

The Multiplicative eccentric Randić index is defined by

$$^{e}R\prod(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_e(u)d_e(v)}}$$

The multiplicative eccentric sum connectivity index is defined by

$$^{e}\chi\prod(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_e(u) + d_e(v)}}$$

The multiplicative eccentric atom bond connectivity index is defined by

$$^{e}ABC\prod(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_e(u) + d_e(v) - 2}{d_e(u)d_e(v)}}$$

then we have the following result:

Theorem 4.1. *Some multiplicative EDBTI for the graph G with $n = 10k + 1$ vertices and $m = 15k$ edges,where k is odd are as follows:*

(1) $^{e}\prod_1(G) = 2^{(45k-6)} * 3^6$

(2) $^{e}\prod_2(G) = 2^{6(10k+1)}$

(3) $^{e}\prod_3(G) = 2^{(75k-6)} * 5^6$

(4) $^{e}GA\prod(G) = \frac{2^9}{3^6}$

(5) $^{e}H\prod_1(G) = 2^{(90k-12)} * 3^{12}$

(6) $^{e}H\prod_2(G) = 2^{(120k+12)}$

(7) $^{e}H\prod_\alpha(G) = 2^{(45k-6)\alpha} * 3^{6\alpha}$

(8) $^{e}R\prod(G) = \left(\frac{1}{2}\right)^{3(10k+1)}$

(9) $^{e}\chi\prod(G) = \frac{1}{3^3} * \left(\frac{1}{2}\right)^{\frac{45k-6}{2}}$

(10) $^{e}ABC\prod(G) = 125 * 6^{\left(\frac{15k-6}{2}\right)} * 2^{-30k}$

Proof. Let $C_{5,k}^3$ be a Pentagonal triple chain of order $n = 10k + 1$ and the size $m = 15k$. Here k is odd. We calculate the eccentric multiplicative topological indices of $C_{5,k}^3$ by using existing multiplicative topological indices are mentioned above and using the eccentric degree sequence of table 1. We get

$$^{e}\prod_1(G) = (4+8)^6 * (4+4)^{(15k-6)} = 2^{(45k-6)} * 3^6$$
$$^{e}\prod_2(G) = (4*8)^6 * (4*4)^{(15k-6)} = 2^{6(10k+1)}$$

Remaining results have been proven as above. □

Theorem 4.2. *Some multiplicative eccentric degree-based topological indices for the graph G with $n = 10k + 1$ vertices and $m = 15k$ edges,where k is even are as follows:*

(1) $^{e}\prod_1(G) = 2^{(45k-12)} * 3^{12}$

(2) $^{e}\prod_2(G) = 2^{60k+12}$

(3) $^{e}\prod_3(G) = 2^{(75k-12)} * 5^{12}$

(4) $^{e}GA\prod(G) = \frac{2^{18}}{3^{12}}$

(5) $^{e}H\prod_1(G) = 2^{(90k-24)} * 3^{24}$

(6) $^{e}H\prod_2(G) = 2^{(120k+24)}$

(7) $^{e}H\prod_{\alpha}(G) = 2^{(45k-12)\alpha} * 3^{12\alpha}$

(8) $^{e}R\prod(G) = \left(\frac{1}{2}\right)^{3(10k+2)}$

(9) $^{e}\chi\prod(G) = \frac{1}{3^{6}} * \left(\frac{1}{2}\right)^{\frac{45k-12}{2}}$

(10) $^{e}ABC\prod(G) = 5^{6} * 6^{(\frac{15k-12}{2})} * 2^{-30k}$

Proof. Let $C^{3}_{5,k}$ be a Pentagonal triple chain of order $n = 10k + 1$ and the size $m = 15k$. Here k is even. Now we calculate the eccentric multiplicative topological indices of $C^{3}_{5,k}$ by using existing multiplicative topological indices are mentioned above and using the eccentric degrees of Table 2. □

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