International Journal of
Mathematics And its Applications
ISSN: 2347-1557

Int. J. Math. And Appl., **13(2)**(2025), 35–43
Available Online: http://ijmaa.in

Applications of Saxena and Gupta Transform in Solving Ordinary Differential Equations

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Abstract

This research paper presents a novel approach to solving ordinary differential equations (ODEs) using the Saxena & Gupta transform, a recently introduced mathematical tool with promising applications in differential equation solving. We apply this transform to various types of ODEs and analyze its effectiveness and give the graphs for different values using Mathematica (version-12.0). The results demonstrate that the Saxena & Gupta transform provides an efficient and accurate technique for solving ODEs. This research enhances the mathematical framework for solving ODEs and has potential applications in engineering, physics, and applied mathematics.

Keywords: Gupta & Saxena transform; Differential equations; Mathematica (version-12.0).

1. Introduction

Integral transforms have become a crucial tool across various fields of science and engineering, particularly in mathematical physics, optics, engineering mathematics, cryptography, and image processing [1–10]. Their significance lies in their ability to simplify complex problems by transforming them from one space to another, making solutions more accessible or reducing the number of independent variables involved. Over time, numerous integral transforms have been developed and widely applied to both theoretical and practical problems. Some of the most extensively used transforms include the Laplace [11,12], Fourier [2], Sumudu [13,14], Elzaki [15–17], Aboodh [18], Natural, and Z transforms [19], each contributing uniquely to solving diverse problems in these fields. This research investigates the application of the Saxena & Gupta transform [20] in solving first-order, first-degree differential equations and give the graphs also. An analysis demonstrates that this transform yields accurate analytical solutions with potentially simpler calculations. Graphical representations using Mathematica (version-12.0), validate the results, highlighting its applicability in engineering, physics, and applied mathematics. This study contributes to the development of integral transforms for solving differential equations efficiently.

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1.1 Some Definitions

Definition 1.1. *A first-order first-degree differential equation is a differential equation that involves the first derivative of the unknown function and the equation is of the first degree. The general form of such an equation is:*

$$F\left(x, y, \frac{dy}{dx}\right) = 0 \tag{1}$$

$y = f(x)$ is the unknown function, and the highest derivative that appears is the first derivative $\frac{dy}{dx}$.

Definition 1.2. *For all real numbers $t \geq 0$, the transform of see in [20]*

$$F(v) = Z[f(t)] = \frac{1}{v}\int_0^\infty f(vt)e^{-t}dt. \tag{2}$$

Definition 1.3 ([20])**.** *First Derivative:*

$$Z\left[F'(t)\right] = \frac{F(v)}{v} - \frac{f(0)}{v^2}, \tag{3}$$

Second Derivative:

$$Z\left[F''(t)\right] = \frac{F(v)}{v^2} - \frac{f(0)}{v^3} - \frac{f'(0)}{v^2}, \tag{4}$$

N^{th} Derivative:

$$Z\left[F^n(t)\right] = \frac{F(v)}{v^n} - \sum_{k=0}^{n-1}\frac{f^k(0)}{v^{n+1-k}}. \tag{5}$$

Definition 1.4 ([20])**.**

S. No.	Function $f(t)$	Laplace transform $L[f(t)]$	Saxena and Gupta transform $Z[f(t)]$
1	1	$\frac{1}{v}$	$\frac{1}{v}$
2	t	$\frac{1}{v^2}$	1
3	t^2	$\frac{2!}{v^3}$	$2v$
4	t^n	$\frac{n!}{s^{n+1}}$	$\frac{v^{n-1}}{\tau(n+1)}$
5	e^{at}	$\frac{1}{s-a}$	$\frac{1}{v(1-av)}$
6	$sinat$	$\frac{a}{a^2+s^2}$	$\frac{a}{1+a^2\ v^2}$
7	$cosat$	$\frac{s}{a^2+s^2}$	$\frac{1}{v(1+a^2v^2)}$
8	$sinhat$	$\frac{a}{s^2-a^2}$	$\frac{a}{1-a^2v^2}$
9	$coshat$	$\frac{s}{s^2-a^2}$	$\frac{1}{v(1-a^2v^2)}$

2. Main Results

We consider some ordinary differential equations (ODEs) and solve using with Saxena & Gupta transform also include the graphs by plotting them using Mathematica (version-12.0).

Example 2.1. *Solve*

$$\frac{d^2y}{dx^2} + y = 0, \tag{6}$$

where $y(0) = 1$ *and* $\frac{dy}{dx} = -1$ *at* $x = 0$.

Solution. By taking Saxena & Gupta transform on both side of given equation (6), we get

$$\begin{aligned}
Z\{y''(x)\} + Z\{y(x)\} &= Z\{0\} \\
\frac{F(v)}{v^2} - \frac{f(0)}{v^3} - \frac{f'(0)}{v^2} + F(v) &= 0 \\
\frac{F(v)}{v^2} - \frac{1}{v^3} - \frac{(-1)}{v^2} + F(v) &= 0 \\
F(v)\left(\frac{1}{v^2} + 1\right) &= \frac{1}{v^3} - \frac{1}{v^2} \\
F(v) &= \frac{1 - v}{v(1 + v^2)} \\
F(v) &= \frac{1}{v(1 + v^2)} - \frac{1}{1 + v^2}
\end{aligned}$$

Now, taking inverse of Saxena & Gupta transform we obtain the required solution

$$y(x) = \cos x - \sin x.$$

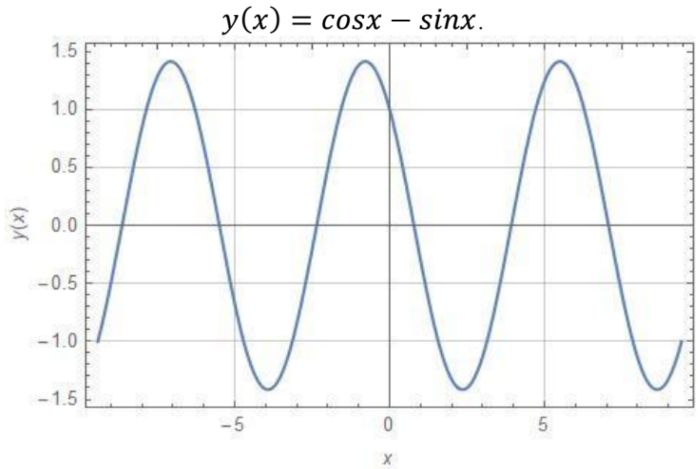

Figure 1: Plot the function $y(x) = \cos x - \sin x$ with respect to x over the range $[-3\pi, 3\pi]$

Example 2.2. *Solve*

$$\frac{dy}{dx} + y = 1, \tag{7}$$

given that $y = 2$ *and when* $x = 0$, $y(0) = 2$.

Solution. By taking Saxena & Gupta transform on both side of equation (7), we obtain

$$\begin{aligned}
Z\{y'(x)\} + z\{y(x)\} &= Z\{1\} \\
\frac{F(v)}{v} - \frac{y(0)}{v^2} + F(v) &= Z\{1\} \\
\frac{F(v)}{v} - \frac{2}{v^2} + F(v) &= \frac{1}{v}
\end{aligned}$$

$$F(v)\left(\frac{1}{v}+1\right)=\frac{1}{v}+\frac{2}{v^2}$$
$$F(v)=\frac{v+2}{v(1+v)}$$
$$F(v)=\frac{v+1+1}{v(1+v)}$$
$$Z\{y(x)\}=\frac{1}{v}+\frac{1}{v(1+v)}$$

Now, taking inverse transform of Saxena & Gupta method, we get the required solution.

$$y(x)=1+e^{-x}.$$

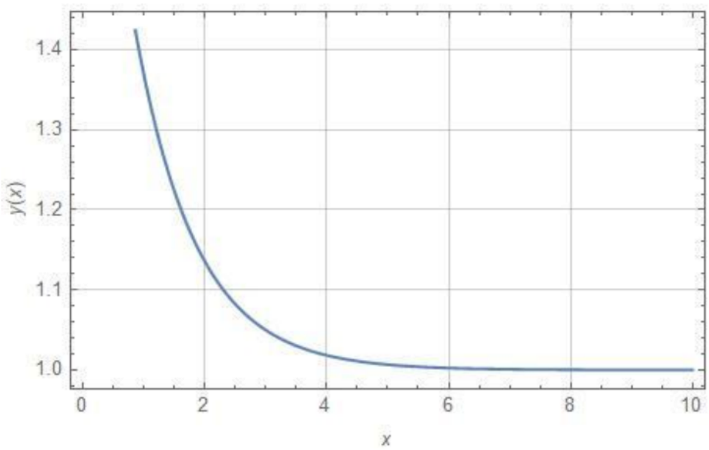

Figure 2: Plot the function $y(x)=1+e^{-x}$ with respect to x over the range $[0,10]$

Example 2.3. *Solve*

$$\frac{d^2y}{dx^2}+y=0, \tag{8}$$

where $y(0)=1$, $y'(0)=0$.

Solution. By taking Saxena & Gupta transform on both side of equation (8), we get

$$Z\{y''(x)\}+Z\{y(x)\}=Z\{0\}$$
$$\frac{F(v)}{v^2}-\frac{y(0)}{v^3}-\frac{y'(0)}{v^2}+F(v)=0$$
$$F(v)\left(\frac{1}{v^2}+1\right)-\frac{1}{v^3}=0$$
$$F(v)\left(\frac{1+v^2}{v^2}\right)=\frac{1}{v^3}$$
$$Z\{y(x)\}=\frac{1}{v(1+v^2)}$$

Now, taking inverse of Saxena & Gupta transform we obtain the require solution

$$y(x)=\cos x.$$

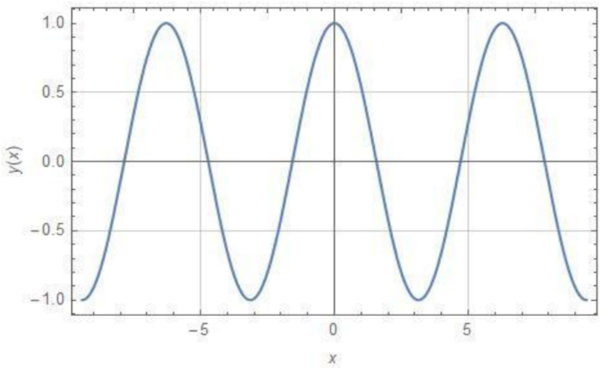

Figure 3: Plot the function $y(x) = \cos x$ with respect to x over the range $[-2\pi, 2\pi]$

Example 2.4. *Solve*

$$\frac{dy}{dx} + y = 0, \tag{9}$$

where $y(0) = 2$.

Solution. By taking Saxena & Gupta transform on both side of equation (9), we get

$$\begin{aligned}
Z\{y'(x)\} + Z\{y(x)\} &= Z\{0\} \\
\frac{F(v)}{v} - \frac{y(0)}{v^2} + F(v) &= 0 \\
F(v)\left(\frac{1}{v} + 1\right) &= \frac{2}{v^2} \\
F(v)\left(\frac{1+v}{v}\right) &= \frac{2}{v^2} \\
Z\{y(x)\} &= \frac{2}{v(1+v)}
\end{aligned}$$

Now, taking inverse of Saxena & Gupta transform we obtain the require solution

$$y(x) = 2e^{-x}.$$

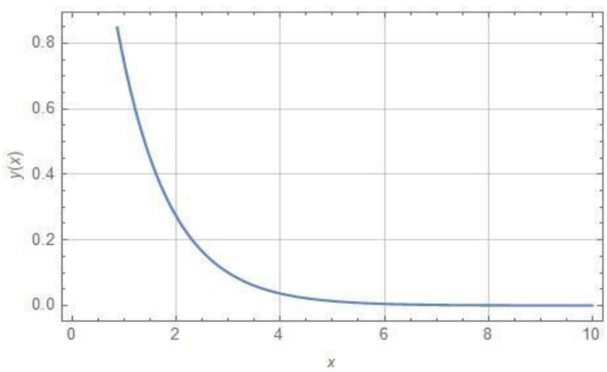

Figure 4: Plot the function $y(x) = 2e^{-x}$ with respect to x over the range $[0, 10]$

Example 2.5. *Solve*

$$\frac{d^2y}{dx^2} + 4y = 0, \tag{10}$$

where $y(0) = -1$, $\frac{dy}{dx}(0) = 0$.

Solution. By taking Saxena & Gupta transform on both sides of equation (10), we get

$$Z\{y''(x)\} + 4\, Z\{y(x)\} = z\{0\}$$
$$\frac{F(v)}{v^2} - \frac{y(0)}{v^3} - \frac{y'(0)}{v^2} + 4F(v) = 0$$
$$F(v)\left(\frac{1}{v^2} + 4\right) - \left(-\frac{1}{v^3}\right) = 0$$
$$F(v)\left(\frac{1+4v^2}{v^2}\right) = -\frac{1}{v^3}$$
$$Z\{y(x)\} = -\frac{1}{v(1+4v^2)}$$

Now, taking inverse of Saxena & Gupta transform we obtain the require solution

$$y(x) = \ -cos(2x).$$

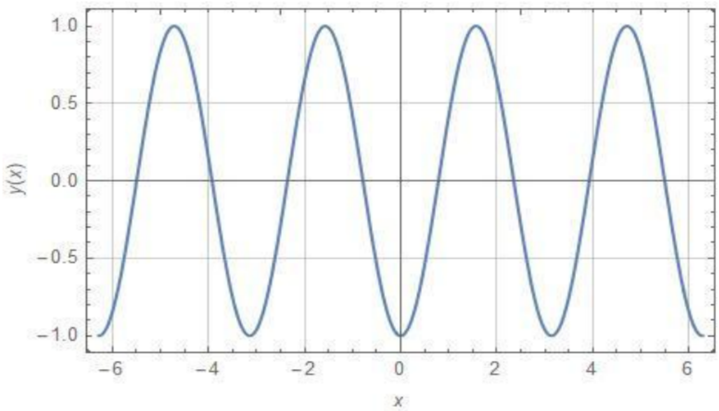

Figure 5: Plot the function $y(x) = -cos(2x)$ with respect to x over the range $[-2\pi, 2\pi]$

Example 2.6. *Solve*

$$\frac{dy}{dx} - 2y = 0, \tag{11}$$

where $y(0) = 4$.

Solution. By taking Saxena & Gupta transform on both sides of equation (11), we get

$$Z\{y'(x)\} - 2\, Z\{y(x)\} = Z\{0\}$$
$$\frac{F(v)}{v} - \frac{y(0)}{v^2} - 2\, F(v) = 0$$
$$F(v)\left(\frac{1}{v} - 2\right) - \frac{4}{v^2} = 0$$

$$F(v)\left(\frac{1-2v}{v}\right)=\frac{4}{v^2}$$
$$Z\{y(x)\}=\frac{4}{v(1-2v)}$$

Now, taking inverse of Saxena & Gupta transform we obtain the require solution

$$y(x)=4e^{2x}.$$

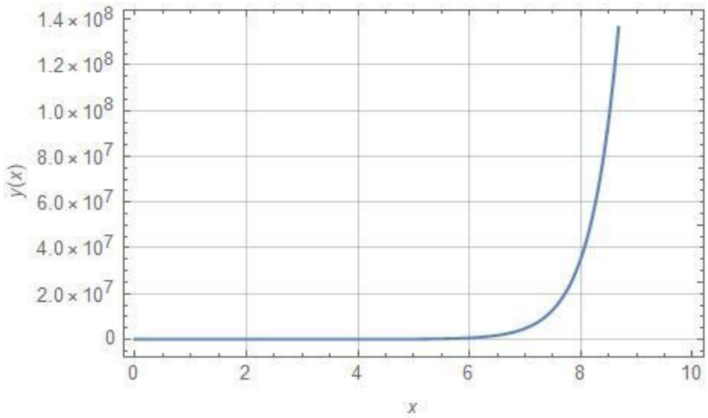

Figure 6: Plot the function $y(x)=4e^{2x}$ with respect to x over the range $[0,10]$

3. Conclusion

The Saxena and Gupta transform Method presents a powerful alternative for solving differential equations. Its unique features and applications make it a significant advancement in the field of mathematical analysis. Additionally, it has been successfully applied in fields such as mathematical physics, engineering mathematics, and control systems. Its capability to solve problems involving exponential, trigonometric, and polynomial functions enhances its applicability, making it a valuable advancement in modern mathematical analysis and applied sciences. Finally, plot solutions of first-order, first-degree differential equations using the **Saxena and Gupta transform** in Mathematica (version-12.0).

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