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An Exposure to Possibilistic Restriction Based Information Analysis Schema

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Abstract

In literature pertaining to Information theory, there is always a contest between importance of *information* or *meaning*. Shannon in his classical paper argued that only syntactic aspects are necessary in information communication, which is very well known as Classical information theory. However, when one considers a hybrid system which has human and machine sub-parts, then the syntactic and semantic, both parts of information become vital. In his recent paper Zadeh states that information is equivalent to a restriction and thus it can be represented as probabilistic and possibilistic restrictions, which can be modified to represent both aspects of information (content + meaning) in a hybrid system. In this paper we discuss some important results on our research on possibilistic modeling of semantic information. We also present a schema for information analysis for hybrid system based on the defined measures.

Keywords: Possibility Distribution; Restriction; hybrid systems; semantic information.
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1. Introduction

Semantic Information theory is a sub-branch of Information theory which studies the communication of meaning of the message [1]. As the technical foundation goes, the meaning of the message does not have any effect in communication of information [2]. However, the meaning part becomes essential when we consider various human - human or human - machine systems, where data (information in very basic terms) and its interpretation both are vital [3]. Example of such system is economic activity, which is regulated by financial / economic data and its interpretations by economic agents. There are many theories to define and quantify semantic content of information or propose changes in mathematical information theory to make it more accommodating for meaning part. The search of proper theory for measuring "meaning" of communicated message was started within the decade of proposal of Mathematical theory of Communication by Claude E. Shannon. In 1948 W. Weaver [2], presented the classification of problems of communication viz. technical, semantic and control.Initiated

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by Bar-Hillel and Carnap [4], the first attempt to modify the approach of Shannon, uses logical probability to measure the semantic information content. In subsequent years this approach was adapted, modified and used to describe how much meaning a message contains. Recent literature pertaining to information theory shows the interest in search of a theory which will enable us to understand information / message or meaning of the communication both [5]. Measuring information through possibility distribution is a prominent theory for the same [6–8].

2. Possibility Distribution and Restrictions

2.1 Possibility Distribution

The concept of *Possibility* was first proposed by Shackle in 1950 as degree of surprise associated with an event, and further formalized by likes of [8–13] and many more. The *possibility distribution* is a function $\pi : \mathcal{P}(X) \to [0,1]$, with following properties ($\mathcal{P}(X)$ being the power set of X) -

1. $\pi() = 0$ and $\pi(X) = 1$.
2. $A \subset B \implies \pi(A) \leq \pi(B)$.
3. $\pi(A \cup B) = \max\{\pi(A), \pi(B)\}$.
4. When $0 \leq \pi(A) \leq 1$, then $A \subset X$ is said to be possibly uncertain.
5. A possibility distribution is said to be *normalized* if $\pi(A_i) = 1$ at least for one $A_i \subset X$.
6. In case of *complete knowledge* $\pi(A_i) = 1$ and $\pi(A_j) = 0$ for $A_i, A_j \subset X$ and $i \neq j$ and in case of *complete ignorance* $\pi(A_i) = 1$, for all $A_i \subset X$.
7. The dual of the possibility is known as *necessity*, and defined by $\nu(A) = 1 - \pi(A^c)$, where A^c is complement of $A \subset X$.

2.2 Fuzzy Set Interpretation of Possibility and Concept of Restriction

In 1978 L. A. Zadeh presented the fuzzy set theory based treatment of possbility theory [14], which asserts that in absence of any other information about X, except a proposition of type X *is* F, where F is a fuzzy set, the possibility distribution of X is numerically equal to the grade membership function of X. The proposition X *is* F is known as a possibilistic restriction and the possibility distribution of X is the collection of all possible values of X [15]. A general form of restriction is X *isr* R, where X and R are variables and r is the restriction type. If r is blank (which gives X *is* R) the restriction becomes possibilistic restriction. When $r = p$ then the restriction is known as probabilistic restriction. Similarly we can form various other restrictions.In most cases variables $X$and R are defined in natural languages, and if R is crisp and singular the restriction becomes an *equation* (Example - x is 9). On the other hand if R is non-singular and fuzzy it denotes information (For example *Ram is handsome*) [16,17].

A possibilistic restriction *"X is A"* represents a possibility distribution defined by $\pi(X = u) = \mu_A(u)$, where A is a fuzzy set with membership function μ_A. According to Zadeh the information conveyed by restriction *"X is A"* is equivalent to the possibility distribution $\Pi = \mu_A$, thus a restriction can be used to denote and measure information. A more *restricted* restriction represents more information (known as *minimum specificity*) [18].

If $P(A_i)$ is the probability and $\pi(A_i)$ is possibility of $A_i \subset X$ the according to Zadeh $P(A_i) \leq \pi(A_i)$, i.e. possibility of an event is always higher than the probability of that event. This property is termed as the phrase *"what is probable, that is possible"*. The difference between Possibility and probability is that for pair of disjoint sets A and B, probability follows *additivity* $P(A \cup B) = P(A) + P(B)$, and possibility follows *maximality* condition $\pi(A \cup B) = \max\{\pi(A), \pi(B)\}$. Probability and possibility also follow a *consistency principle* [14], which states that for a constant C and $A_i \subset X$.

$$\sum_{i}^{n} P(A_i)\pi(A_i) = C \tag{1}$$

2.3 Problems Related to Information Represented by Restriction

In recent paper from Zadeh [19], it has been proposed that information is equivalent to restriction. Generally information is represented by possibilistic restriction defined in natural language. Human mind has a remarkable ability to convert information in possibilistic restriction and solve problems on the basis of restrictions formed. For example, consider two statements -

- *Usually it takes half an hour to reach railway station.*
- *At peak time it takes longer than usual in traffic.*

Both these statements are example of information conveyed by possibilistic restriction. Both these restriction are defined on the variable time with fuzzy limits (*usual* and *longer than usual*), and hence the solution will be in terms of time or an interval of time. To imitate this process in machines is bit difficult due to following reasons -

1. The process of fitting a possibility distribution is not well formalized as that of a probability distribution [20,21].
2. The Fitting a restriction on data set is difficult as compare to finding a restriction on data set. This process is known as precisiation of restriction.
3. There are no formal methods to form a restriction on data set and precisiate it, specially when data is in continuous form [16,22].
4. Current systems of analysis of information have limited capabilities of handling data of specific types [17,23].

In view of above problems information representation by restriction has not been used widely despite of being capable of handling vagueness and uncertainty well.

2.4 A New Restriction Based Method

As it is clear by above discussion that restriction representation of information captures the vagueness and semantic part of information communication very well. This property of restrictions make it an appropriate tool for representing information in a hybrid system [24,25]. We first recall a hybrid system which has both human and machine sub-parts. Human part communicate in natural language (verbal) and gestures (non-verbal) and thus generate various types of data. The machine sub-part communicates in data and machine language. Both parts inter communicate by command - response method in which human part generates data as command and machine sub-part responses with action. Thus if we can teach machine to read data as restriction and make it enable to analyse information through possibilistic restriction, the intercommunication will be more effective and efficient.

2.5 Defining New Measure

Again to achieve our goal we define new measures for a hybrid system. Suppose $X = \{x_1, x_2...x_n\}$ is the set of observation and $F = \{f_1, f_2...f_n\}$ is the set of respective frequencies. The system starts with selecting focal elements, which are the decision variables in the system. The confirmation stage is the stage at which the number of sufficient observations (n) is confirmed. One more criterion will be discussed later after defining the measures.Let $N = \sum f_i$ and the mode observation-frequency duo is (x_M, f_M), which is unique. The ordered pair of observation (X, F) is what Zadeh termed as *Explanatory Database* (ED) [19]. We first define a natural probability measure $P : (X, F) \to [0, 1]$ by

$$P(x_i) = \frac{f_i}{N} \tag{2}$$

Second, we define a measure of natural possibility $\pi_M :\to [0, 1]$ by

$$\pi_M(x_i) = \frac{f_i}{f_M} \tag{3}$$

The measure defined by equation (2) is the classic definition of probability. This definition is also used in the confirmation phase, that is, confirm the minimum required observation n such that $N = \sum\limits_{i=1}^{n} f_i$ and

$$\sum_{i=1}^{n} P(x_i) = \sum_{i=1}^{n} (\frac{f_i}{N}) = 1 \tag{4}$$

Similarly measure defined by equation (3) is a measure which follows possibility distribution on (X, F). The sufficient condition for this is that, the modal frequency (maximum frequency) f_M is unique and much higher than other frequencies ($f_i \ll f_M$). We shall show that an ED following above stated sufficient conditions follows the properties of a possibility distribution -

- *Normalization*- Since for mode x_M, $\pi_M(x_M) = \frac{f_M}{f_M} = 1$, hence the distribution is normalized.

- *Maximality*- Let $f_i \vee f_j$ denote frequencies of observation $x_i \vee x_j$, where $f_i \wedge f_j$ is nill, then by condition f_M is unique and much higher than other frequencies ($f_i \ll f_M$),we have without loss of generality

$$\begin{aligned}
\pi_M(x_i \vee x_j) &= \frac{f_i \vee f_j}{f_M} \\
&= \frac{f_i + f_j}{f_M} && f_i \wedge f_j = 0 \\
&= \frac{f_i}{f_M} + \frac{f_j}{f_M} && \text{Since } f_i \ll f_M \text{ it is sufficient to assume} \\
\pi_M(x_i \vee x_j) &= \max\{\pi_M(x_i), \pi_M(x_j)\} && \text{this proves the maximality.}
\end{aligned}$$

- *Probability-Possibility Relation* - By definition from equations (2) & (3)and the fact that $N > f_M$, it is clear that

$$\begin{aligned}
\frac{f_i}{N} &\leq \frac{f_i}{f_M} \\
P(x_i) &\leq \pi_M(x_i)
\end{aligned}$$

It is clear from the above discussion that the measure defined by equation (3) is a possibility measure. Also, the ED (X, F) is restricted by the modal frequency, the restriction x is x_M, where x_M being the mode. The condition $f_i \ll f_M$ is also significant in real world situation because the mode is the most probable and most possible of all the outcomes or observations. Hence this schema mimics the human ability to calculate and compute information by restriction. Now consider the sum (this constitutes the decision phase)

$$\begin{aligned}
\sum_{i=1}^{n} \pi_M(x_i) &= \pi_M(x_1) + \pi_M(x_2) + ... + \pi_M(x_M) + ... + \pi_M(x_n) \\
&= \frac{f_1}{f_M} + \frac{f_2}{f_M} + ... + \frac{f_M}{f_M} + ... + \frac{f_n}{f_M} \\
&= \frac{f_1}{f_M} + \frac{f_2}{f_M} + ... + 1 + ... + \frac{f_n}{f_M} \\
&= \{\frac{f_1}{f_M} + \frac{f_2}{f_M} + ... + \frac{f_n}{f_M}\} + 1 \\
&= \frac{f_1 + f_2 + ... + f_n}{f_M} + 1 \\
\sum_{i=1}^{n} \pi_M(x_i) - 1 &= \frac{\bar{N}}{f_M} && \text{The sum } \bar{N} \text{ in denominator does not contain } f_M
\end{aligned}$$

Without loss of generality we can write by equation (4) that, $\sum_{i=1}^{n} P(x_i) = 1$.

$$\sum_{i=1}^{n} \pi_M(x_i) - \sum_{i=1}^{n} P(x_i) = \frac{\bar{N}}{f_M} \tag{5}$$

Above equation (5) provides a useful measure of difference between sum of all possibilities and sum of probabilities for the ED (X, F). Since the RHS of above equation is sum of frequencies, and hence non negative, this implies that the difference between possibility and probability is positive value. This also proves that $P(x_i) \leq \pi(x_i)$. Thus based on above measures the information analysis system progresses through following steps or phases -

- *Initiation Phase* - For a problem, various decision variables are listed, x_1, x_2...., which constitutes the observation set X. The variables can be multi-dimensional.This phase is also a *learning phase* for system.
- *Confirmation Phase* - For each variable the frequencies are measured.In this phase the minimum number of observations n is also confirmed, hence an explanatory database is formed (X, F).
- *Analysis Phase* - In this phase we define and evaluate measures defined by equations (2),(3) & (5). If the sufficient conditions of unique much greater mode frequency is not met, we can break (X, F) in disjoint subsets in which the condition is satisfied.
- *Decision Phase* - In this phase the criterion laid out by equation (5) is used to follow minimum difference (Precisiation of restriction). Together with minimum specificity the decisions can be made.The steps are repeated until the desired result is obtained.The system can progress through time and newer observations can be added and old observations can be removed.The*Principle of minimum specificity* dictates the selection of newer observations to add.

This system mimics the human process of decision making without relying on data sufficiency and complete knowledge. Only requirement is the accuracy of observations. The above process tries to resolve the difficulty of fitting possibility distribution when data is given or data set is insufficient.

2.6 Conclusion and Further Work

In above discussion it is shown that restrictions are useful tool to represent and measure information and on the basis of data set we can construct a natural possibility distribution not based on probability [7,26]. Also various properties of possibility measure are proved for defined novel measure with sufficient conditions in place. Finally we have defined a difference measure for possibility and possibility which works as the decision criterion for the model. Further we will work on non-unique modal (X, F) and refine the system. Some applications of this model will also be researched. Suggestions for improvement from scientific community are welcome.

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