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Vector Basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-Cordial Labeling of $D(T_n)\odot mK_1$ and $D(Q_n)\odot mK_1$

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Abstract

Let G be a (p,q) graph. Let V be an inner product space with basis S. We denote the inner product of the vectors x and y by $< x,y >$. Let $\phi : V(G) \to S$ be a function. For edge uv assign the label $< \phi(u),\phi(v) >$. Then ϕ is called a vector basis S-cordial labeling of G if $|\phi_x - \phi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where ϕ_x denotes the number of vertices labeled with the vector x and γ_i denotes the number of edges labeled with the scalar i. A graph which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph. In this paper, we prove that the graphs $D(T_n)\odot mK_1$ and $D(Q_n)\odot mK_1$ admit a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial.

Keywords: triangular snake; quadrilateral snake; double triangular snake; double quadrilateral snake; star graph.

2020 Mathematics Subject Classification: 05C78, 05C38.

1. Introduction

In this paper, the graph $G(V,E)$ mean a finite, simple and undirected. The paper written by Euler on the seven bridges of Konigsberg and published in 1736 is regarded as the first paper in the history of graph theory. Graph labeling problems was first introduced by Rosa in 1967 [15]. The progress and uses of graph labeling are significant when compared other fields of mathematics. Graph labeling is a dynamic field of study within graph theory that has primarily developed due to its various applications in mobile telecommunications systems, optimal circuit designs, graph decomposition problems, coding theory and communication networks. Gowri and Jayapriya [4] have explored the HMC labeling behavior of triangular snake, alternate triangular snake, double triangular snake and alternate double triangular snake. Moreover the quotient labeling number of quadrilateral snake, double quadrilateral snake, alternate triangular snake, alternate double triangular snake, subdivision of triangular snake and subdivision of quadrilateral snake have been investigated by Sumathi and Rathi [17]. The concept of heronian mean labeling was introduced by santhiya et al. and moreover

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studied the heronian mean labeling of triangular snake, double triangular snake, quadrilateral snake and double quadrilateral snake in [16].

The innovative concept of cordial labeling was introduced by Cahit [1]. Pair mean cordial labeling of triangular snake, alternate triangular snake, quadrilateral snake and alternate quadrilateral snake, hexagonal snake, irregular quadrilateral snake and triple triangular snake was discussed in [10,11]. The innovative idea of group mean cordial labeling was introduced by Rajalekshmi and Kala and they have computed that the triangular snake, alternate triangular snake, double triangular snake, alternate double triangular snake, quadrilateral snake, alternate quadrilateral snake, double quadrilateral snake and alternate double quadrilateral snake are group mean graphs [13,14]. H_k-cordial labeling of triangular snake, double triangular snake, triple triangular snake, alternate triangular snake, irregular triangular snake, quadrilateral snake, double quadrilateral snake, alternate quadrilateral snake, irregular quadrilateral snake have been studied in [2]. Ponraj et al. [12] computed that the prism, Mongolian tent, book, young tableau, $K_m \times P_2$, torus grids, n-cube graphs are difference cordial. Prime cordial and 3-equitable prime cordial graphs was discussed in [18]. The standard terminology and notations that we follow by Harary [5] and Hertein [6].

Various types of graph labeling have been studied in an excellent survey of graph labeling by Gallian [3]. Ponraj and Jeya [7] have introduced the vector basis S-cordial labeling of graphs and the vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling of behavior of the path, cycle, star, comb, complete graph, generalized friendship graph, tadpole graph and gear graph and thorn related graphs have been investigated in [7–9]. In this paper, we show that the graphs $D(T_n) \odot mK_1$ and $D(Q_n) \odot mK_1$ are a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial.

2. Preliminaries

In this section, we state a few definitions which are relevant for proving the main results.

Definition 2.1 ([16]). *The triangular snake T_n is obtained from a path $P_n : u_1u_2 \ldots u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \le i \le n-1$. That is every edge of a path is replaced by a triangle.*

Definition 2.2 ([16]). *The double triangular snake $D(T_n)$ is obtained from a path $P_n : u_1u_2 \ldots u_n$ by joining u_i and u_{i+1} to a new vertices v_i and w_i for $1 \le i \le n-1$. That is it consists of two triangular snakes that have a common path.*

Definition 2.3 ([16]). *The quadrilateral snake Q_n is obtained from a path $P_n : u_1u_2 \ldots u_n$ by joining u_i and u_{i+1} to a new vertices v_i and w_i respectively and adding edges v_iw_i for $1 \le i \le n-1$. That is every edge of a path is replaced by a cycle C_4.*

Definition 2.4 ([16]). *The double quadrilateral snake $D(Q_n)$ is obtained from two quadrilateral snakes that have a common path.*

Definition 2.5 ([3]). *The corona graph $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and n copies of G_2 and joining i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2, where G_1 is graph of order n.*

In this paper, we consider the inner product space R^n and the standard inner product $\langle x, y\rangle = x_1y_1 + x_2y_2 + \cdots + x_ny_n$ where $x = (x_1, x_2, \ldots, x_n), y = (y_1, y_2, \ldots, y_n)$, $x_i, y_i \in R$.

3. Vector Basis S-Cordial Labeling

Let G be a (p, q) graph. Let V be an inner product space with basis S. We denote the inner product of the vectors x and y by $\langle x, y\rangle$. Let $\phi : V(G) \to S$ be a function. For edge uv assign the label $\langle \phi(u), \phi(v)\rangle$. Then ϕ is called a vector basis S-cordial labeling of G if $|\phi_x - \phi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where ϕ_x denotes the number of vertices labeled with the vector x and γ_i denotes the number of edges labeled with the scalar i. A graph which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph. An illustration for the vector basis $\{(1, 1, 1, 1), (1, 1, 1, 0), (1, 1, 0, 0), (1, 0, 0, 0)\}$-cordial labeling of graph is shown in Figure 1.

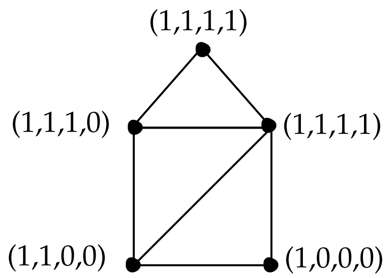

Figure 1. A vector basis $\{(1, 1, 1, 1), (1, 1, 1, 0), (1, 1, 0, 0), (1, 0, 0, 0)\}$-cordial graph

4. Main Results

In this section, we discuss the existence of the vector basis $\{(1, 1, 1, 1), (1, 1, 1, 0), (1, 1, 0, 0), (1, 0, 0, 0)\}$-cordial labeling of $D(T_n) \odot mK_1$ and $D(Q_n) \odot mK_1$.

Theorem 4.1. *Every graph is a subgraph of a connected vector basis $\{(1, 1, 1, 1), (1, 1, 1, 0), (1, 1, 0, 0), (1, 0, 0, 0)\}$-cordial graph.*

Proof. Let G be a (p, q) graph. Consider the four copies of the complete graph K_p. Denote the i^{th} copy of K_p by K_p^i, $i = 1, 2, 3, 4$. Let $V(K_p^i) = \{u_1^i, u_2^i, \ldots, u_p^i\}$. We construct the new graph which is a super graph G^* as follows: Let $V(G^*) = V(K_p^1) \cup V(K_p^2) \cup V(K_p^3) \cup V(K_p^4)$ and $E(G^*) = E(K_p^1) \cup E(K_p^2) \cup E(K_p^3) \cup E(K_p^4) \cup \{u_1^1u_1^2, u_1^2u_1^3, u_1^3u_1^4\}$. Now we assign the labels to the vertices of $V(G^*)$ as follows: First assign the vector $(1, 0, 0, 0)$ to all the vertices of K_p^1 and assign the vector $(1, 1, 0, 0)$ to all the vertices of K_p^2. Then assign the vector $(1, 1, 1, 0)$ to all the vertices of K_p^3 and finally assign the vector $(1, 1, 1, 1)$ to all the vertices of K_p^4. Clearly $\phi_{(1,0,0,0)} = p$, $\phi_{(1,1,0,0)} = p$, $\phi_{(1,1,1,0)} = p$ and $\phi_{(1,1,1,1)} = p$. Hence $\gamma_1 = \binom{p}{2} + 1$, $\gamma_2 = \binom{p}{2} + 1$, $\gamma_3 = \binom{p}{2} + 1$ and $\gamma_4 = \binom{p}{2}$. Clearly this vertex labeling is a vector basis $\{(1, 1, 1, 1), (1, 1, 1, 0), (1, 1, 0, 0), (1, 0, 0, 0)\}$-cordial. □

Example 4.2. *We know that C_4 is not vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial graph [7]. But it is a subgraph of following graph (figure 2) which is a vector basis {(1,1,1,1),(1,1,1,0), (1,1,0,0),(1,0,0,0)}-cordial graph. Here $\phi_{(1,0,0,0)} = \phi_{(1,1,0,0)} = \phi_{(1,1,1,0)} = \phi_{(1,1,1,1)} = 4$. Hence $\gamma_1 = \gamma_2 = \gamma_3 = \binom{4}{2} + 1 = 7$ and $\gamma_4 = \binom{4}{2} = 6$.*

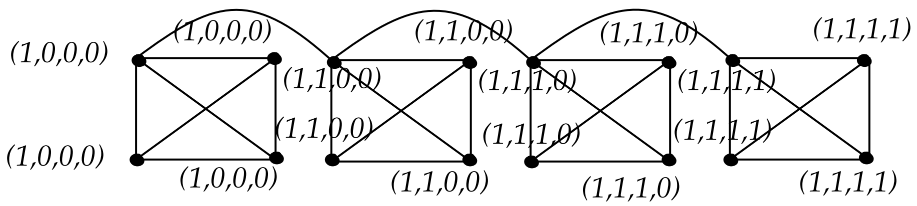

Figure 2. A vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial graph

Theorem 4.3. *The graph $D(T_n) \odot mK_1$ is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial for all $n, m \geq 2$.*

Proof. Denote by $V(D(T_n) \odot mK_1) = \{u_i, u_{ij} \mid 1 \leq i \leq n \text{ and } 1 \leq j \leq m\} \cup \{v_i, w_i, v_{ij}, w_{ij} \mid 1 \leq i \leq n-1 \text{ and } 1 \leq j \leq m\}$ and $E(D(T_n) \odot mK_1) = \{u_iu_{i+1}, u_iv_i, u_{i+1}v_i, u_iw_i, u_{i+1}w_i, v_iv_{ij}, w_iw_{ij} \mid 1 \leq i \leq n-1 \text{ and } 1 \leq j \leq m\} \cup \{u_iu_{ij} \mid 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}$ respectively the vertex and edge sets of $D(T_n) \odot mK_1$. Clearly

$$p = |V(D(T_n) \odot mK_1)| = (3n-2)(m+1)$$

and $q = |E(D(T_n) \odot mK_1)| = (3n-2)(m+1) + 2n - 3$. Assign the vectors to the vertices in the following order u_1, u_2, v_1, w_1, u_3, v_2, w_2, u_4, v_3, w_3, $\ldots$, u_{n-1}, u_n, v_{n-1}, w_{n-1} and u_{11}, u_{12}, $\ldots$, u_{1m}, v_{11}, v_{12}, $\ldots$, v_{1m}, w_{11}, w_{12}, $\ldots$, w_{1m}, u_{21}, u_{22}, $\ldots$, u_{2m}, v_{21}, v_{22}, $\ldots$, v_{2m}, w_{21}, w_{22}, $\ldots$, w_{2m}, $\ldots$, $v_{n-1,1}$, $v_{n-1,2}$, $\ldots$, $v_{n-1,m}$, $w_{n-1,1}$, $w_{n-1,2}$, $\ldots$, $w_{n-1,m}$, $u_{n,1}$, $u_{n,2}$, $\ldots$, u_{nm}. We have consider the following four cases:

Case (i): $2n - 2 \equiv 0 \pmod 4$

Let $2n-2 = 4s_1$, $s_1 > 0$. We get $n = 2s_1 + 1$, $p = (6s_1+1)(m+1)$, and $q = (6s_1+1)(m+1) + 4s_1 - 1$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $\frac{3s_1}{2} + 1$ vertices. Then assign the vector $(1,1,1,0)$ to the next $\frac{3s_1}{2}$ vertices and assign the vector $(1,1,0,0)$ to the next $\frac{3s_1}{2}$ vertices. Finally assign the vector $(1,0,0,0)$ to the next $\frac{3s_1}{2}$ vertices. From these vertex labeling, we get $\frac{5s_1}{2}, \frac{5s_1}{2}, \frac{5s_1}{2}, \frac{5s_1}{2}$ edges with edge label 4,3,2,1 respectively.

When s_1 is odd, assign the vector (1,1,1,1) to the first $\frac{3(s_1+1)}{2}$ vertices. Then assign the vector (1,1,1,0) to the next $\frac{3(s_1+1)}{2} - 2$ vertices and assign the vector (1,1,0,0) to the next $\frac{3(s_1-1)}{2} + 2$ vertices. Finally assign the vector $(1,0,0,0)$ to the next $\frac{3(s_1-1)}{2} + 1$ vertices. From these vertex labeling, we obtain $\frac{5(s_1-1)}{2} + 3, \frac{5(s_1-1)}{2} + 2, \frac{5(s_1-1)}{2} + 3, \frac{5(s_1-1)}{2} + 2$ edges with edge label $4,3,2,1$ respectively.

Subcase (A): $m \equiv 0 \pmod 4$

Let $m = 4s_2$, $s_2 > 0$. We get $p = 24s_1s_2 + 6s_1 + 4s_2 + 1$, and $q = 24s_1s_2 + 10s_1 + 4s_2$. If s_1 is even, assign the vector (1,1,1,1) to the first $6s_1s_2 + s_2$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $6s_1t_2 + s_2$ pendent vertices and assign the vector (1,1,0,0) to the next $6s_1s_2 + s_2$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $6s_1s_2+s_2$ pendent vertices. We get $\phi_{(1,1,1,1)} = 6s_1s_2+s_2+\frac{3s_1}{2}+1$, $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2+s_2+\frac{3s_1}{2}$. Clearly $\gamma_1=\gamma_2=\gamma_3=\gamma_4 = 6s_1s_2+s_2+\frac{5s_1}{2}$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $6s_1s_2+s_2-1$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $6s_1s_2+s_2+1$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $6s_1s_2+s_2$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $6s_1s_2+s_2$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = 6s_1s_2+s_2+\frac{3(s_1+1)}{2}-1$, $\phi_{(1,1,0,0)} = 6s_1s_2+s_2+\frac{3(s_1-1)}{2}+2$ and $\phi_{(1,0,0,0)} = 6s_1s_2+s_2+\frac{3(s_1-1)}{2}+1$. Clearly $\gamma_1=\gamma_4 = 6s_1s_2+s_2+\frac{5(s_1-1)}{2}+2$ and $\gamma_2=\gamma_3 = 6s_1s_2+s_2+\frac{5(s_1-1)}{2}+3$.

Subcase (B): $m \equiv 1 \pmod 4$

Let $m = 4s_2+1$, $s_2 > 0$. We get $p = 24s_1s_2+12s_1+4s_2+2$, and $q = 24s_1s_2+16s_1+4s_2+1$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $6s_1s_2+s_2+\frac{3s_1}{2}$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $6s_1s_2+s_2+\frac{3s_1}{2}$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $6s_1s_2+s_2+\frac{3s_1}{2}$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $6s_1s_2+s_2+\frac{3s_1}{2}+1$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,0,0,0)} = 6s_1s_2+s_2+3s_1+1$, $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = 6s_1s_2+s_2+3s_1$. Clearly $\gamma_2=\gamma_3=\gamma_4 = 6s_1s_2+s_2+4s_1$ and $\gamma_1 = 6s_1s_2+s_2+4s_1+1$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $6s_1s_2+s_2+\frac{3(s_1-1)}{2}+1$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $6s_1s_2+s_2+\frac{3(s_1-1)}{2}+2$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $6s_1s_2+s_2+\frac{3(s_1+1)}{2}-2$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $6s_1s_2+s_2+\frac{3(s_1+1)}{2}$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,0,0,0)} = 6s_1s_2+s_2+3s_1+1$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = 6s_1s_2+s_2+3s_1$. Clearly $\gamma_4=\gamma_3=\gamma_2 = 6s_1s_2+s_2+4s_1$ and $\gamma_1 = 6s_1s_2+s_2+4s_1+1$.

Subcase (C): $m \equiv 2 \pmod 4$

Let $m = 4s_2+2$, $s_2 \geq 0$. We get $p = 24s_1s_2+18s_1+4s_2+3$, and $q = 24s_1s_2+22s_1+4s_2+2$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $6s_1s_2+s_2+3s_1$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $6s_1s_2+s_2+3s_1$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $6s_1s_2+s_2+3s_1+1$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $6s_1s_2+s_2+3s_1+1$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2+s_2+\frac{9s_1}{2}+1$ and $\phi_{(1,1,1,0)} = 6s_1s_2+s_2+\frac{9s_1}{2}$. Clearly $\gamma_4=\gamma_3 = 6s_1s_2+s_2+\frac{11s_1}{2}$ and $\gamma_2=\gamma_1 = 6s_1s_2+s_2+\frac{11s_1}{2}+1$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $6s_1s_2+s_2+3s_1$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $6s_1s_2+s_2+3s_1+1$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $6s_1s_2+s_2+3s_1$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $6s_1s_2+s_2+3s_1+1$ pendent vertices. We have $\phi_{(1,1,1,1)} = 6s_1s_2+s_2+\frac{9s_1+3}{2}$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2+s_2+\frac{9s_1+1}{2}$. Clearly $\gamma_4=\gamma_2=\gamma_1 = 6s_1s_2+s_2+\frac{11s_1+1}{2}$ and $\gamma_3 = 6s_1s_2+s_2+\frac{11s_1-1}{2}$.

Subcase (D): $m \equiv 3 \pmod 4$

Let $m = 4s_2+3$, $s_2 \geq 0$. We get $p = 24s_1s_2+24s_1+4s_2+4$, and $q = 24s_1s_2+28s_1+4s_2+3$. If s_1 is even, assign the vector (1,1,1,1) to the first $6s_1s_2+s_2+\frac{9s_1}{2}$ pendent vertices. Then assign the vector (1,1,1,0) to the next $6s_1s_2+s_2+\frac{9s_1}{2}+1$ pendent vertices and assign the vector $(1,1,0,0)$ to the next

$6s_1s_2 + s_2 + \frac{9s_1}{2} + 1$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $6s_1s_2 + s_2 + \frac{9s_1}{2} + 1$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2 + s_2 + 6s_1 + 1$. Clearly $\gamma_4 = 6s_1s_2 + s_2 + 7s_1$ and $\gamma_3 = \gamma_2 = \gamma_1 = 6s_1s_2 + s_2 + 7s_1 + 1$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $6s_1s_2 + s_2 + \frac{9s_1-1}{2}$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $6s_1s_2 + s_2 + \frac{9s_1+1}{2}$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $6s_1s_2 + s_2 + \frac{9s_1+1}{2}$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $6s_1s_2 + s_2 + \frac{9s_1+3}{2}$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2 + s_2 + 6s_1 + 1$. Clearly $\gamma_4 = 6s_1s_2 + s_2 + 7s_1$ and $\gamma_3 = \gamma_2 = \gamma_1 = 6s_1s_2 + s_2 + 7s_1 + 1$.

Case (ii): $2n - 2 \equiv 1 \pmod 4$

Let $2n - 2 = 4s_1 + 1$, $s_1 \geq 0$. We get $n = 2s_1 + 2$, $p = (6s_1 + 4)(m + 1)$, and $q = (6s_1 + 4)(m + 1) + 4s_1 + 1$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $\frac{3s_1}{2} + 3$ vertices. Then assign the vector $(1,1,1,0)$ to the next $\frac{3s_1}{2} + 1$ vertices and assign the vector $(1,1,0,0)$ to the next $\frac{3s_1}{2}$ vertices. Finally assign the vector $(1,0,0,0)$ to the next $\frac{3s_1}{2}$ vertices. From these vertex labeling, we get $\frac{5s_1}{2} + 3, \frac{5s_1}{2} + 2, \frac{5s_1}{2}, \frac{5s_1}{2}$ edges with edge label $4,3,2,1$ respectively.

When s_1 is odd, assign the vector $(1,1,1,1)$ to the first $\frac{3(s_1+1)}{2} + 1$ vertices. Then assign the vector $(1,1,1,0)$ to the next $\frac{3(s_1+1)}{2}$ vertices and assign the vector $(1,1,0,0)$ to the next $\frac{3(s_1-1)}{2} + 2$ vertices. Finally assign the vector $(1,0,0,0)$ to the next $\frac{3(s_1-1)}{2} + 1$ vertices. From these vertex labeling, we obtain $\frac{5(s_1+1)}{2}, \frac{5(s_1+1)}{2}, \frac{5(s_1-1)}{2} + 3, \frac{5(s_1-1)}{2} + 2$ edges with edge label $4,3,2,1$ respectively.

Subcase (A): $m \equiv 0 \pmod 4$

Let $m = 4s_2$, $s_2 > 0$. We get $p = 24s_1s_2 + 6s_1 + 16s_2 + 4$, and $q = 24s_1s_2 + 10s_1 + 16s_2 + 5$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $6s_1s_2 + 4s_2 - 2$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $6s_1s_2 + 4s_2$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $6s_1s_2 + 4s_2 + 1$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $6s_1s_2 + 4s_2 + 1$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2 + 4s_2 + \frac{3s_1}{2} + 1$. Clearly $\gamma_1 = \gamma_2 = \gamma_4 = 6s_1s_2 + 4s_2 + \frac{5s_1}{2} + 1$ and $\gamma_3 = 6s_1s_2 + 4s_2 + \frac{5s_1}{2} + 2$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $6s_1s_2 + 4s_2 - 1$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $6s_1s_2 + 4s_2 - 1$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $6s_1s_2 + 4s_2 + 1$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $6s_1s_2 + 4s_2 + 1$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,0,0)} = 6s_1s_2 + 4s_2 + \frac{3(s_1+3)}{2}$ and $\phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2 + s_2 + \frac{3(s_1+1)}{2} + 2$. Clearly $\gamma_4 = \gamma_3 = \gamma_2 = 6s_1s_2 + 4s_2 + \frac{5(s_1+3)}{2}$ and $\gamma_1 = 6s_1s_2 + s_2 + \frac{5(s_1+1)}{2}$.

Subcase (B): $m \equiv 1 \pmod 4$

Let $m = 4s_2 + 1$, $s_2 > 0$. We get $p = 24s_1s_2 + 12s_1 + 16s_2 + 8$, and $q = 24s_1s_2 + 16s_1 + 16s_2 + 9$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $6s_1s_2 + 4s_2 + \frac{3s_1}{2} - 1$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $6s_1s_2 + 4s_2 + \frac{3s_1}{2} + 1$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $6s_1s_2 + 4s_2 + \frac{3s_1}{2} + 2$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $6s_1s_2 + 4s_2 + \frac{3s_1}{2} + 2$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2 + 4s_2 + 3s_1 + 2$. Clearly $\gamma_4 = \gamma_2 = \gamma_1 = 6s_1s_2 + 4s_2 + 4s_1 + 2$ and $\gamma_3 = 6s_1s_2 + 4s_2 + 4s_1 + 3$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $6s_1s_2 + s_2 + \frac{3(s_1-1)}{2} + 1$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $6s_1s_2 + s_2 + \frac{3(s_1-1)}{2} + 2$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $6s_1s_2 + s_2 + \frac{3(s_1+1)}{2}$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $6s_1s_2 + s_2 + \frac{3(s_1+1)}{2} + 1$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2 + 4s_2 + 3s_1 + 2$. Clearly $\gamma_4 = \gamma_2 = \gamma_1 = 6s_1s_2 + 4s_2 + 4s_1 + 2$ and $\gamma_3 = 6s_1s_2 + s_2 + 4s_1 + 3$.

Subcase (C): $m \equiv 2 \pmod 4$

Let $m = 4s_2 + 2$, $s_2 \geq 0$. We get $p = 24s_1s_2 + 18s_1 + 16s_2 + 12$, and $q = 24s_1s_2 + 22s_1 + 16s_2 + 13$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $6s_1s_2 + 4s_2 + 3s_1$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $6s_1s_2 + 4s_2 + 3s_1 + 2$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $6s_1s_2 + 4s_2 + 3s_1 + 3$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $6s_1s_2 + 4s_2 + 3s_1 + 3$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2 + 4s_2 + \frac{9s_1}{2} + 3$. Clearly $\gamma_4 = \gamma_2 = \gamma_1 = 6s_1s_2 + 4s_2 + \frac{11s_1}{2} + 3$ and $\gamma_3 = 6s_1s_2 + 4s_2 + \frac{11s_1}{2} + 4$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $6s_1s_2 + 4s_2 + 3s_1 + 1$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $6s_1s_2 + 4s_2 + 3s_1 + 1$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $6s_1s_2 + 4s_2 + 3s_1 + 2$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $6s_1s_2 + 4s_2 + 3s_1 + 4$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,0,0,0)} = 6s_1s_2 + 4s_2 + \frac{9s_1+7}{2}$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = 6s_1s_2 + 4s_2 + \frac{9s_1+5}{2}$. Clearly $\gamma_4 = \gamma_3 = \gamma_1 = 6s_1s_2 + 4s_2 + \frac{11s_1+7}{2}$ and $\gamma_2 = 6s_1s_2 + 4s_2 + \frac{11s_1+5}{2}$.

Subcase (D): $m \equiv 3 \pmod 4$

Let $m = 4s_2 + 3$, $s_2 \geq 0$. We get $p = 24s_1s_2 + 24s_1 + 16s_2 + 16$, and $q = 24s_1s_2 + 28s_1 + 16s_2 + 17$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $6s_1s_2 + 4s_2 + \frac{9s_1}{2} + 1$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $6s_1s_2 + 4s_2 + \frac{9s_1}{2} + 3$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $6s_1s_2 + 4s_2 + \frac{9s_1}{2} + 4$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $6s_1s_2 + 4s_2 + \frac{9s_1}{2} + 4$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2 + 4s_2 + 6s_1 + 4$. Clearly $\gamma_4 = \gamma_2 = \gamma_1 = 6s_1s_2 + 4s_2 + 7s_1 + 4$ and $\gamma_3 = 6s_1s_2 + 4s_2 + 7s_1 + 5$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $6s_1s_2 + 4s_2 + \frac{9s_1+3}{2}$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $6s_1s_2 + 4s_2 + \frac{9s_1+5}{2}$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $6s_1s_2 + 4s_2 + \frac{9s_1+7}{2}$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $6s_1s_2 + 4s_2 + \frac{9s_1+9}{2}$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 6s_1s_2 + 4s_2 + 6s_1 + 4$. Clearly $\gamma_4 = \gamma_2 = \gamma_1 = 6s_1s_2 + 4s_2 + 7s_1 + 4$ and $\gamma_3 = 6s_1s_2 + 4s_2 + 7s_1 + 5$.

Hence the vertex labeling ϕ is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling of $D(T_n) \odot mK_1$ for all $n, m \geq 2$. □

Example 4.4. *An illustration for the vector basis* $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$*-cordial labeling of* $D(T_6) \odot 4K_1$ *for the case when* $n \equiv 2 \pmod 4$ *and* $m \equiv 0 \pmod 4$ *is shown in Figure 3.*

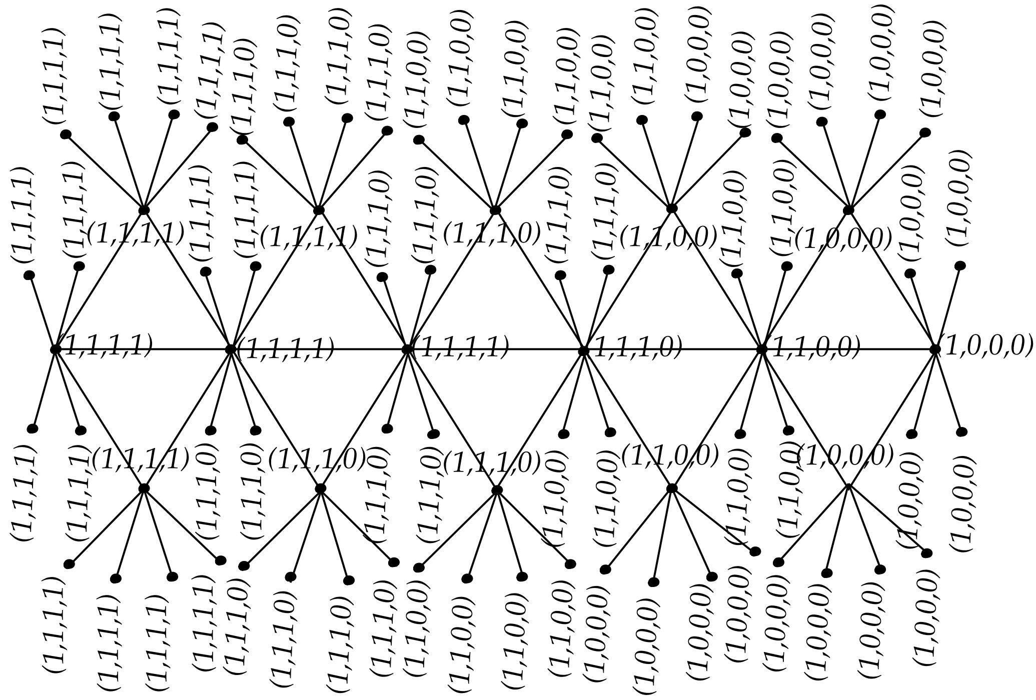

Figure 3. A vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of $D(T_6) \odot 4K_1$

Theorem 4.5. *The graph $D(Q_n) \odot mK_1$ is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial for all $n, m \geq 2$.*

Proof. Denote by $V(D(Q_n) \odot mK_1) = \{u_i, u_{ij} \mid 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}$ $\cup\{v_i, w_i, v_{ij}, w_{ij}, x_i, y_i, x_{ij}, y_{ij} \mid 1 \leq i \leq n-1$ and $1 \leq j \leq m\}$ and $E(D(Q_n) \odot mK_1)$ $= \{u_iu_{i+1}, u_iv_i, u_ix_i, v_iw_i, x_iy_i, u_{i+1}w_i, u_{i+1}y_i, v_iv_{ij}, w_iw_{ij}, x_ix_{ij}, y_iy_{ij} \mid 1 \leq i \leq n-1 \text{ and } 1 \leq j \leq m\} \cup \{u_iu_{ij} \mid 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}$ respectively the vertex and edge sets of $D(Q_n) \odot mK_1$. Clearly

$$p = |V(D(Q_n) \odot mK_1)| = (5n-4)(m+1) \quad and \quad q = |E(D(Q_n) \odot mK_1)| = (5n-4)(m+1) + 2n - 3$$

Assign the vectors to the vertices in the following order: u_1, u_2, w_1, v_1, x_1, y_1, u_3, w_2, v_2, x_2, y_2, u_4, w_3, v_3, x_3, y_3, $\ldots$, u_{n-1}, u_n, w_{n-1}, v_{n-1}, x_{n-1}, y_{n-1} and u_{11}, u_{12}, $\ldots$, u_{1m}, v_{11}, v_{12}, $\ldots$, v_{1m}, w_{11}, w_{12}, $\ldots$, w_{1m}, x_{11}, x_{12}, $\ldots$, x_{1m}, y_{11}, y_{12}, $\ldots$, y_{1m}, u_{21}, u_{22}, $\ldots$, u_{2m}, v_{21}, v_{22}, $\ldots$, v_{2m}, w_{21}, w_{22}, $\ldots$, w_{2m}, x_{21}, x_{22}, $\ldots$, x_{2m}, y_{21}, y_{22}, $\ldots$, y_{2m}, $\ldots$, $v_{n-1,1}$, $v_{n-1,2}$, $\ldots$, $v_{n-1,m}$, $w_{n-1,1}$, $w_{n-1,2}$, $\ldots$, $w_{n-1,m}$, $x_{n-1,1}$, $x_{n-1,2}$, $\ldots$, $x_{n-1,m}$, $y_{n-1,1}$, $y_{n-1,2}$, $\ldots$, $y_{n-1,m}$, u_{n1}, u_{n2}, $\ldots$, u_{nm}. We have consider the following four cases:

Case (i): $2n - 2 \equiv 0 \pmod 4$

Let $2n-2 = 4s_1$, $s_1 > 0$. We get $n = 2s_1+1$, $p = (10s_1+1)(m+1)$, and $q = (10s_1+1)(m+1)+4s_1-1$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $\frac{5s_1}{2}+1$ vertices. Then assign the vector $(1,1,1,0)$ to the next $\frac{5s_1}{2}$ vertices and assign the vector $(1,1,0,0)$ to the next $\frac{5s_1}{2}$ vertices. Finally assign the vector $(1,0,0,0)$ to the next $\frac{5s_1}{2}$ vertices. From these vertex labeling, we get $\frac{7s_1}{2}, \frac{7s_1}{2}, \frac{7s_1}{2}, \frac{7s_1}{2}$ edges with edge label $4,3,2,1$ respectively.

When s_1 is odd, assign the vector $(1,1,1,1)$ to the first $\frac{5(s_1-1)}{2}+4$ vertices. Then assign the vector

$(1,1,1,0)$ to the next $\frac{5(s_1-1)}{2}+2$ vertices and assign the vector $(1,1,0,0)$ to the next $\frac{5(s_1-1)}{2}+3$ vertices. Finally assign the vector $(1,0,0,0)$ to the next $\frac{5(s_1-1)}{2}+2$ vertices. From these vertex labeling, we obtain $\frac{7(s_1-1)}{2}+4$, $\frac{7(s_1-1)}{2}+3$, $\frac{7(s_1-1)}{2}+4$, $\frac{7(s_1-1)}{2}+3$ edges with edge label $4,3,2,1$ respectively.

Subcase (A): $m \equiv 0 \pmod 4$

Let $m = 4s_2$, $s_2 > 0$. We get $p = 40s_1s_2 + 10s_1 + 4s_2 + 1$, and $q = 40s_1s_2 + 14s_1 + 4s_2$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $10s_1s_2 + s_2$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + s_2$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + s_2$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $10s_1s_2 + s_2$ pendent vertices. We get $\phi_{(1,1,1,1)} = 10s_1s_2 + s_2 + \frac{5s_1}{2} + 1$, $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 10s_1s_2 + s_2 + \frac{5s_1}{2}$. Clearly $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 10s_1s_2 + s_2 + \frac{7s_1}{2}$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $10s_1s_2 + s_2 - 1$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + s_2$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + s_2$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $10s_1s_2 + s_2 + 1$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 10s_1s_2 + s_2 + \frac{5(s_1-1)}{2} + 3$ and $\phi_{(1,1,1,0)} = 10s_1s_2 + s_2 + \frac{5(s_1-1)}{2} + 2$. Clearly $\gamma_4 = \gamma_3 = 10s_1s_2 + s_2 + \frac{7(s_1-1)}{2} + 3$ and $\gamma_2 = \gamma_1 = 10s_1s_2 + s_2 + \frac{7(s_1-1)}{2} + 4$.

Subcase (B): $m \equiv 1 \pmod 4$

Let $m = 4s_2 + 1$, $s_2 > 0$. We get $p = 40s_1s_2 + 20s_1 + 4s_2 + 2$, and $q = 40s_1s_2 + 24s_1 + 4s_2 + 1$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $10s_1s_2 + s_2 + \frac{5s_1}{2}$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + s_2 + \frac{5s_1}{2}$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + s_2 + \frac{5s_1}{2}$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $10s_1s_2 + s_2 + \frac{5s_1}{2} + 1$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,0,0,0)} = 10s_1s_2 + s_2 + 5s_1 + 1$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = 10s_1s_2 + s_2 + 5s_1$. Clearly $\gamma_4 = \gamma_3 = \gamma_2 = 10s_1s_2 + s_2 + 6s_1$ and $\gamma_1 = 10s_1s_2 + s_2 + 6s_1 + 1$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $10s_1s_2 + s_2 + \frac{5(s_1-1)}{2} + 2$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + s_2 + \frac{5(s_1-1)}{2} + 3$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + s_2 + \frac{5(s_1-1)}{2} + 2$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $10s_1s_2 + s_2 + \frac{5(s_1-1)}{2} + 4$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,0,0,0)} = 10s_1s_2 + s_2 + 5s_1 + 1$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = 10s_1s_2 + s_2 + 5s_1$. Clearly $\gamma_4 = \gamma_3 = \gamma_2 = 10s_1s_2 + s_2 + 6s_1$ and $\gamma_1 = 10s_1s_2 + s_2 + 6s_1 + 1$.

Subcase (C): $m \equiv 2 \pmod 4$

Let $m = 4s_2 + 2$, $s_2 \geq 0$. We get $p = 40s_1s_2 + 30s_1 + 4s_2 + 3$, and $q = 40s_1s_2 + 34s_1 + 4s_2 + 2$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $10s_1s_2 + s_2 + 5s_1$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + s_2 + 5s_1$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + s_2 + 5s_1 + 1$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $10s_1s_2 + s_2 + 5s_1 + 1$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 10s_1s_2 + s_2 + \frac{15s_1}{2} + 1$ and $\phi_{(1,1,1,0)} = 10s_1s_2 + s_2 + \frac{15s_1}{2}$. Clearly $\gamma_4 = \gamma_3 = 10s_1s_2 + s_2 + \frac{17s_1}{2}$ and $\gamma_2 = \gamma_1 = 10s_1s_2 + s_2 + \frac{17s_1}{2} + 1$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $10s_1s_2 + s_2 + 5s_1$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + s_2 + 5s_1 + 1$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + s_2 + 5s_1$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $10s_1s_2 + s_2 + 5s_1 + 1$ pendent vertices. We have $\phi_{(1,1,1,1)} = 10s_1s_2 + s_2 + \frac{15s_1+3}{2}$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 10s_1s_2 + s_2 + \frac{15s_1+1}{2}$. Clearly $\gamma_4 = \gamma_2 = \gamma_1 = 10s_1s_2 + s_2 + \frac{17s_1+1}{2}$ and $\gamma_3 = 10s_1s_2 + s_2 + \frac{17s_1-1}{2}$.

Subcase (D): $m \equiv 3 \pmod 4$

Let $m = 4s_2 + 3$, $s_2 \geq 0$. We get $p = 40s_1s_2 + 40s_1 + 4s_2 + 4$, and $q = 40s_1s_2 + 44s_1 + 4s_2 + 3$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $10s_1s_2 + s_2 + \frac{15s_1}{2}$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + s_2 + \frac{15s_1}{2} + 1$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + s_2 + \frac{15s_1}{2} + 1$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $10s_1s_2 + s_2 + \frac{15s_1}{2} + 1$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 10t_1s_2 + s_2 + 10s_1 + 1$. Clearly $\gamma_4 = 10s_1s_2 + s_2 + 11s_1$ and $\gamma_3 = \gamma_2 = \gamma_1 = 10s_1s_2 + s_2 + 11s_1 + 1$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $10s_1s_2 + s_2 + \frac{15s_1-1}{2}$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + s_2 + \frac{15s_1+3}{2}$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + s_2 + \frac{15s_1+1}{2}$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $10s_1s_2 + s_2 + \frac{15s_1+3}{2}$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 10s_1s_2 + s_2 + 10s_1 + 1$. Clearly $\gamma_4 = 10s_1s_2 + s_2 + 11s_1$ and $\gamma_3 = \gamma_2 = \gamma_1 = 10s_1s_2 + s_2 + 11s_1 + 1$.

Case (ii): $2n - 2 \equiv 1 \pmod 4$

Let $2n - 2 = 4s_1 + 1$, $s_1 \geq 0$. We get $n = 2s_1 + 2$, $p = (10s_1 + 6)(m + 1)$, and $q = (10s_1 + 6)(m + 1) + 4s_1 + 1$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $\frac{5s_1}{2} + 4$ vertices. Then assign the vector (1,1,1,0) to the next $\frac{5s_1}{2} + 2$ vertices and assign the vector $(1,1,0,0)$ to the next $\frac{5s_1}{2}$ vertices. Finally assign the vector $(1,0,0,0)$ to the next $\frac{5s_1}{2}$ vertices. From these vertex labeling, we get $\frac{7s_1}{2} + 4$, $\frac{7s_1}{2} + 3$, $\frac{7s_1}{2}$, $\frac{7s_1}{2}$ edges with edge label $4,3,2,1$ respectively.

When s_1 is odd, assign the vector $(1,1,1,1)$ to the first $\frac{5(s_1+1)}{2} + 1$ vertices. Then assign the vector $(1,1,1,0)$ to the next $\frac{5(s_1+1)}{2}$ vertices and assign the vector $(1,1,0,0)$ to the next $\frac{5(s_1-1)}{2} + 3$ vertices. Finally assign the vector $(1,0,0,0)$ to the next $\frac{5(s_1-1)}{2} + 2$ vertices. From these vertex labeling, we obtain $\frac{7(s_1+1)}{2}$, $\frac{7(s_1+1)}{2}$, $\frac{7(s_1-1)}{2} + 4$, $\frac{7(s_1-1)}{2} + 3$ edges with edge label $4,3,2,1$ respectively.

Subcase (A): $m \equiv 0 \pmod 4$

Let $m = 4s_2$, $s_2 > 0$. We get $p = 40s_1s_2 + 10s_1 + 24s_2 + 6$ and $q = 40s_1s_2 + 14s_1 + 24s_2 + 7$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $10s_1s_2 + 6s_2 - 2$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + 6s_2 - 1$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + 6s_2 + 1$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $10s_1s_2 + 6s_2 + 2$ pendent vertices. We get $\phi_{(1,1,1,1)} = \phi_{(1,0,0,0)} = 10s_1s_2 + 6s_2 + \frac{5s_1}{2} + 2$ and $\phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = 10s_1s_2 + 6s_2 + \frac{5s_1}{2} + 1$. Clearly $\gamma_1 = \gamma_2 = \gamma_4 = 10s_1s_2 + 6s_2 + \frac{7s_1}{2} + 1$ and $\gamma_3 = 10s_1s_2 + 6s_2 + \frac{7s_1}{2} + 2$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $10s_1s_2 + 6s_2 - 2$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + 6s_2 - 1$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + 6s_2 + 1$ pendent vertices and assign the vector $(1,0,0,0)$ to the next

$10s_1s_2+6s_2+2$ pendent vertices. We have $\phi_{(1,1,1,1)}=\phi_{(1,1,1,0)}=10s_1s_2+6s_2+\frac{5(s_1+1)}{2}-1$ and $\phi_{(1,1,0,0)}=\phi_{(1,0,0,0)}=10s_1s_2+6s_2+\frac{5(s_1-1)}{2}+4$. Clearly $\gamma_4=\gamma_3=10s_1s_2+6s_2+\frac{7(s_1+1)}{2}-2$ and $\gamma_2=\gamma_1=10s_1s_2+6s_2+\frac{7(s_1-1)}{2}+5$.

Subcase (B): $m\equiv 1 \pmod 4$

Let $m=4s_2+1$, $s_2>0$. We get $p=40s_1s_2+20s_1+24s_2+12$ and $q=40s_1s_2+20s_1+24s_2+13$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $10s_1s_2+6s_2+\frac{5s_1}{2}-1$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $10s_1s_2+6s_2+\frac{5s_1}{2}+1$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $10s_1s_2+6s_2+\frac{5s_1}{2}+3$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $10s_1s_2+6s_2+\frac{5s_1}{2}+3$ pendent vertices. We get $\phi_{(1,1,1,1)}=\phi_{(1,1,1,0)}=\phi_{(1,1,0,0)}=\phi_{(1,0,0,0)}=10s_1s_2+6s_2+5s_1+3$. Clearly $\gamma_4=\gamma_2=\gamma_1=10s_1s_2+6s_2+6s_1+3$ and $\gamma_3=10s_1s_2+6s_2+6s_1+3$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $10s_1s_2+6s_2+\frac{5(s_1-1)}{2}+2$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $10s_1s_2+6s_2+\frac{5(s_1-1)}{2}+3$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $10s_1s_2+6s_2+\frac{5(s_1+1)}{2}$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $10s_1s_2+6s_2+\frac{5(s_1+1)}{2}+1$ pendent vertices. We have $\phi_{(1,1,1,1)}=\phi_{(1,1,1,0)}=\phi_{(1,1,0,0)}=\phi_{(1,0,0,0)}=10s_1s_2+6s_2+5s_1+3$. Clearly $\gamma_4=\gamma_2=\gamma_1=10s_1s_2+6s_2+6s_1+3$ and $\gamma_3=10s_1s_2+6s_2+6s_1+4$.

Subcase (C): $m\equiv 2 \pmod 4$

Let $m=4s_2+2$, $s_2\geq 0$. We get $p=40s_1s_2+30s_1+24s_2+18$, and $q=40s_1s_2+34s_1+24s_2+19$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $10s_1s_2+6s_2+5s_1+1$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $10s_1s_2+6s_2+5s_1+4$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $10s_1s_2+6s_2+5s_1+4$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $10s_1s_2+6s_2+5s_1+5$ pendent vertices. We get $\phi_{(1,1,1,1)}=\phi_{(1,0,0,0)}=10s_1s_2+6s_2+\frac{15s_1}{2}+5$ and $\phi_{(1,1,1,0)}=\phi_{(1,1,0,0)}=10s_1s_2+6s_2+\frac{15s_1}{2}+4$. Clearly $\gamma_4=\gamma_2=\gamma_1=10s_1s_2+6s_2+\frac{17s_1}{2}+5$ and $\gamma_3=6s_1s_2+4s_2+\frac{17s_1}{2}+4$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $10s_1s_2+6s_2+5s_1+1$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $10s_1s_2+6s_2+5s_1+2$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $10s_1s_2+6s_2+5s_1+4$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $10s_1s_2+6s_2+5s_1+5$ pendent vertices. We have $\phi_{(1,1,1,1)}=\phi_{(1,1,1,0)}=\phi_{(1,1,0,0)}=\phi_{(1,0,0,0)}=10s_1s_2+6s_2+\frac{15s_1+9}{2}$. Clearly $\gamma_4=\gamma_3=\gamma_1=10s_1s_2+6s_2+\frac{17s_1+9}{2}$ and $\gamma_2=10s_1s_2+6s_2+\frac{17s_1+11}{2}$.

Subcase (D): $m\equiv 3 \pmod 4$

Let $m=4s_2+3$, $s_2\geq 0$. We get $p=40s_1s_2+40s_1+24s_2+24$ and $q=40s_1s_2+44s_1+24s_2+25$. If s_1 is even, assign the vector $(1,1,1,1)$ to the first $10s_1s_2+6s_2+\frac{15s_1}{2}+2$ pendent vertices. Then assign the vector $(1,1,1,0)$ to the next $10s_1s_2+6s_2+\frac{15s_1}{2}+4$ pendent vertices and assign the vector $(1,1,0,0)$ to the next $10s_1s_2+6s_2+\frac{15s_1}{2}+6$ pendent vertices. Finally assign the vector $(1,0,0,0)$ to the next $10s_1s_2+6s_2+\frac{15s_1}{2}+6$ pendent vertices. We get $\phi_{(1,1,1,1)}=\phi_{(1,1,1,0)}=\phi_{(1,1,0,0)}=\phi_{(1,0,0,0)}=10s_1s_2+6s_2+10s_1+6$. Clearly $\gamma_4==\gamma_2=\gamma_1=10s_1s_2+6s_2+11s_1+6$ and $\gamma_3=10s_1s_2+6s_2+11s_1+7$.

When s_1 is odd. Then assign the vector $(1,1,1,1)$ to the first $10s_1s_2+6s_2+\frac{15s_1+5}{2}$ pendent vertices and assign the vector $(1,1,1,0)$ to the next $10s_1s_2 + 6s_2 + \frac{15s_1+7}{2}$ pendent vertices. Thereafter assign the vector $(1,1,0,0)$ to the next $10s_1s_2 + 6s_2 + \frac{15s_1+11}{2}$ pendent vertices and assign the vector $(1,0,0,0)$ to the next $10s_1s_2 + 6s_2 + \frac{9s_1+13}{2}$ pendent vertices. We have $\phi_{(1,1,1,1)} = \phi_{(1,1,1,0)} = \phi_{(1,1,0,0)} = \phi_{(1,0,0,0)} = 10s_1s_2 + 6s_2 + 10s_1 + 6$. Clearly $\gamma_4 = \gamma_2 = \gamma_1 = 10s_1s_2 + 6s_2 + 11s_1 + 6$ and $\gamma_3 = 10s_1s_2 + 6s_2 + 11s_1 + 7$. Hence the vertex labeling ϕ is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling of$D(Q_n) \odot mK_1$ for all $n, m \geq 2$. □

5. Conclusion

In this paper, we have investigated the existence of a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling of $D(T_n) \odot mK_1$ and $D(Q_n) \odot mK_1$. Investigating the vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial labeling behaviour of corona product of some other family of graphs is our future work.

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