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An Approach to Find the Solution of Assignment Problem

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Abstract

In this paper we introduce an approach to give a solution for travelling salesman problem(Assignment problem). We consider a salesman problem if he wants to visit a certain number of cities allotted to work. He knows the distance (or cost or time) of journey between every pair of cities, usually denoted by c_{ij}, i.e., from city i to city j. His problem is to select such a route that starts from his home city, passes through each city once and only once, and returns to his home city in the shortest possible distance and its goal would be to minimize the total distance traveled so that the cost or time will be optimal.

Keywords: assignment problem; optimization; travelling salesman; puzzles; algorithm.

1. Introduction

The assignment problem is a special type of linear programming problem where assignees are being assigned to perform tasks. Also, the travelling salesman problem is one of the problems considered as puzzles and it is a classical algorithm problem in the field of operations research. In the view of delivery vehicles delivering packages, the travelling salesman problem can be used to determine the most efficient route for each vehicle. This would involve defining the cities as the delivery locations and the salesman as the delivery vehicle. Although the name assignment problem seems to have first appeared in a 1952 paper by Votaw and Orden [1], what is generally recognized to be the beginning of the development of practical solution methods for and variations on the classic assignment problem,hence after (referred to as the assignment problem), was the publication 1955 of Kuhn's article on the Hungarian method for its solution [2].
The problem may be classified in two forms:

1. **Symmetrical.** The problem is said to be symmetrical if the distance (or cost or time) between every pair of cities is independent of the direction of his journey.

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2. **Asymmetrical.** The problem is said to be asymmetrical if, for one or more pair of cities, the distance (or cost or time) changes with the direction.

Furthermore, if number of cities is only two, obviously there is no choice. If number of cities become three, say P, Q and R, one of them (say P) is the home base, then there are two possible routes:

$$P \to Q \to R \to P \quad \text{and} \quad P \to R \to Q \to P.$$

For four cities P, Q, R and S, there are $3! = 6$ possible routes, i.e.,

$$P \to Q \to R \to S \to P, \quad P \to Q \to S \to R \to P, \quad P \to R \to Q \to S \to P,$$
$$P \to R \to S \to Q \to P, \quad P \to S \to Q \to R \to P, \quad \text{and} \quad P \to S \to R \to Q \to P.$$

But, if number of cities is increased to 21, there are $20! = 2,402,902,008,176,640,000$ different routes.it is practically impossible to find the best route by trying each one. In general, if there are n cities, there are $(n-1)!$ possible routes. At present, the best procedure is to solve the problem as if it were an assignment problem with the additional restriction on his choice of route. Different types of method have been presented of the assignment problem and various article have been published in this subject [3], and [4].

2. Mathematical Formulation of Travelling-Salesman Problem

Suppose c_{ij} is the distance (or cost or time) from city i to city j, and $x_{ij} = 1$ if the salesman goes directly from city i to city j, and $x_{ij} = 0$ otherwise. Then minimize $\sum_i \sum_j c_{ij} x_{ij}$ with the additional restriction that the x_{ij} must be so chosen that no city is visited twice before the tour of all cities is completed. In particular, he cannot go directly from city i to city i itself. This possibility may be avoided in the minimization process by adopting the convention $c_{ii} = \infty$ which ensures that x_{ii} can never be unity. Alternatively, omit the variable x_{ii} from the problem specification. It is also important to that only single x_{ij} for each value of i and j. The distance (or cost or time) matrix for this problem is given in the following table.

From\To	A_1	A_2	$\cdots$	$\cdots$	$\cdots$	A_n
A_1	∞	c_{12}				C_{1n}
A_2	C_{21}	∞				C_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_n	C_{n1}	C_{n2}				∞

3. Solution Procedure

Let us consider an example, given the matrix of set of cost is

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	2	5	7	1
A_2	6	∞	3	8	2
A_3	8	7	∞	4	7
A_4	12	4	6	∞	5
A_5	1	3	2	8	∞

and we have to find an order in which products are produced so that the set-up costs are minimized, is an asymmetrical travelling-salesman problem. The change-over/set-up costs between products must be produced once and only once and production must return to the first product. Initially we solve the problem as an assignment problem

Table 1(Given)

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	2	5	7	1
A_2	6	∞	3	8	2
A_3	8	7	∞	4	7
A_4	12	4	6	∞	5
A_5	1	3	2	8	∞

Step 1: First, subtract the smallest element from each row to get the reduced matrix in Table 2.

Table 2

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	1	4	6	0
A_2	4	∞	1	6	0
A_3	4	3	∞	0	3
A_4	8	0	2	∞	1
A_5	0	2	1	7	∞

Step 2: Also subtract the smallest element in each column to get the reduced the matrix in Table 3.

Table 3

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	1	3	6	0
A_2	4	∞	0	6	0
A_3	4	3	∞	0	3
A_4	8	0	1	∞	1
A_5	0	2	0	7	∞

Step 3: The zeros of this matrix (marked ◯) give a solution to the assignment problem

Table 4.

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	1	3	6	(0)
A_2	4	∞	(0)	6	~~0~~
A_3	4	3	∞	(0)	3
A_4	8	(0)	1	∞	1
A_5	(0)	2	~~0~~	7	∞

it is ($A_1 \to A_5$, $A_5 \to A_1$, $A_2 \to A_3$, $A_3 \to A_4$ and $A_4 \to A_2$) but this is not a solution of the travelling-salesman problem. It indicates to produce the products A_1, then A_5 and then again A_1, without producing the products A_2, A_3 and A_4, thereby violating the additional restriction of producing each product once and only once before returning to the first product.

Step 4: Again, examine the matrix for some of the 'next best' solutions to the assignment problem, and try to find out one solution which satisfies the additional restriction. The smallest element other than zero is 1, so try the effect of putting such an element in the solution.

Table 5

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	[1]	3	6	0
A_2	4	∞	[0]	6	0
A_3	4	3	∞	[0]	3
A_4	8	0	1	∞	[1]
A_5	[0]	2	0	7	∞

Start by making unity-assignment in the cell (1, 2) instead of zero-assignment in the cell (1, 5), as shown in the above matrix .

Although there is no solution to the assignment problem among zeros, it is easy to see the best solution lies in the marked "□" elements. Thus, the required solution of the problem is **Step 4.** $A_1 \to A_2 \to A_3 \to A_4 \to A_5 \to A_1$.where as another solution $A_1 \to A_5 \to A_3 \to A_4 \to A_2 \to A_1$.

It is seen that any solution where the 'cost' exceeds 2 is not optimal. Now, only examine solutions containing the element 1 in the cell (1, 2) not so far used to see if a better solution exists. Hence, the most suitable sequence is $A_1 \to A_2 \to A_3 \to A_4 \to A_5$ and the minimum set-up cost = $2 + 3 + 4 + 5 + 1 =$ Rs.15.

Consider another Example (2)

A person has to visit five village A_1, A_2, A_3, A_4 and A_5. The distances between five village in km are as follows:

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	7	6	8	4
A_2	7	∞	8	5	6
A_3	6	8	∞	9	7
A_4	8	5	9	∞	8
A_5	4	6	7	8	∞

If the person starts village A_1 and come back to the village, A_1 we have to find which route be select so the distance will be optimal. Using procedure each step from example (1), we get

Table 1 (Given)

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	7	6	8	4
A_2	7	∞	8	5	6
A_3	6	8	∞	9	7
A_4	8	5	9	∞	8
A_5	4	6	7	8	∞

Table 2

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	3	2	4	0
A_2	2	∞	3	0	1
A_3	0	2	∞	3	1
A_4	3	0	4	∞	3
A_5	0	2	3	4	∞

Table 3

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	3	0	4	0
A_2	2	∞	1	0	1
A_3	0	2	∞	3	1
A_4	3	0	2	∞	3
A_5	0	2	1	4	∞

Table 4

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	3	~~0~~	4	(0)
A_2	3	∞	1	(0)	1
A_3	(0)	1	∞	2	~~0~~
A_4	4	(0)	2	∞	3
A_5	~~0~~	1	(0)	3	∞

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	3	(0)	4	~~0~~
A_2	3	∞	1	(0)	1
A_3	~~0~~	1	∞	2	(0)
A_4	4	(0)	2	∞	3
A_5	(0)	1	~~0~~	3	∞

Table 5

From\To	A_1	A_2	A_3	A_4	A_5
A_1	∞	3	0	4	[0]
A_2	3	∞	1	[0]	1
A_3	[0]	1	∞	2	0
A_4	4	0	[2]	∞	3
A_5	0	[1]	0	3	∞

$A_1 \rightarrow A_5 \rightarrow A_2 \rightarrow A_4 \rightarrow A_3 \rightarrow A_1$ and minimum distance $= 4+6+5+9+6 = 30\ km$, whereas other possibilities of solutions also exist, but that is not optimal.

4. Conclusion

We have seen the Hungarian method is not sufficient to solve this type of problem. Also, there are many other algorithm for the travelling salesman problem, including computer programs trying to improve the results that have been optimal. Here, we have been getting better results for a given problem.

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