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Satellites: Equation of Motion in Nechvill's Co-ordinate System

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Abstract: This paper deals with the system of non-linear, non-autonomous and non-homogeneous equations of motion in terms of the Nechvill's Co-ordinate system have been derived. The general solution is beyond our reach.

Keywords: System, Nonlinear, Homogeneous, Frame of reference, constraints.

1. Introduction

The physical properties of the celestial bodies is generally faced with two types of problems namely gravity gradient stabilization and altitude stabilization of the satellites. Gravity gradient stabilization means that the portion carrying the instrument in the satellites is always pointed towards the surface of the earth. This resulted in the formulation of the problem of the passive altitude stabilization of the satellites in the orbit.

2. Main Results

We transform the equations of motion of the particle of mass m_1 in terms of Nechvill's co-ordinate system using the Nechvill's transformation given by.

$$\left.\begin{aligned} \xi &= \rho x \\ \eta &= \rho y \\ \zeta &= \rho z \end{aligned}\right\} \tag{1}$$

Where

$$\rho = \frac{R}{r} = \frac{1}{1+e\cos v} \tag{2}$$

Here r and p are semi latus rectum (focal parameter) eccentricity of the orbit of the centre of mass of the system. For our further study, we shall choose the true anomaly $v$of the centre of mass as an independent variable which is given by the differential equation.

$$v = \frac{dv}{at} = \frac{\sqrt{\mu_r}}{r^2}\frac{1}{\rho^2} \tag{3}$$

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Where $\frac{\sqrt{\mu_r}}{r^2}$ remains constant for a given orbit. Let dash denote differentiations with respect to the variable v

$$\begin{aligned}\dot{\xi} &= \frac{d}{at}(\rho x) = \frac{d}{dv}(\rho x)\frac{dv}{at} \\ &= \left(\rho' x + \rho x'\right)\dot{v} \\ \dot{\eta} &= \left(\rho' y + \rho y'\right)\dot{v} \\ \zeta &= \left(\rho' z + \rho z'\right)\dot{v}\end{aligned} \tag{4}$$

Differentiating (4) again we have

$$\begin{aligned}\ddot{\xi} &= \left(\rho'' ex + 2\rho' x' + \rho x''\right)\dot{v}^2 + \left(\rho' x + \rho x'\right)\dot{v}\dot{v}' \\ \ddot{\eta} &= \left(\rho'' y + 2\rho' y' + \rho y''\right)\dot{v}^2 + \left(\rho' y + \rho y'\right)\dot{v}\dot{v}' \\ \ddot{\zeta} &= \left(\rho'' z + 2\rho' z' + \rho z''\right)\dot{v}^2 + \left(\rho' z + \rho z'\right)\dot{v}\dot{v}'\end{aligned} \tag{5}$$

From (3) we have

$$\begin{aligned}\dot{v} &= \frac{\sqrt{\mu r}}{r^2}\frac{1}{\rho^2} \\ \ddot{v} &= \frac{\sqrt{\mu r}}{r^2}\left(\frac{-2\rho\rho'\dot{v}}{\rho^4}\right) \\ &= \frac{-\sqrt{\mu r}}{r^2}.\frac{2p'}{p^3}v'\end{aligned}$$

$$2\dot{v}^2\rho' + \ddot{v}e = 0 \tag{6}$$

But $\dot{v}' = \frac{-2\sqrt{\mu\rho}}{r^2}\frac{\rho'}{\rho^3}$; $\ddot{v} = \dot{v}^1\dot{v}$. Putting $\ddot{v} = v^1\dot{v}$ in (6) we get

$$\begin{aligned}\dot{v}\left(2\dot{v}\rho^1 + \rho v^1\right) &= 0 \\ 2\dot{v}\rho^1 + \rho v^1 &= 0\end{aligned} \tag{7}$$

Let us choose $\frac{\sqrt{\mu r}}{r^2}$ = constant $= a$ (say). Then $\dot{v} = \frac{a}{\rho^2}$.

$$\dot{v}^1 = \frac{2a}{p^3}\rho^1$$

Also $\rho' = \frac{1}{1 + e\cos v} \Rightarrow \rho^1 = e\rho' \sin v$

$$\begin{aligned}\rho'' &= 2e\rho\rho \sin v + ep^2\cos v \\ \rho''\dot{v} - \rho\dot{v} + \rho'\dot{v} &= \left(ep^2\cos v + 2e\rho\rho' \sin v\right)\frac{a}{\rho^2} - \frac{pa}{\rho^2} + e\rho^2 \sin v\left(\frac{-2a}{\rho^3}\rho^1\right) \\ &= a\left(\rho\cos v - \frac{1}{\rho}\right) \\ &= -a\frac{-\sqrt{\mu r}}{\gamma^2}\end{aligned}$$

$$\rho''\dot{v} - \rho\dot{v} + \rho'\dot{v} = \frac{-\sqrt{\mu r}}{\gamma^2} \tag{8}$$

Also

$$\rho''\dot{v} + \rho'\dot{v}' = \frac{d}{dv}\left(\rho'\dot{v}\right)$$

$$
\begin{aligned}
&= \frac{d}{dv}\left(\rho' . \frac{a}{\rho^2}\right) \\
&= \frac{d}{dv}\left(ae\sin v\right) \\
&= ae\cos v \\
&= a\left(\frac{1}{a} - 1\right) \qquad (9)
\end{aligned}
$$

and

$$
\begin{aligned}
R &= \rho\rho \\
&= \rho\frac{d}{dv}\left(\frac{1}{1+e\cos v}\right)\frac{dv}{al} \\
&= r\rho'\dot{v} \qquad (10)
\end{aligned}
$$

Using (1), (2), (3), (4), (5) and (10) in equation of system. We get

$$
\begin{aligned}
\rho\dot{v}^2x'' + \left(2\rho'\dot{v} + \rho v'\right)x'\dot{v} - \left(\ddot{v}\rho + 2\dot{v}^2\rho'\right)y - 2\rho\dot{v}^2y + \left(\rho''\dot{v} - \rho'\dot{v} + \rho'\dot{v}'\right)\dot{v}x + a_1\rho\rho'\dot{v}\frac{-2\mu x}{r^3\rho^2} - \frac{12\mu k_2 x}{\rho^4 r^5} \\
+ \lambda a\left[1 - \frac{\iota_0}{\rho\sqrt{x^2+y^2+z^2}}\right]\rho x = -\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\frac{\mu_E}{\gamma^2\rho^2}\dot{v}\cos i \qquad (11)
\end{aligned}
$$

using (7) and (8) in (11) we get

$$
\rho\dot{v}^2x'' - 2\rho\dot{v}^2y' + a_1y\rho^2\dot{v} - \frac{2\mu x}{r^3\rho^2} - \frac{12\mu k_2 x}{\rho^4 r^5} - \frac{\sqrt{\mu r}}{r^2}vx = -\lambda x\left[1 - \frac{\iota_0}{\rho\sqrt{x^2+y^2+z^2}}\right]\rho x - \left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\frac{\mu_E}{r^2\rho^2}v\cos i
$$

Dividing through out by $\rho\dot{v}^2$ we get

$$
x'' - 2y' - \frac{\sqrt{\mu r}}{r^2\rho^2}\frac{x}{\rho\dot{v}} + \frac{a_1 r\rho^1}{\rho\dot{v}}\frac{2\mu x}{r^3\rho^3\dot{v}^2}\frac{-12\mu k_2 x}{\rho^5 r^5\dot{v}^2} = \frac{-\lambda a}{\dot{v}^2}\left[1 - \frac{\iota_0}{\rho\sqrt{x^2+y^2+z^2}}\right]x - \left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\frac{\mu_E}{r^2\rho^3\dot{v}}Cosi
$$

Putting the value of $\dot{v}$ we obtain the above equation of motion in the form

$$
x'' - 2y^1 - 3\rho x\frac{12k_2x}{\rho r^2} + \frac{a_1r^3\rho\rho^1}{\sqrt{\mu\gamma}} = \frac{-\lambda a r^3}{\mu}\rho^4\left[1 - \frac{\iota_0}{\rho\sqrt{x^2+y^2+z^2}}\right]x - \left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\frac{\mu_E}{\sqrt{\mu p}}\frac{1}{\rho}\cos i
$$

i.e.

$$
x'' - 2y^1 - 3x\rho - \frac{12k_2x}{\rho r^2} + f\rho\rho' = -\lambda a\rho^4\left[1 - \frac{\iota_0}{\rho\sqrt{x^2+y^2+z^2}}\right]x - \frac{B}{\rho\cos i} \qquad (12)
$$

Where

$$
\begin{aligned}
f &= \frac{a_1r^3}{\sqrt{\mu r}} \\
\overline{\lambda}a &= \frac{r^3}{\mu}\lambda a \qquad (13) \\
B &= \left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\frac{\mu_E}{\sqrt{\mu r}}
\end{aligned}
$$

Similarly 2nd and 3rd equation of (13) can be written as

$$
y'' + 2x' + f\rho^2 + \frac{3k_2}{\rho r^2}y = -\overline{\lambda}a\left[1 - \frac{\iota_0}{\rho\sqrt{x^2+y^2+z^2}}\right]\rho^4 y - \frac{B\rho'}{\rho^2}\cos i.......... \qquad (14)
$$

and

$$
\begin{aligned}
z'' + z + \frac{3k_2 z}{\rho r^2} &= -\lambda a p^4 \left[1 - \frac{\iota_0}{\rho\sqrt{x^2+y^2+z^2}} \right] z \\
&= B \Big/ \rho \left[\frac{\rho^1}{\rho} \cos(v+w) + \frac{1}{\mu_E}(3\rho^3 r^3 - \mu_E) / - \mu_E \sin(v+w) \right] \sin i
\end{aligned} \tag{15}
$$

Where $v + w =$ argument of the latitude of the particle. Thus we get a new set of differential equation character sing the motion of the particle o mass m_1 as

$$
\begin{aligned}
x'' - 2y' - 3x\rho + \frac{4A}{\rho} x + f\rho\rho^1 &= \overline{\lambda} a \rho^4 \left[1 - \frac{\iota_0}{\rho\sqrt{x^2+y^2+z^4}} \right] x - \frac{B}{\rho} \cos i \\
y'' + 2x^1 + f\rho^2 - \frac{A}{\rho}.y &= -\lambda a \rho^4 \left[1 - \frac{\iota_0}{\rho\sqrt{x^2+y^2+z^2}} \right] y - \frac{B\rho^1}{\rho^2} \cos i \\
z'' + z - \frac{A}{\rho z} &= -\overline{\lambda} a \rho^4 \left[1 - \frac{\iota_0}{\rho\sqrt{x^2+y^2+z^2}} \right] z \\
&\quad - \frac{B}{\rho} \left[\frac{\rho^1}{\rho} \cos(v-w) + \frac{1}{\mu_E}(3r^3\rho^2 - \mu E) \sin(v+w) \right] \sin i
\end{aligned} \tag{16}
$$

Where $A = \dfrac{-3\mu k_2}{r^2}$ and $f, \lambda\overline{a}$ and B are given by (13). The condition of constraint takes the form

$$
x^2 + y^2 + z^2 \leq \frac{\iota_0}{\rho^2} \tag{17}
$$

3. Conclusion

The system of equation (16) describes the motion of the particle of mass m_1 in rotating frame of reference in Nechvill's co-ordinates which is a non-linear, non-homogeneous and non-continuous whose general solution is beyond our reach.

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