International Journal of
Mathematics And its Applications
ISSN: 2347-1557

Int. J. Math. And Appl., **13(3)**(2025), 51–60
Available Online: http://ijmaa.in

Applications of Fuzzy Baire Spaces

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Abstract

The fuzzy sets, in which the elements of the universe have their membership and non-membership degrees in [0,1] is of Zadeh's fuzzy set. In this paper fuzzy sets are used as tools for assessment and decision making. This is useful in cases where one is not sure about the suitability of the linguistic characterizations assigned to each element of the universal set. Applications illustrating our results are also presented.

Keywords: Fuzzy nowhere desne set; fuzzy first category set; fuzzy Baire space; Fuzziness and Vagueness; Fuzzy interior and closure; Fuzzy Decision making; Fuzzy information systems.

1. Introduction

Assessment is a very important process connected to all human and machine activities, because it helps to improve the performance by avoiding mistakes or weak behaviours of the past. In many cases the assessment is performed using qualitative grades instead of numerical scores. This happens either because the existing data are not exact, or for reasons of elasticity. Obviously in such cases the mean performance of a group cannot be assessed calculating the mean value of the individual scores of its members.

Decision making the other hand, which is a fundamental activity of human cognition, can be defined as the process of choosing a solution among two or more alternatives, with the help of suitable criteria, among to obtained the best possible outcomes for a given problem. The recent technological process and the rapid changes happening to human society increased the complexity of the decision making problems. Thus from the 1950's a systematic approach has been started for decision making known as Statistical Decision Theory, which is based on a synthesis of principles of several scientific fields, including Probability Theory, Statistics, Economics, Psychology.

Frequently in everyday life situations, however the decision making process takes place under conditions of uncertainty, which makes the applications of the traditional Decision Making methods

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for the solution of the corresponding problems impossible. The application of fuzzy sets to assessment process is developed in section 4 also their application to decision making. Finally, includes the general conclusions and a short discussion for further research.

2. Preliminaries

Definition 2.1 ([4]). *Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). We define $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$. For any fuzzy set in a fuzzy topological space (X, T), it is easy to see that $1\check{}cl(\lambda) = int(1\check{}\lambda)$ and $1\check{}int(\lambda) = cl(1\check{}\lambda)$.*

Definition 2.2 ([4]). *A fuzzy set λ in a fuzzy topological space X is called fuzzy open if $\lambda \leq clint(\lambda)$ and fuzzy closed if $intcl(\lambda) \leq \lambda$.*

Definition 2.3 ([4]). *A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy G_δ-set in (X, T) if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$.*

Definition 2.4 ([4]). *A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy F_σ -set in (X, T) if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in T$.*

Definition 2.5 ([4]). *A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$.*

Definition 2.6 ([3]). *A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy first category set if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.*

Definition 2.7 ([3]). *Let λ be a fuzzy first category set in a fuzzy topological space (X, T). Then $1 - \lambda$ is called a fuzzy residual set in (X, T).*

Definition 2.8 ([4]). *Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called fuzzy σ-first category if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)]$, where λ_i's are fuzzy σ-nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be fuzzy σ-second category in (X, T).*

Definition 2.9 ([4]). *A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy σ-nowhere dense set, if λ is a fuzzy F_σ-set in (X, T) such that $int(\lambda) = 0$.*

Definition 2.10 ([3]). *Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire space if $int\left[\bigvee_{i=1}^{\infty}(\lambda_i)\right] = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T).*

Definition 2.11. *[4] Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy σ-Baire space if $int\left[\bigvee_{i=1}^{\infty}(\lambda_i)\right] = 0$, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T).*

3. Application

Fuzzy Baire spaces are important in areas where uncertainty or vagueness is inherent, such as

- Decision theory
- Fuzzy control systems
- Approximate reasoning
- Fuzzy neural networks
- Optimization problems with imprecise data.

In such systems, preserving properties like "non-meagerness" (i.e., avoiding the space being a countable union of fuzzy nowhere-dense sets) crucial to ensure the existence of meaningful solutions or stable states.

4. Relevance of fuzzy Baire spaces in Decision Theory

Decision Theory involves choosing the best option under uncertainty. Traditional models assume Crisp, precise, information: however, real-world decisions often involve vague, imprecise, or incomplete information, which is better handled by fuzzy set theory.
A fuzzy Baire space ensures that the space of possible decision is "rich" or "non - meager", meaning viable decision aren't lost due to overly narrow or restrictive criteria. This concept is crucial in ensuring preferences or data don't eliminate all acceptable outcomes.

Example 4.1. *A company wants to employ a new person among the seven candidates A,B,C,D,E,F and G. The ideal qualifications for the new employee are to have satisfactory previous experience and other categories [eg, to strength of subjective, to be driving license and to be young]. Assume that A, B, G are the candidate with satisfactory previous experience B,C,D,E,F are the strength of subjective, D, G. are the holders of a driving license and D is the only young candidate. Find the best decision for the company.*

Solution. Let $T = \{A, B, C, D, E, F, G\}$ and let $X = \{\lambda, \mu, \gamma, \delta\}$ be the set of parameters $\lambda =$ well experienced, $\mu =$ subject strength, $\gamma =$ holder of a driving license and $\delta =$ young candidate. $X : [0, 1]$ is defined as

$$\lambda = \{A, B, G\}$$

$$\mu = \{B, C, D, E, F\}$$

$$\gamma = \{D, G\}$$

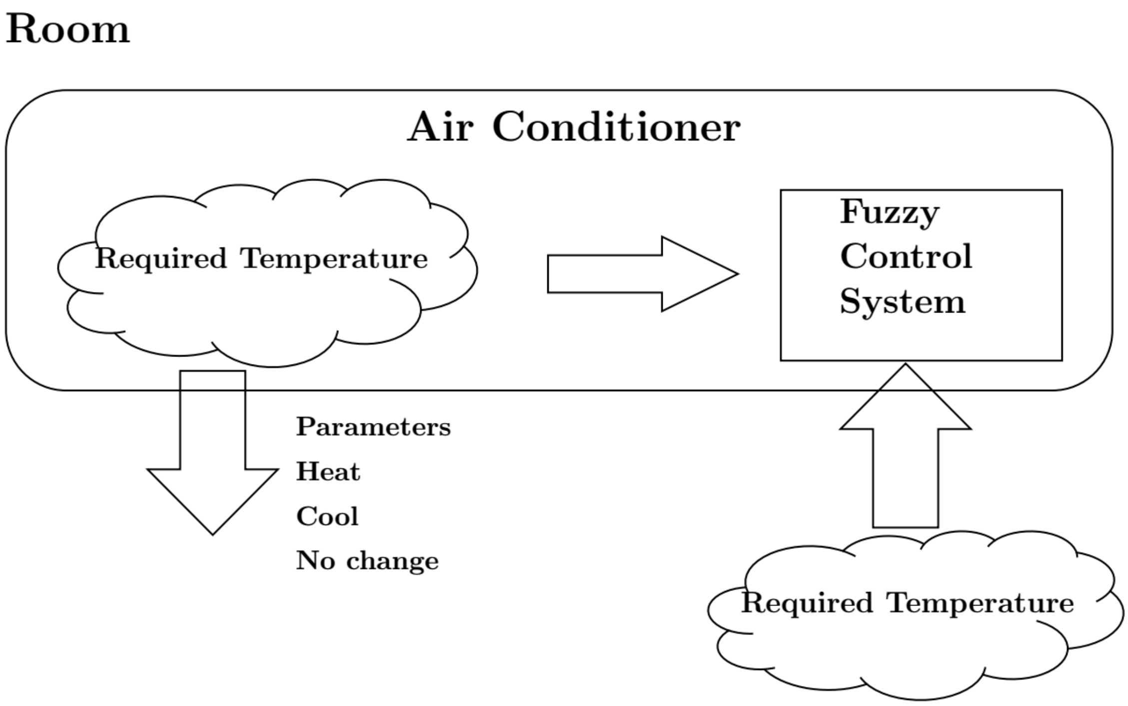
$$\delta = \{D\}$$

Define tabular matrix of the fuzzy set.

	λ	μ	γ	δ
A	1	0	0	0
B	1	1	0	0
C	0	1	0	0
D	0	1	1	1
E	0	1	0	0
F	0	1	0	0
G	1	0	1	0

Thus, the candidates A, C, E, and F have choice value 1 B, G have choice value 2 and D have choice value 3. Therefore the company must choose the Candidate D.

Example 4.2. *Let us consider an air conditioning system 5 level fuzzy control system. This system adjusts the temperature of air conditioner by comparing the room temperature and the highest temperature required value.*



The algorithm of fuzzy control system is mentioned below:

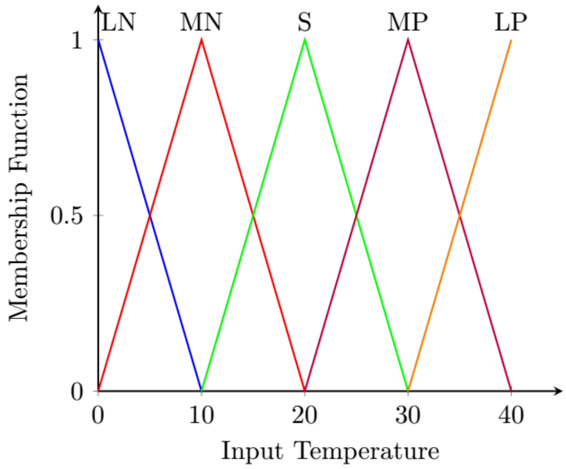
Step 1: Define Linguistic Variable and meaning. Linguistic variables are input and output variables. For room temperature, cold, warm, heat are linguistic terms.

Step 2: To construct membership function

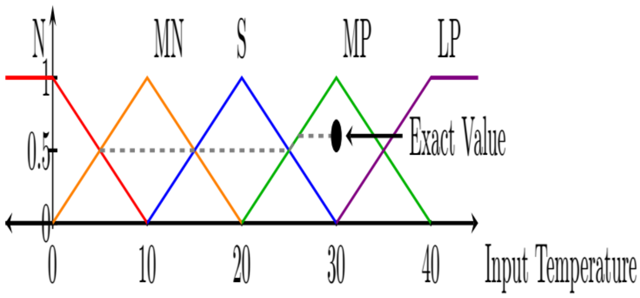
Step 3: Construct room temperature base rules

Room temperature required	Heavy Cold	Cold	Warm	Heat	Very Heat
Heavy Cold	—	Heat	Heat	Heat	Heat
Cold	Cool	—	Heat	Heat	Heat
Warm	Cool	Cool	—	Heat	Heat
Heat	Cool	Cool	Cool	—	Heat
Very Heat	Cool	Cool	Cool	Cool	—

S.No	Condition	Sloution
1.	Temperature = Cold (or) very cold	Heat
2.	Temperature = Heat (or) very Heat	Cool
3.	Temperature = Warm	No change



Conclusion:



Note:

Step	Description
LP	X value is large positive
MP	X value is medium positive
S	X value is small
MN	X value is Medium negative
LN	X value is large negative

Example 4.3. *We derive the Room Temperature control system*

Input: Temperature (°C)

Output: Fan speed (RPM)

Linguistic terms

I. *Temperature ranges*

1. *Low* $(0 - 20°C)$
2. *Medium* $(15 - 30°C)$
3. *High* $(25 - 40°C)$

II. Fan Speed level

1. *Slow*
2. *Medium*
3. *Fast*

Rules:

1. *If temperature is low then fan speed is slow.*
2. *If temperature is medium then fan speed is medium.*
3. *If temperature is high then fan speed is fast.*

Input: Temperature =26°C $\longrightarrow$ falls partially into "medium" and "high".
Output: Fan speed = 1600RPM
Fan Speed(RPM)

1) *Slow : 200 - 600*
2) *Medium : 600 - 1200*
3) *Fast : 1200 - 2000.*

Membership Functions:
Temperature = 26°C
μ - low(26)=0
μ - medium(26)=0.4
μ - high(26)=0.6
Apply Rules:
Rule 1: Not active (μ=0)
Rule 2: Activated with 0.4 - medium speed
Rule 3: Activated with 0.6 - high speed
Calculation:

1) *Medium speed (600 - 1200 RPM) $\mu = 0.4$ at 1200(RPM).*
2) *High speed (1200 - 2200 RPM) $\mu = 0.6$ at 1600(RPM).*

Example 4.4. *Fuzzy decision theory in Medical treatment.*
Question: Fuzzy decision for diagnosing fever.
1. Body temperature (°C)
2. Cough Intensity (Scale 0-10).
Step 1: *Define fuzzy sets*

Input 1: Temperature(T)

we define a normal and high temperature in fuzzy sets by

$$\alpha_{Normal}(T) = \begin{cases} 1 & T \leq 36 \\ \frac{37-T}{1} & 36 < T < 37 \\ 1 & T \geq 37 \end{cases} \tag{1}$$

$$\alpha_{Normal}(T) = \begin{cases} 0 & T \leq 37 \\ \frac{T-37}{1} & 37 < T < 40 \\ 1 & T \geq 39 \end{cases} \tag{2}$$

Input 2: Cough Intensity (C)

we define a mild and severe cough in fuzzy sets by

$$\gamma_{mild}(C) = \begin{cases} 1 & C \leq 3 \\ \frac{5-C}{2} & 3 < C < 5 \\ 1 & C \geq 5 \end{cases} \tag{3}$$

$$\eta_{severe}(C) = \begin{cases} 0 & C \leq 4 \\ \frac{C-4}{2} & 4 < C < 6 \\ 1 & C \geq 6 \end{cases} \tag{4}$$

***Step 2:** Taking Fuzzy Rules*

S. No	Rules
L1	If temperature is high and cough is severe then fever is severe (0.9)
L2	If temperature is high and cough is mild then fever is average (0.6)
L3	If temperature is high and cough is mild then fever is average (0.2)

***Step 3:** Taking Fuzzy Rules*

1. $T = 39°C$

2. $C = 4.3$ *(out of 10)*

***Step 4:** We define membership values in the above data*

$\alpha_{Normal}(39) = 0$

$\beta_{High}(39) = \frac{39-37}{2} = 1.0$

$\gamma_{Mild}(4.3) = \frac{5-4.3}{2} = 0.35$

$\eta_{severe}(4.3) = \frac{4.3-4}{2} = \frac{0.3-2}{2} = 0.15$

$L_1(severe)$: $\min(1.0, 0.15) = 0.15$

$L_2(average) : \min(1.0, 0.35) = 0.35$

$L_3(none) : \min(0, 0.35) = 0$

Average for fever = $\frac{0.15*0.9+0.35*0.6}{0.15+0.35} = \frac{0.135+0.21}{0.50} = \frac{0.345}{0.50} = 0.69.$

Conclusion : Fever symptoms = 0.69 (medium fever but not severe)

Example 4.5. *Fuzzy decision making in traffic signal*

Question: Control the traffic using green light duration.

(a) Traffic density (vehicles per lane)

(b) Waiting time (in seconds)

Step 1: *Define fuzzy sets*

a) Traffic Density (D)

(i) Low

$$\lambda_{low}(D) = \begin{cases} 1 & D \leq 15 \\ \frac{25-D}{5} & 5 < D < 25 \\ 0 & D \geq 25 \end{cases} \tag{5}$$

(ii) High

$$\mu_{high}(D) = \begin{cases} 0 & D \leq 20 \\ \frac{D-20}{5} & 20 < D < 30 \\ 1 & D \geq 30 \end{cases} \tag{6}$$

b) Waiting Time (w)

(i) Minimum

$$\alpha_{min}(W) = \begin{cases} 1 & W \leq 20 \\ \frac{60-W}{30} & 20 < D < 60 \\ 0 & W \geq 60 \end{cases} \tag{7}$$

$$\beta_{max}(W) = \begin{cases} 0 & W \leq 50 \\ \frac{W-50}{30} & 50 < W < 30 \\ 1 & W \geq 80 \end{cases} \tag{8}$$

Step 2: *Fuzzy Rules*

Rule 1: If Density is high and waiting time is maximum, then green light time is maximum (90 seconds).

Rule 2: If Density is low and waiting time is minimum, then green light time is minimum (40 seconds).

Rule 3: If Density is high and waiting time is minimum, then green light time is moderate (60 seconds).

Step 3: *Input Data*

a) Traffic density = 20 vehicles

b) Waiting time = 63 seconds

Step 4: *Construct Membership Values*

$\mu_{low}(20) = \frac{25-20}{5} = \frac{5}{5} = 1$

$\mu_{high}(20) = \frac{20-20}{5} = 0$

$\alpha_{\min}(63) = \frac{63-60}{30} = \frac{3}{30} = 0.1$

$\beta_{\max}(63) = \frac{63-50}{30} = \frac{13}{30} = 0.43$

Step 5: *Discussion Rules*

Rule 1: $(High, \max) : \min(0, 0.43) = 0$

Rule 2: $(low, \min) : \min(1, 0.1 = 0.1$

Rule 3: $(high, \min) = \min(0, 0.1) = 0$

Step 6: *Average for green light time dutation =* $\frac{0\times 90+0+1\times 40+0\times 60}{0+0.1+0} = \frac{40}{0.1} = 40$ *seconds.*

Conclusion: Green light time duration is 40 seconds. (traffic control)

Example 4.6. *The new coach of a Cricket team is not sure yet about the quality of he players. He characterized, therefore, the good players as follows.*P_1 *and* P_2 *by (1,0),* P_3, P_4 *and* P_5 *by (0.7, 0.2),*P_6, P_7 *and* P_8 *by (0.6,0.3),* P_8, P_9 *and* P_{10} *by (0.7, 0.4),*P_{11} *by (0.5,0.1)* P_{12}*and* P_{13} *by (0.6,0.3),* P_{14} *by (0.3,0.6),* P_{15} *and* P_{16} *by (0.3,0.9),* P_{17} *by (0.1,0.8) and the remaining three players*$[P_{18}, P_{19}$ *and* $P_{20}]$ *by (0,1). In other words, the coach believes that* P_1 *and* P_2 *are good players, he is 80%.* P_3, P_4 *and* P_5 *are good players too, but he has 10%. may not be good players and a 10% hesitation to decide about it, and so on. For the last three players the coach absolutely sure that they are not good players. Estimate the overall quality of the team according to the opinion of the coach about the individual quality of his players.*

Solution. The overall quality of the cricket team can be estimated by Calculating the Average values say A: (m,n) to the elements of the Sorry set of the good players of the team.

$$A = (m, n) = 1/20[2(1,0) + 3(0.7, 0.2) + 3(0.6, 0.3) + 3(0.7, 0.4) + 1(0.5, 0.1) + 2(0.6, 0.3) + 1(0.3, 0.6) + 2(0.3, 0.9) + 1(0.1, 0.8) + 3(0, 1)]$$

$1/20(9.9, 7.3) = (0.495, 0.365)$

A random player of the team is a good player by 49.5%. but there is a 36.5%. chance to be not a good player and a 14%. hesitation for deciding whether he is a good player or not these outcomes give a quite good idea about the overall quality of the Cricket team.

5. Conclusion

Applications of fuzzy Baire spaces extend across various fields such as decision theory, control fuzzy systems, data analysis and artificial Intelligence, where handling imprecise information is crucial.

There role to the development of fuzzy topology, fuzzy function spaces and generalized compactness further highlights their importance in both theoretical and applied mathematics. Overall, fuzzy Baire spaces contribute significantly to the advancement of fuzzy set theory and its integration into real world Problem solving Contexts.

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