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A Study on Gourava Bipartite Domination Topological Indices of Some Graphs

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Abstract

In this paper, we study the first and second Gourava bipartite domination indices and their corresponding polynomials of a graph. Further, we complete these indices and their corresponding polynomials for some standard graphs, French windmill graphs, friendship graphs, book graphs.

Keywords: Gourava bipartite domination degree; Gourava bipartite domination polynomial graph; standard graphs.

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1. Introduction

The graph $G = (V, E)$, where V be the vertex set and E be the edge set. The concept of domination and topological index hold great significance within the relation of graph theory. Therefore, it is pertinent to merge these concepts to derive the domination index of a graph. The domination index is defined by Kavya and Sunitha [4]. The gourava domination indices of a graph was introduced by Kulli [3]. The notion of domination topological index in graph was introduced by Hanan et. al [1]. Motivated from these ideas this article introduces a novel idea of a bipartite domination index using minimal bipartite dominating sets. One of fast developing fields in graph theory is the study of domination and related concepts. For instance, Bachstein et al., [2] published about the bipartite domination in graph. For a nontrivial connected graph G, anon empty set $S \subseteq V(G)$ is a bipartite dominating set of a graph G, if the subgraph $G[S]$ induced by S is bipartite and for every vertex not in S is dominated by any vertex in S. The bipartite domination number γ_{bp} of a graph G is the minimum cardinality of a bipartite dominating set of G.

Definition 1.1. *Let G be a graph and $v \in V$, then the bipartite domination degree of a vertex v is defined as the minimum number of vertices in minimal bipartite dominating set (MBDS) containing v, that is, $d_{b\gamma}$ ={$min|S|$: is a MBDS containing v}.*

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The degree of bipartite domination, both minimum and maximum are denoted by $\delta_{b\gamma}(G) = \delta_{b\gamma}$ *and* $\triangle_{b\gamma}(G) = \triangle_{b\gamma}$*, respectively, where* $\delta_{b\gamma} = min\{d_{b\gamma}(v) : v \in V(G)\}$ *and* $\triangle_{b\gamma} = max\{d_{b\gamma}(v) : v \in V(G)\}$.

Definition 1.2. *The first and second Gourava domination indices of a graph G are,*

$$GOD(G) = \sum_{u,v \in E(G)} [(d_d(u) + d_d(v)) + (d_d(u).d_d(v))],$$
$$GOD_2(G) = \sum_{u,v \in E(G)} [(d_d(u) + d_d(v)).(d_d(u).d_d(v))].$$

Considering the first and second Gourava domination indices, the first and second Gourava domination polynomial of a graph G are as follows.

$$GOD_1(G) = \sum_{u,v \in E(G)} x^{[(d_d(u)+d_d(v))+(d_d(u).d_d(v))]},$$
$$GOD_2(G) = \sum_{u,v \in E(G)} x^{[(d_d(u)+d_d(v)).(d_d(u).d_d(v))]}.$$

Motivated from this idea, in this chapter we introduced the following definition.

Definition 1.3. *The first and second Gourava bipartite domination indices defined as*

$$GOBD_1(G) = \sum_{u,v \in E(G)} [(d_{b\gamma}(u) + d_{b\gamma}(v)) + (d_{b\gamma}(u).d_{b\gamma}(v))],$$
$$GOBD_2(G) = \sum_{u,v \in E(G)} [(d_{b\gamma}(u) + d_{b\gamma}(v)).(d_{b\gamma}(u).d_{b\gamma}(v))].$$

Definition 1.4. *The first and second Gourava bipartite domination indices of polynomials are defined as*

$$GOBD_1(G,x) = \sum_{u,v \in E(G)} x^{[(d_{b\gamma}(u)+d_{b\gamma}(v))+(d_{b\gamma}(u).d_{b\gamma}(v))]},$$
$$GOBD_2(G,x) = \sum_{u,v \in E((G)} x^{[(d_{b\gamma}(u)+d_{b\gamma}(v)).(d_{b\gamma}(u).d_{b\gamma}(v))]}.$$

2. Gourava Bipartite Domination Indices of Some Standard Graphs

Proposition 2.1. *Let* K_n *be a complete graph,* $n \geq 2$*. Then*

$$GOBD_1(K_n) = 4n(n-1),$$
$$GOBD_2(K_n) = 8n(n-1).$$

Proof. If K_n is a complete graph, with $n \geq 2$. Then $d_{b\gamma}(x) = 2, \forall\ x \in V(K_n)$. From definition, we have

$$GOBD_1(K_n) = \frac{n(n-1)}{2}.[(2+2) + (2.2)],$$

$$GOBD_1(K_n) = 4n(n-1),$$
$$GOBD_2(K_n) = \frac{n(n-1)}{2}.[(2+2).(2.2)],$$
$$GOBD_2(K_n) = 8n(n-1).$$

□

Proposition 2.2. *Let $K_{r,t}$ be a complete bipartite graph. Then*

$$GOBD_1(K_{r,t}) = 8rt,$$
$$GOBD_2(K_{r,t}) = 16rt.$$

Proof. Let $G \cong K_{r,t}$. Then $d_{b\gamma}(x) = 2, \forall\ x \in V(K_{r,t})$. From definitions, we have

$$GOBD_1(G) = \sum_{u,v \in E(G)} [(d_{b\gamma}(u) + d_{b\gamma}(v)) + (d_{b\gamma}(u).d_{b\gamma}(v))],$$
$$GOBD_1(G) = (rt)[(2+2) + (2.2)] = 8rt,$$
$$GOBD_2(G) = (rt)[(2+2).(2.2)] = 16rt.$$

□

Proposition 2.3. *Let G be wheel graph W_n with $n \geq 4$. Then*

$$GOBD_1(G) = 16n,$$
$$GOBD_2(G) = 32n.$$

Proof. Let $G \cong W_n, n \geq 4$. Then, $d_{b\gamma}(G) = 2, \forall\ x \in V$.

$$GOBD_1(G) = (2n)[(2+2) + (2.2)] = 16n,$$
$$GOBD_2(G) = (2n)[(2+2).(2.2)] = 32n.$$

□

Proposition 2.4. *For any friendship graph F_n, where $n \geq 2$,*

$$GOBD_1(G) = 16n,$$
$$GOBD_2(G) = 32n.$$

Proof. Let $G \cong F_n$ be a friendship graph on $2n+1$ vertices and $2n$ edges. Then $d_{b\gamma} = 2, \forall\ x \in F_n$, we have

$$GOBD_1(G) = (2n)[(2+2)+(2.2)] = 16n,$$
$$GOBD_2(G) = (2n)[(2+2).(2.2)] = 32n.$$

□

Proposition 2.5. *For any firecracker graph $F(m,n)$, where $n \geq 2$,*

$$GOBD_1(G) = 4m(m+1)(mn-1),$$
$$GOBD_2(G) = 16m^3(mn-1).$$

Proof. Let $G \cong F(m,n)$ be a firecracker graph on mn vertices with $(mn-1)$ edges. Then $d_{b\gamma}(G) = 2m$, $\forall \ x \in F(m,n)$, we have

$$GOBD_1(G) = (mn-1)[(2m+2m)+(2m.2m)],$$
$$GOBD_1(G) = 4m(m+1)(mn-1),$$

and

$$GOBD_1(G) = (mn-1)[(2m+2m).(2m.2m)]$$
$$GOBD_2(G) = 16m^3(mn-1).$$

□

Proposition 2.6. *For any diamond snake graph D_k, where $k \geq 1$, then*

$$GOBD_1(G) = 16k^2(k+1),$$
$$GOBD_2(G) = 64k^4.$$

Proof. Let $G \cong D_k, k \geq 1$ be a diamond snake graph on $3k+1$ vertices and $4k$ edges. Then, $d_{b\gamma}(G) = 2k$, $\forall \ G \in D_k$,

$$GOBD_1(G) = (4k)[(2k+2k)+(2k.2k)] = 16k^2(k+1)$$
$$GOBD_2(G) = (4k)[(2k+2k).(2k.2k)] = 64k^4.$$

□

Proposition 2.7. *For any book graph B_k, where $k \geq 1$, then*

$$GOBD_1(G) = 6k^2 + 10k + 8,$$
$$GOBD_2(G) = k^3 + 10k^2 + 13k + 8.$$

Proof. Let $G \cong B_k, k \geq 1$ be a book graph on $2k+2$ vertices and $3k+1$ edges. Then,

$$d_{b\gamma}(G) = \begin{cases} 2 & \text{if is the center vertex} \\ k+1 & \text{Otherwise} \end{cases}$$

There are three types of edges in the book graph. Let E_1 represents for the set of k edges of type $(k+1, k+1)$, E_2 represents for the set of$2k$ edges of type $(2, k+1)$ and E_3 represents 1 edge of type $(2,2)$.

$$GOBD_1(G) = 1.[(2+2) + (2.2)] + (2k).[2 + (k+1) + (2.(k+1))],$$
$$GOBD_1(G) = 6k^2 + 10k + 8,$$
$$GOBD_2(G) = 1.[(2+2) + (2.2)] + (2k)[(k+1) + (k+1).(k+1)(k+1)]$$
$$+ k.[2 + (k+1)(k+1).2.(k+1).(k+1)],$$
$$GOBD_2(G) = 4k^4 + 14k^3 + 20k^2 + 10k + 8$$

□

Proposition 2.8. *Let G be a helm graph H_n. Then*

$$GOBD_1(G) = 12n^2(4n+1),$$
$$GOBD_2(G) = 48n^4.$$

Proof. Let $G \cong H_n$, be a helm graph on $2n$ vertices and $3n$ edges. Then, $d_{b\gamma}(G) = 2n, \forall \ x \in H_n$, we have

$$GOBD_1(G) = (3n).[(2n+2n) + n.2n)],$$
$$GOBD_1(G) = 12n^2(4n+1),$$
$$GOBD_2(G) = (3n).[(2n+2n).(4n^2)],$$
$$GOBD_2(G) = 48n^4.$$

□

Theorem 2.9 ([?]). *Let C_n be a cycle graph on n vertices. Then*

$$\gamma_{bp}(G) = \begin{cases} \frac{n}{4}, & if \quad n \equiv 0(mod4), \\ \frac{n+1}{4}, & if \quad n \equiv 1,3(mod4), \\ \frac{n+2}{4}, & if \quad n \equiv 2(mod4), \end{cases}$$

Theorem 2.10. *Given a cycle C_n, $n \geq 3$. Then*

$$(i)\ GOBD_1(C_n) = \begin{cases} \frac{n^2(n+4)}{4}, & if \quad n \equiv 0(mod4), \\ \frac{n(n^2+6n+1)}{4}, & if \quad n \equiv 1,3(mod4), \\ \frac{n(n^2+8n+12)}{4}, & if \quad n \equiv 2(mod4), \end{cases}$$

$$(ii)\ GOBD_2(C_n) = \begin{cases} \frac{n^4}{4}, & if \quad n \equiv 0(mod4), \\ \frac{n(n+1)^3}{4}, & if \quad n \equiv 1,3(mod4),. \\ \frac{n(n+2)^3}{4}, & if \quad n \equiv 2(mod4) \end{cases}$$

Proof. Let C_n be a cycle graph on n vertices. Then by Theorem ?, we have

$$d_{b\gamma}(x) = \begin{cases} \frac{n}{4}, & if \quad n \equiv 0(mod4), \\ \frac{n+1}{4}, & if \quad n \equiv 1,3(mod4), \\ \frac{n+2}{4}, & if \quad n \equiv 2(mod4), \end{cases}$$

By the definition of Gourava indices, we have

Case (i): If $n \equiv 0(mod4)$, then

$$GOBD_1(C_n) = \sum_{u,v \in E(G)} [(d_{b\gamma}(u) + d_{b\gamma}(v)) + (d_{b\gamma}(u).d_{b\gamma}(v))]$$
$$GOBD_1(C_n) = n. \left[\left(\frac{n}{2} + \frac{n}{2}\right) + \left(\frac{n}{2}.\frac{n}{2}\right) \right]$$
$$GOBD_1(C_n) = \frac{n^2(n+4)}{4}$$

If $n \equiv 1,3(mod4)$, then

$$GOBD_1(C_n) = n. \left\{ \left[\left(\frac{n+1}{2} + \frac{n+1}{2}\right) + \left(\frac{n+1}{2}.\frac{n+1}{2}\right) \right] \right\}$$
$$GOBD_1(C_n) = \frac{n(n+1)(n+5)}{4}$$

If $n \equiv 2(mod4)$, then

$$GOBD_1(C_n) = \sum_{u,v \in E(G)} [(d_{b\gamma}(u) + d_{b\gamma}(v)) + (d_{b\gamma}(u).d_{b\gamma}(v))]$$

If $n \equiv 2(mod4)$, then

$$GOBD_1(C_n) = n.\left[\left(\frac{n+2}{2} + \frac{n+2}{2}\right) + \left(\frac{n+2}{2}.\frac{n+2}{2}\right)\right]$$
$$GOBD_1(C_n) = \frac{n(n+2)(n+6)}{4}$$

Case (ii): If $n \equiv 0(mod4)$, then

$$GOBD_2(C_n) = \sum_{u,v \in E(G)} [(d_{b\gamma}(u) + d_{b\gamma}(v)).(d_{b\gamma}(u).d_{b\gamma}(v))]$$
$$GOBD_2(C_n) = n.\left[\left(\frac{n}{2} + \frac{n}{2}\right) \cdot \left(\frac{n}{2}.\frac{n}{2}\right)\right]$$
$$GOBD_2(C_n) = \frac{n^4}{4}$$

If $n \equiv 1, 3(mod4)$, then

$$GOBD_2(C_n) = n.\left\{\left(\frac{n+1}{2} + \frac{n+1}{2}\right) \cdot \left(\frac{n+1}{2}.\frac{n+1}{2}\right)\right\}$$
$$GOBD_2(C_n) = \frac{n(n+1)^3}{4}$$

If $n \equiv 2(mod4)$, then

$$GOBD_2(C_n) = \sum_{u,v \in E(G)} \left[\left(d_{b\gamma}(u) + d_{b\gamma}(v)\right) + \left(d_{b\gamma}(u).d_{b\gamma}(v)\right)\right]$$

If $n \equiv 2(mod4)$, then

$$GOBD_2(C_n) = n.\left[\left(\frac{n+2}{2} + \frac{n+2}{2}\right) \cdot \left(\frac{n+2}{2}.\frac{n+2}{2}\right)\right]$$
$$GOBD_2(C_n) = \frac{n(n+2)^3}{4}$$

□

In the following proposition, by using definition, we obtain the first and second Gourava bipartite domination polynomials of $K_n, K_{r,t}, W_n, F_n, F_{m,n}, D_k, k \geq 1, B_k$ and H_n.

2.1 Gourava bipartite domination polynomials of some class of graphs.

The first and second Gourava bipartite domination polynomials of $K_n, K_{r,t}, W_n, F_n, F_{m,n}, D_k, k \geq 1, B_k$ and H_n.

1. $GOBD_1(K_n, x) = \frac{n(n-1)}{2}.x^8$
2. $GOBD_1(K_{r,t}, x) = (rt).x^8$
3. $GOBD_1(W_n, x) = (2n).x^8$
4. $GOBD_1(F_n, x) = (2n).x^8$
5. $GOBD_1(F_{m,n}, x) = (mn-1).x^{4m(m+1)}$
6. $GOBD_1(D_k, x) = (4k).x^{4k(k+1)}$
7. $GOBD_1(B_k, x) = x^8 + (2k).x^{(k+1)(k+3)}$
8. $GOBD_1(H_n, x) = (3n).x^{4n^2+4n}$
9. $(i)\ GOBD_1(C_n) = \begin{cases} n.x^{\frac{n(n+9)}{16}}, & if \quad n \equiv 0(mod4), \\ n.x^{\frac{(n+1)(n^2+6n+1)}{16}}, & if \quad n \equiv 1,3(mod4), \\ n.x^{\frac{(n+2)(n+10)}{16}}, & if \quad n \equiv 2(mod4), \end{cases}$
10. $GOBD_2(K_n, x) = \frac{n(n-1)}{2}.x^{16}$
11. $GOBD_2(K_{r,t}, x) = (rt).x^{16}$
12. $GOBD_2(W_n, x) = (2n).x^{16}$
13. $GOBD_2(F_n, x) = (2n).x^{16}$
14. $GOBD_2(F_{m,n}, x) = (mn-1).x^{16m^3}$
15. $GOBD_2(D_k, x) = (4k)x^{16k^3}$
16. $GOBD_2(B_k, x) = x^{16} + (2k).x^{2(k+1)^3}$
17. $GOBD_2(H_n, x) = (3n).x^{16n^3}$
18. $GOBD_1(C_n) = \begin{cases} n.x^{\frac{(n+2)(n+10)}{32}}, & if \quad n \equiv 0(mod4), \\ n.x^{\frac{(n+1)^3}{32}}, & if \quad n \equiv 1,3(mod4), \\ n.x^{\frac{(n+2)^3}{4}}, & if \quad n \equiv 2(mod4), \end{cases}$

3. Conclusion

In this work, the first and second Gourava domination indices and bipartite domination indices are calculated. The idea behind this problem is the general problem started by V. R. Kulli in 2022. He calculated some formula, called Gourava index, to determine results. Here, such ideas are calculated for some standard graphs.

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