International Journal of
Mathematics And its Applications
ISSN: 2347-1557

Int. J. Math. And Appl., **13(3)**(2025), 33–39
Available Online: http://ijmaa.in

Matrix Representations of Group Algebras of 16 Order Abelian Groups

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Abstract

Group algebras of abelian groups of order 16 are represented in terms of block circulant matrices.

Keywords: Group Algebra; Circulant Matrix; Abelian Group.

2020 Mathematics Subject Classification: 16G99.

1. Preliminaries

There are fourteen groups of order 16, five of them are abelian the other nine are nonabelian [3]. The abelian groups of order 16 are:

(i) $C_{16} = \langle \alpha : \alpha^{16} = 1 \rangle$

(ii) $C_8 \times C_2 = \langle \alpha, \beta : \alpha^8 = \beta^2 = 1, \beta\alpha = \alpha\beta \rangle$

(iii) $C_4 \times C_4 = \langle \alpha, \beta : \alpha^4 = \beta^4 = 1, \beta\alpha = \alpha\beta \rangle$

(iv) $C_4 \times C_2 \times C_2 = \langle \alpha, \beta, \gamma : \alpha^4 = \beta^2 = \gamma^2 = 1, \beta\alpha = \alpha\beta, \gamma\alpha = \alpha\gamma, \gamma\beta = \beta\gamma \rangle$

(v) $C_2 \times C_2 \times C_2 \times C_2 = \langle \alpha, \beta, \gamma, \delta : \alpha^2 = \beta^2 = \gamma^2 = \delta^2 = 1, \beta\alpha = \alpha\beta, \gamma\alpha = \alpha\gamma, \gamma\beta = \beta\gamma, \delta\alpha = \alpha\delta, \delta\beta = \beta\delta, \delta\gamma = \gamma\delta \rangle$.

The results in [1] and [2] are used to find the representations of these groups.
Let F be a field. A ring A with unity is an algebra over F ($F-$algebra) if A is a vector space over F and the following compatibility condition holds $(sa).b = s(a.b) = a.(sb)$ for any $s \in F$. A is also called associative algebra (over F). The dimension of the algebra A is the dimension of A as a vector space over F.

Theorem 1.1 ([4]). *Let A be a $n-$dimensional algebra over a field F. Then there is a one to one algebra homomorphism from A into $M_n(F)$, the algebra of $n-$matrices over F.*

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Let $G = \{g_1 = 1, g_2, \ldots, g_n\}$ be a finite group of order n and F a field. Define $FG = \{a_1 g_1 + a_2 g_2 + \cdots + a_n g_n : a_i \in F\}$. FG is $n-$dimensional vector space over F with basis G. Multiplication of G can be extended linearly to FG. Thus, FG becomes an algebra over F of dimension n. FG is called group algebra. The following identifications should be realized:

(i) $0_F g_G = 0_{FG} = 0$ for any $g \in G$

(ii) $1_F g_G = g_{FG} = g$ for any $g \in G$. In particular $1_F g_G = 1_{FG} = 1$

(iii) $a_F 1_G = a_{FG}$ for any $a \in G$

A circulant matrix M on parameters $a_0, a_1, \ldots, a_{n-1}$ is defined as follows:

$$M\left(a_0, a_1, \ldots, a_{n-1}\right) = \begin{bmatrix} a_0 & a_{n-1} & \cdots & a_1 \\ a_1 & a_0 & \cdots & a_2 \\ \vdots & \vdots & \cdots & \vdots \\ a_{n-1} & a_{n-1} & \cdots & a_0 \end{bmatrix}$$

This matrix may be denoted in terms of its columns by $[col\left(a_0\right)|col\left(a_{n-1}\right)|\ldots|col(a_0)]$. M is said to be circulant block matrix if it is if the form $M(M_1, M_2, \ldots, M_n)$. i.e, it is circulant blockwise on the blocks $M_1, M_2, \ldots, M_n$. Thus,

$$M = \begin{bmatrix} M_1 & M_n & \cdots & M_2 \\ M_2 & M_1 & \cdots & M_3 \\ \vdots & \vdots & \cdots & \vdots \\ M_n & M_{n-1} & \cdots & M_1 \end{bmatrix}$$

2. Main Results

Theorem 2.1 ([1]). *Let F be a field and $G = \langle \alpha : \alpha^n = 1 \rangle$ a cyclic group of order n. Then any element $a_1 1 + a_2\alpha + \cdots + a_n\alpha^{n-1}$ of FG can be represented with respect to the order basis $\left\{1, \alpha, \ldots, \alpha^{n-1}\right\}$ by the circulant matrix* $M\left(a_1, a_2, \ldots, a_n\right) = \begin{bmatrix} a_1 & a_n & \cdots & a_2 \\ a_2 & a_1 & \cdots & a_3 \\ \vdots & \vdots & \cdots & \vdots \\ a_n & a_{n-1} & \cdots & a_1 \end{bmatrix}$.

Theorem 2.2 ([1]). *Let F be a field and G a split metacyclic group [2]. The representation of the general element $\sum_{j=0}^{m-1}\sum_{i=0}^{n-1} a_{ij}\alpha^i\beta^j$ in FG is given by the circulant matrix $M\left(M(a_{i0}), M^{\beta}(a_{i1}), \ldots, M^{\beta^{m-1}}(a_{im-1})\right); i = 0, 1, \ldots, n-1$.*

Corollary 2.3 ([1]). *Let F be a field. Matrix representation of $F(C_n \times C_m)$, where $(m, n) \neq 1$ is given by $M\left((a_{i0}), M(a_{i1}), \ldots, M(a_{im-1})\right); i = 0, 1, \ldots, n-1$ and $a_{ij} \in F$.*

Note that if the order of the basis elements is changed, we obtain a different matrix of representation. The new matrix is obtained by suitable interchanging of the columns of the matrix $M\left(a_0, a_1, \ldots, a_{n-1}\right)$. In [2] the representation is done for the non-split metacyclic group. For more complicated finite groups we use the circulant block matrices to do the required representations.

3. Applications

(i) $C_{16}\langle\alpha\rangle = \left\langle \alpha : \alpha^{16} = 1 \right\rangle = \left\{1, \alpha, \alpha^2, \ldots, \alpha^{15}\right\}$

$$w = a_1 1 + a_2\alpha + a_2\alpha^2 + \cdots + a_{16}\alpha^{16}$$

$$[w] = M\left(a_1,\ a_2, \ldots,\ a_{16}\right) = \begin{bmatrix} a_1 & a_{16} & \cdots & a_2 \\ a_2 & a_1 & \cdots & a_3 \\ \vdots & \vdots & \cdots & \vdots \\ a_{16} & a_{15} & \cdots & a_1 \end{bmatrix}$$

(ii) $C_8\langle\alpha\rangle \times C_2\langle\beta\rangle = \left\langle \alpha, \beta : \alpha^8 = \beta^2 = 1,\ \beta\alpha = \alpha\beta \right\rangle = \left\{1, \alpha, \alpha^2, \ldots, \alpha^7, \beta, \alpha\beta, \alpha^2\beta, \ldots, \alpha^7\beta\right\}$

$$w = a_1 1 + a_2\alpha + a_3\alpha^2 + \cdots + a_8\alpha^7 + a_9\beta + a_{10}\alpha\beta + a_{11}\alpha^2\beta + \cdots + a_{16}\alpha^7\beta$$

$$[w] = M\left(M\left(a_1, \ldots, a_8\right), M\left(a_9, \ldots, a_{16}\right)\right) = \begin{bmatrix} M\left(a_1, \ldots, a_8\right) & \vdots & M\left(a_9, \ldots, a_{16}\right) \\ \ldots\ldots\ldots\ldots & \cdots & \ldots\ldots\ldots\ldots \\ M\left(a_9, \ldots, a_{16}\right) & \vdots & M\left(a_1, \ldots, a_8\right) \end{bmatrix}$$

$$\begin{bmatrix}
a_1 & a_8 & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & \vdots & a_9 & a_{16} & a_{15} & a_{14} & a_{13} & a_{12} & a_{11} & a_{10} \\
a_2 & a_1 & a_8 & a_7 & a_6 & a_5 & a_4 & a_3 & \vdots & a_{10} & a_9 & a_{16} & a_{15} & a_{14} & a_{13} & a_{12} & a_{11} \\
a_3 & a_2 & a_1 & a_8 & a_7 & a_6 & a_5 & a_4 & \vdots & a_{11} & a_{10} & a_9 & a_{16} & a_{15} & a_{14} & a_{13} & a_{12} \\
a_4 & a_3 & a_2 & a_1 & a_8 & a_7 & a_6 & a_5 & \vdots & a_{12} & a_{11} & a_{10} & a_9 & a_{16} & a_{15} & a_{14} & a_{13} \\
a_5 & a_4 & a_3 & a_2 & a_1 & a_8 & a_7 & a_6 & \vdots & a_{13} & a_{12} & a_{11} & a_{10} & a_9 & a_{16} & a_{15} & a_{14} \\
a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_8 & a_7 & \vdots & a_{14} & a_{13} & a_{12} & a_{11} & a_{10} & a_9 & a_{16} & a_{15} \\
a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_8 & \vdots & a_{15} & a_{14} & a_{13} & a_{12} & a_{11} & a_{10} & a_9 & a_{16} \\
a_8 & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & \vdots & a_{16} & a_{15} & a_{14} & a_{13} & a_{12} & a_{11} & a_{10} & a_9 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_9 & a_{16} & a_{15} & a_{14} & a_{13} & a_{12} & a_{11} & a_{10} & \vdots & a_1 & a_8 & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 \\
a_{10} & a_9 & a_{16} & a_{15} & a_{14} & a_{13} & a_{12} & a_{11} & \vdots & a_2 & a_1 & a_8 & a_7 & a_6 & a_5 & a_4 & a_3 \\
a_{11} & a_{10} & a_9 & a_{16} & a_{15} & a_{14} & a_{13} & a_{12} & \vdots & a_3 & a_2 & a_1 & a_8 & a_7 & a_6 & a_5 & a_4 \\
a_{12} & a_{11} & a_{10} & a_9 & a_{16} & a_{15} & a_{14} & a_{13} & \vdots & a_4 & a_3 & a_2 & a_1 & a_8 & a_7 & a_6 & a_5 \\
a_{13} & a_{12} & a_{11} & a_{10} & a_9 & a_{16} & a_{15} & a_{14} & \vdots & a_5 & a_4 & a_3 & a_2 & a_1 & a_8 & a_7 & a_6 \\
a_{14} & a_{13} & a_{12} & a_{11} & a_{10} & a_9 & a_{16} & a_{15} & \vdots & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_8 & a_7 \\
a_{15} & a_{14} & a_{13} & a_{12} & a_{11} & a_{10} & a_9 & a_{16} & \vdots & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_8 \\
a_{16} & a_{15} & a_{14} & a_{13} & a_{12} & a_{11} & a_{10} & a_9 & \vdots & a_8 & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1
\end{bmatrix}$$

(iii) $C_4\langle\alpha\rangle \times C_4\langle\beta\rangle = \{1, \alpha, \alpha^2, \alpha^3, \beta, \alpha\beta, \alpha^2\beta, \alpha^3\beta, \beta^2, \alpha\beta^2, \alpha^2\beta^2, \alpha^3\beta^2, \beta^3, \alpha\beta^3, \alpha^2\beta^3, \alpha^3\beta^3\}$.

$$w = a_1 1 + a_2\alpha + a_3\alpha^2 + a_4\alpha^3 + a_5\beta + a_6\alpha\beta + a_7\alpha^2\beta + a_8\alpha^3\beta + a_9\beta^2 + a_{10}\alpha\beta^2 + a_{11}\alpha^2\beta^2 + a_{12}\alpha^3\beta^2 + a_{13}\beta^3$$
$$+ a_{14}\alpha\beta^3 + a_{15}\alpha^2\beta^3 + a_{16}\alpha^3\beta^3$$

$$[w] = M\left(M(a_1, a_2, a_3, a_4), M(a_5, a_6, a_7, a_8), M(a_9, a_{10}, a_{11}, a_{12}), M(a_{13}, a_{14}, a_{15}, a_{16})\right)$$

$$\begin{bmatrix}
M(a_1, a_2, a_3, a_4) & \vdots & M(a_{13}, a_{14}, a_{15}, a_{16}) & \vdots & M(a_9, a_{10}, a_{11}, a_{12}) & \vdots & M(a_5, a_6, a_7, a_8) \\
\cdots\cdots\cdots & \vdots & \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\
M(a_5, a_6, a_7, a_8) & \vdots & M(a_1, a_2, a_3, a_4) & \vdots & M(a_{13}, a_{14}, a_{15}, a_{16}) & \vdots & M(a_9, a_{10}, a_{11}, a_{12}) \\
\cdots\cdots\cdots & \vdots & \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\
M(a_9, a_{10}, a_{11}, a_{12}) & \vdots & M(a_5, a_6, a_7, a_8) & \vdots & M(a_1, a_2, a_3, a_4) & \vdots & M(a_{13}, a_{14}, a_{15}, a_{16}) \\
\cdots\cdots\cdots & \vdots & \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\
M(a_{13}, a_{14}, a_{15}, a_{16}) & \vdots & M(a_9, a_{10}, a_{11}, a_{12}) & \vdots & M(a_5, a_6, a_7, a_8) & \vdots & M(a_1, a_2, a_3, a_4)
\end{bmatrix}$$

$$
\left[\begin{array}{ccccccccccccccccccc}
a_1 & a_4 & a_3 & a_2 & \vdots & a_{13} & a_{16} & a_{15} & a_{14} & \vdots & a_9 & a_{12} & a_{11} & a_{10} & \vdots & a_5 & a_8 & a_7 & a_6 \\
a_2 & a_1 & a_4 & a_3 & \vdots & a_{14} & a_{13} & a_{16} & a_{15} & \vdots & a_{10} & a_9 & a_{12} & a_{11} & \vdots & a_6 & a_5 & a_8 & a_7 \\
a_3 & a_2 & a_1 & a_4 & \vdots & a_{15} & a_{14} & a_{13} & a_{16} & \vdots & a_{11} & a_{10} & a_9 & a_{12} & \vdots & a_7 & a_6 & a_5 & a_8 \\
a_4 & a_3 & a_2 & a_1 & \vdots & a_{16} & a_{15} & a_{14} & a_{13} & \vdots & a_{12} & a_{11} & a_{10} & a_9 & \vdots & a_8 & a_7 & a_6 & a_5 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_5 & a_8 & a_7 & a_6 & \vdots & a_1 & a_4 & a_3 & a_2 & \vdots & a_{13} & a_{16} & a_{15} & a_{14} & \vdots & a_9 & a_{12} & a_{11} & a_{10} \\
a_6 & a_5 & a_8 & a_7 & \vdots & a_2 & a_1 & a_4 & a_3 & \vdots & a_{14} & a_{13} & a_{16} & a_{15} & \vdots & a_{10} & a_9 & a_{12} & a_{11} \\
a_7 & a_6 & a_5 & a_8 & \vdots & a_3 & a_2 & a_1 & a_4 & \vdots & a_{15} & a_{14} & a_{13} & a_{16} & \vdots & a_{11} & a_{10} & a_9 & a_{12} \\
a_8 & a_7 & a_6 & a_5 & \vdots & a_4 & a_3 & a_2 & a_1 & \vdots & a_{16} & a_{15} & a_{14} & a_{13} & \vdots & a_{12} & a_{11} & a_{10} & a_9 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_9 & a_{12} & a_{11} & a_{10} & \vdots & a_5 & a_8 & a_7 & a_6 & \vdots & a_1 & a_4 & a_3 & a_2 & \vdots & a_{13} & a_{16} & a_{15} & a_{14} \\
a_{10} & a_9 & a_{12} & a_{11} & \vdots & a_6 & a_5 & a_8 & a_7 & \vdots & a_2 & a_1 & a_4 & a_3 & \vdots & a_{14} & a_{13} & a_{16} & a_{15} \\
a_{11} & a_{10} & a_9 & a_{12} & \vdots & a_7 & a_6 & a_5 & a_8 & \vdots & a_3 & a_2 & a_1 & a_4 & \vdots & a_{15} & a_{14} & a_{13} & a_{16} \\
a_{12} & a_{11} & a_{10} & a_9 & \vdots & a_8 & a_7 & a_6 & a_5 & \vdots & a_4 & a_3 & a_2 & a_1 & \vdots & a_{16} & a_{15} & a_{14} & a_{13} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_{13} & a_{16} & a_{15} & a_{14} & \vdots & a_9 & a_{12} & a_{11} & a_{10} & \vdots & a_5 & a_8 & a_7 & a_6 & \vdots & a_1 & a_4 & a_3 & a_2 \\
a_{14} & a_{13} & a_{16} & a_{15} & \vdots & a_{10} & a_9 & a_{12} & a_{11} & \vdots & a_6 & a_5 & a_8 & a_7 & \vdots & a_2 & a_1 & a_4 & a_3 \\
a_{15} & a_{14} & a_{13} & a_{16} & \vdots & a_{11} & a_{10} & a_9 & a_{12} & \vdots & a_7 & a_6 & a_5 & a_8 & \vdots & a_3 & a_2 & a_1 & a_4 \\
a_{16} & a_{15} & a_{14} & a_{13} & \vdots & a_{12} & a_{11} & a_{10} & a_9 & \vdots & a_8 & a_7 & a_6 & a_5 & \vdots & a_4 & a_3 & a_2 & a_1
\end{array}\right]
$$

(iv) $C_4\langle\alpha\rangle \times C_2\langle\beta\rangle \times C_2\langle\gamma\rangle = \{1, \alpha, \alpha^2, \alpha^3, \beta, \alpha\beta, \alpha^2\beta, \alpha^3\beta, \gamma, \alpha\gamma, \alpha^2\gamma, \alpha^3\gamma, \beta\gamma, \alpha\beta\gamma, \alpha^2\beta\gamma, \alpha^3\beta\gamma\}$.

$$
\begin{aligned}
w &= a_1 1 + a_2\alpha + a_3\alpha^2 + a_4\alpha^3 + a_5\beta + a_6\alpha\beta + a_7\alpha^2\beta + a_8\alpha^3\beta + a_9\gamma + a_{10}\alpha\gamma + a_{11}\alpha^2\gamma + a_{12}\alpha^3\gamma + a_{13}\beta\gamma \\
&\quad + a_{14}\alpha\beta\gamma + a_{15}\alpha^2\beta\gamma + a_{16}\alpha^3\beta\gamma \\
[w] &= M\left(M\left(M\left(a_1, a_2, a_3, a_4\right), M\left(a_5, a_6, a_7, a_8\right)\right), M\left(a_9, a_{10}, a_{11}, a_{12}\right), M\left(a_{13}, a_{14}, a_{15}, a_{16}\right)\right).
\end{aligned}
$$

$$
\left[\begin{array}{ccccccc}
M(a_1, a_2, a_3, a_4) & \vdots & M(a_5, a_6, a_7, a_8) & \vdots & M(a_9, a_{10}, a_{11}, a_{12}) & \vdots & M(a_{13}, a_{14}, a_{15}, a_{16}) \\
\cdots\cdots & \vdots & \cdots\cdots & \vdots & \cdots\cdots & \vdots & \cdots\cdots \\
M(a_5, a_6, a_7, a_8) & \vdots & M(a_1, a_2, a_3, a_4) & \vdots & M(a_{13}, a_{14}, a_{15}, a_{16}) & \vdots & M(a_9, a_{10}, a_{11}, a_{12}) \\
\cdots\cdots & \vdots & \cdots\cdots & \vdots & \cdots\cdots & \vdots & \cdots\cdots \\
M(a_9, a_{10}, a_{11}, a_{12}) & \vdots & M(a_{13}, a_{14}, a_{15}, a_{16}) & \vdots & M(a_1, a_2, a_3, a_4) & \vdots & M(a_5, a_6, a_7, a_8) \\
\cdots\cdots & \vdots & \cdots\cdots & \vdots & \cdots\cdots & \vdots & \cdots\cdots \\
M(a_{13}, a_{14}, a_{15}, a_{16}) & \vdots & M(a_9, a_{10}, a_{11}, a_{12}) & \vdots & M(a_5, a_6, a_7, a_8) & \vdots & M(a_1, a_2, a_3, a_4)
\end{array}\right]
$$

$$
\begin{bmatrix}
a_1 & a_4 & a_3 & a_2 & \vdots & a_5 & a_8 & a_7 & a_6 & \vdots & a_9 & a_{12} & a_{11} & a_{10} & \vdots & a_{13} & a_{16} & a_{15} & a_{14} \\
a_2 & a_1 & a_4 & a_3 & \vdots & a_6 & a_5 & a_8 & a_7 & \vdots & a_{10} & a_9 & a_{12} & a_{11} & \vdots & a_{14} & a_{13} & a_{16} & a_{15} \\
a_3 & a_2 & a_1 & a_4 & \vdots & a_7 & a_6 & a_5 & a_8 & \vdots & a_{11} & a_{10} & a_9 & a_{12} & \vdots & a_{15} & a_{14} & a_{13} & a_{16} \\
a_4 & a_3 & a_2 & a_1 & \vdots & a_8 & a_7 & a_6 & a_5 & \vdots & a_{12} & a_{11} & a_{10} & a_9 & \vdots & a_{16} & a_{15} & a_{14} & a_{13} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_5 & a_8 & a_7 & a_6 & \vdots & a_1 & a_4 & a_3 & a_2 & \vdots & a_{13} & a_{16} & a_{15} & a_{14} & \vdots & a_9 & a_{12} & a_{11} & a_{10} \\
a_6 & a_5 & a_8 & a_7 & \vdots & a_2 & a_1 & a_4 & a_3 & \vdots & a_{14} & a_{13} & a_{16} & a_{15} & \vdots & a_{10} & a_9 & a_{12} & a_{11} \\
a_7 & a_6 & a_5 & a_8 & \vdots & a_3 & a_2 & a_1 & a_4 & \vdots & a_{15} & a_{14} & a_{13} & a_{16} & \vdots & a_{11} & a_{10} & a_9 & a_{12} \\
a_8 & a_7 & a_6 & a_5 & \vdots & a_4 & a_3 & a_2 & a_1 & \vdots & a_{16} & a_{15} & a_{14} & a_{13} & \vdots & a_{12} & a_{11} & a_{10} & a_9 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_9 & a_{12} & a_{11} & a_{10} & \vdots & a_{13} & a_{16} & a_{15} & a_{14} & \vdots & a_1 & a_4 & a_3 & a_2 & \vdots & a_5 & a_8 & a_7 & a_6 \\
a_{10} & a_9 & a_{12} & a_{11} & \vdots & a_{14} & a_{13} & a_{16} & a_{15} & \vdots & a_2 & a_1 & a_4 & a_3 & \vdots & a_6 & a_5 & a_8 & a_7 \\
a_{11} & a_{10} & a_9 & a_{12} & \vdots & a_{15} & a_{14} & a_{13} & a_{16} & \vdots & a_3 & a_2 & a_1 & a_4 & \vdots & a_7 & a_6 & a_5 & a_8 \\
a_{12} & a_{11} & a_{10} & a_9 & \vdots & a_{16} & a_{15} & a_{14} & a_{13} & \vdots & a_4 & a_3 & a_2 & a_1 & \vdots & a_8 & a_7 & a_6 & a_5 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_{13} & a_{16} & a_{15} & a_{14} & \vdots & a_9 & a_{12} & a_{11} & a_{10} & \vdots & a_5 & a_8 & a_7 & a_6 & \vdots & a_1 & a_4 & a_3 & a_2 \\
a_{14} & a_{13} & a_{16} & a_{15} & \vdots & a_{10} & a_9 & a_{12} & a_{11} & \vdots & a_6 & a_5 & a_8 & a_7 & \vdots & a_2 & a_1 & a_4 & a_3 \\
a_{15} & a_{14} & a_{13} & a_{16} & \vdots & a_{11} & a_{10} & a_9 & a_{12} & \vdots & a_7 & a_6 & a_5 & a_8 & \vdots & a_3 & a_2 & a_1 & a_4 \\
a_{16} & a_{15} & a_{14} & a_{13} & \vdots & a_{12} & a_{11} & a_{10} & a_9 & \vdots & a_8 & a_7 & a_6 & a_5 & \vdots & a_4 & a_3 & a_2 & a_1
\end{bmatrix}
$$

(v) $C_2\langle\alpha\rangle \times C_2\langle\beta\rangle \times C_2\langle\gamma\rangle \times C_2\langle\delta\rangle = \langle \alpha,\beta,\gamma,\delta : \alpha^2 = \beta^2 = \gamma^2 = \delta^2 = 1, \beta\alpha = \alpha\beta, \gamma\alpha = \alpha\gamma, \gamma\beta = \beta\gamma, \delta\alpha = \alpha\delta, \delta\beta = \beta\delta, \delta\gamma = \gamma\delta \rangle = \{1, \alpha, \beta, \alpha\beta, \gamma, \alpha\gamma, \beta\gamma, \alpha\beta\gamma, \delta, \alpha\delta, \beta\delta, \alpha\beta\delta, \gamma\delta, \alpha\gamma\delta, \beta\gamma\delta, \alpha\ \beta\gamma\delta\}$

$$
\begin{aligned}
w &= a_1 1 + a_2\alpha + a_3\beta + a_4\alpha\beta + a_5\gamma + a_6\alpha\gamma + a_7\beta\gamma + a_8\alpha\beta\gamma + a_9\delta + a_{10}\alpha\delta + a_{11}\beta\delta + a_{12}\alpha\beta\delta + a_{13}\gamma\delta \\
&\quad + a_{14}\alpha\gamma\delta + a_{15}\beta\gamma\delta + a_{16}\alpha\beta\gamma\delta
\end{aligned}
$$

$$
\begin{aligned}
[w] &= M\left(M\left(M\left(M\left(a_1,a_2\right), M\left(a_3,a_4\right)\right), M\left(M\left(a_5,a_6\right), M\left(a_7,a_8\right)\right)\right), M\left(M\left(M\left(a_9,a_{10}\right), M\left(a_{11},a_{12}\right)\right),\right.\right. \\
&\quad \left.\left.M\left(M\left(a_{13},a_{14}\right), M\left(a_{15},a_{16}\right)\right)\right)\right).
\end{aligned}
$$

$$
\begin{bmatrix}
M(M(M(a_1,a_2), M(a_3,a_4)), M(M(a_5,a_6), M(a_7,a_8))) & \vdots & M(M(M(a_9,a_{10}), M(a_{11},a_{12})), M(M(a_{13},a_{14}), M(a_{15},a_{16}))) \\
\cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\
M(M(M(a_9,a_{10}), M(a_{11},a_{12})), M(M(a_{13},a_{14}), M(a_{15},a_{16}))) & \vdots & M(M(M(a_1,a_2), M(a_3,a_4)), M(M(a_5,a_6), M(a_7,a_8))) \\
\cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\
M(M(M(a_9,a_{10}), M(a_{11},a_{12})), M(M(a_{13},a_{14}), M(a_{15},a_{16}))) & \vdots & M(M(M(a_1,a_2), M(a_3,a_4)), M(M(a_5,a_6), M(a_7,a_8))) \\
\cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\
M(M(M(a_1,a_2), M(a_3,a_4)), M(M(a_5,a_6), M(a_7,a_8))) & \vdots & M(M(M(a_9,a_{10}), M(a_{11},a_{12})), M(M(a_{13},a_{14}), M(a_{15},a_{16})))
\end{bmatrix}
$$

$$
\begin{bmatrix}
a_1 & a_2 & \vdots & a_3 & a_4 & \vdots & a_5 & a_6 & \vdots & a_7 & a_8 & \vdots & a_9 & a_{10} & \vdots & a_{11} & a_{12} & \vdots & a_{13} & a_{14} & \vdots & a_{15} & a_{16} \\
a_2 & a_1 & \vdots & a_4 & a_3 & \vdots & a_6 & a_5 & \vdots & a_8 & a_7 & \vdots & a_{10} & a_9 & \vdots & a_{12} & a_{11} & \vdots & a_{14} & a_{13} & \vdots & a_{16} & a_{15} \\
\cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\
a_3 & a_4 & \vdots & a_1 & a_2 & \vdots & a_7 & a_8 & \vdots & a_5 & a_6 & \vdots & a_{11} & a_{12} & \vdots & a_9 & a_{10} & \vdots & a_{15} & a_{16} & \vdots & a_{13} & a_{14} \\
a_4 & a_3 & \vdots & a_2 & a_1 & \vdots & a_8 & a_7 & \vdots & a_6 & a_5 & \vdots & a_{12} & a_{11} & \vdots & a_{10} & a_9 & \vdots & a_{16} & a_{15} & \vdots & a_{14} & a_{13} \\
\cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\
a_5 & a_6 & \vdots & a_7 & a_8 & \vdots & a_1 & a_2 & \vdots & a_3 & a_4 & \vdots & a_{13} & a_{14} & \vdots & a_{15} & a_{16} & \vdots & a_9 & a_{10} & \vdots & a_{11} & a_{12} \\
a_6 & a_5 & \vdots & a_8 & a_7 & \vdots & a_2 & a_1 & \vdots & a_4 & a_3 & \vdots & a_{14} & a_{13} & \vdots & a_{16} & a_{15} & \vdots & a_{10} & a_9 & \vdots & a_{12} & a_{11} \\
\cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\
a_7 & a_8 & \vdots & a_5 & a_6 & \vdots & a_3 & a_4 & \vdots & a_1 & a_2 & \vdots & a_{15} & a_{16} & \vdots & a_{13} & a_{14} & \vdots & a_{11} & a_{12} & \vdots & a_9 & a_{10} \\
a_8 & a_7 & \vdots & a_6 & a_5 & \vdots & a_4 & a_3 & \vdots & a_2 & a_1 & \vdots & a_{16} & a_{15} & \vdots & a_{14} & a_{13} & \vdots & a_{12} & a_{11} & \vdots & a_{10} & a_9 \\
\cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\
a_9 & a_{10} & \vdots & a_{11} & a_{12} & \vdots & a_{13} & a_{14} & \vdots & a_{15} & a_{16} & \vdots & a_1 & a_2 & \vdots & a_3 & a_4 & \vdots & a_5 & a_6 & \vdots & a_7 & a_8 \\
a_{10} & a_9 & \vdots & a_{12} & a_{11} & \vdots & a_{14} & a_{13} & \vdots & a_{16} & a_{15} & \vdots & a_2 & a_1 & \vdots & a_4 & a_3 & \vdots & a_6 & a_5 & \vdots & a_8 & a_7 \\
\cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\
a_{11} & a_{12} & \vdots & a_9 & a_{10} & \vdots & a_{15} & a_{16} & \vdots & a_{13} & a_{14} & \vdots & a_3 & a_4 & \vdots & a_1 & a_2 & \vdots & a_7 & a_8 & \vdots & a_5 & a_6 \\
a_{12} & a_{11} & \vdots & a_{10} & a_9 & \vdots & a_{16} & a_{15} & \vdots & a_{14} & a_{13} & \vdots & a_4 & a_3 & \vdots & a_2 & a_1 & \vdots & a_8 & a_7 & \vdots & a_6 & a_5 \\
\cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\
a_{13} & a_{14} & \vdots & a_{15} & a_{16} & \vdots & a_9 & a_{10} & \vdots & a_{11} & a_{12} & \vdots & a_5 & a_6 & \vdots & a_7 & a_8 & \vdots & a_1 & a_2 & \vdots & a_3 & a_4 \\
a_{14} & a_{13} & \vdots & a_{16} & a_{15} & \vdots & a_{10} & a_9 & \vdots & a_{12} & a_{11} & \vdots & a_6 & a_5 & \vdots & a_8 & a_7 & \vdots & a_2 & a_1 & \vdots & a_4 & a_3 \\
\cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\
a_{15} & a_{16} & \vdots & a_{13} & a_{14} & \vdots & a_{11} & a_{12} & \vdots & a_9 & a_{10} & \vdots & a_7 & a_8 & \vdots & a_5 & a_6 & \vdots & a_3 & a_4 & \vdots & a_1 & a_2 \\
a_{16} & a_{15} & \vdots & a_{14} & a_{13} & \vdots & a_{12} & a_{11} : & \vdots & a_{10} & a_9 & \vdots & a_8 & a_7 & \vdots & a_6 & a_5 & \vdots & a_4 & a_3 & \vdots & a_2 & a_1
\end{bmatrix}
$$

Acknowledgment

Acknowledgment is due to Professor Kahtan H. Alzubaidy for his suggestions and comments.

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