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Relation between Luhn Algorithm and Level sets

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Abstract

The Luhn algorithm, a widely used checksum formula, ensures the validity of identification numbers such as credit cards and IMEI numbers by detecting accidental errors. Operating through a simple yet effective process of digit manipulation and summation, it enhances data integrity in financial and technological systems. Though it does not prevent fraud, its efficiency in identifying common mistakes makes it a crucial tool in electronic payment processing. Additionally, the algorithm's mathematical foundation relates to level sets in differential geometry, offering a deeper perspective on credit card number validation. Its continued relevance and public accessibility underscore its enduring impact across industries that require accurate and reliable numeric verification.

Keywords: Credit cards; Identification number; Luhn algorithm; Level set, Differential geometry.

1. Introduction

The validation of identification numbers is a critical aspect of maintaining data integrity in various industries, including finance, telecommunications and government services. One widely adopted solution for ensuring the accuracy of these numbers is the Luhn algorithm, a simple yet effective checksum formula. Developed in the late 1960s, the Luhn algorithm has become a cornerstone of error detection in numerous applications, from credit card validation to IMEI number verification. The Luhn algorithm's widespread adoption can be attributed to its ease of implementation, effectiveness in detecting accidental errors, and public domain status. By leveraging a calculated check digit, the algorithm provides a robust mechanism for verifying the validity of identification numbers. Its ability to detect single-digit errors and most adjacent transpositions has significantly enhanced the reliability of systems that rely on accurate data entry. To gain a deeper understanding of the Luhn algorithm and its uses, consulting relevant literature is suggested [1–3]. Level sets are a fundamental concept in differential geometry, providing a powerful tool for analysing and understanding geometric structures. In essence, a level set is a set of points that satisfy a particular condition or equation, often represented

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as a hypersurface in a higher-dimensional space. In [4–8] we get deep insight of level sets. The mathematical structure of level sets can be related to the Luhn algorithm's checksum calculation. By exploring this connection, we can gain insights into the algorithm's effectiveness and robustness in validating identification numbers. This paper aims to explore the relation between Luhn algorithm and level sets.

2. The Luhn Algorithm and the Level Set

The concepts of Luhn algorithm and level sets have been widely studied and documented. In this section we explain about both these concepts.

Definition 2.1 ([3]). *The Luhn algorithm*

Step 1: *Double Every Second Digit: Starting from the rightmost digit of the number, double the value of every second digit.*

Step 2: *Adjust Two-Digit Results: If doubling produces a two-digit number (greater than 9), reduce it to a single digit. This can be done by subtracting 9 from the product or by summing the digits of the result.*

Step 3: *Sum the Transformed Digits: Add all the resulting single-digit values from step 2.*

Step 4: *Add Remaining Digits: Add all the digits that were not doubled (those in odd positions from right to left).*

Step 5: *Combine the Totals: Sum the results of steps 3 and 4.*

Step 6: *Check Divisibility by 10: If the final sum is divisible by 10, the number is syntactically valid according to the Luhn algorithm. Otherwise, it is invalid.*

This systematic process is both efficient and easy to implement, making it an invaluable tool for validating identification numbers such as credit cards and other numeric codes.

Example 2.2. *Consider a credit card number: 7992235683564519.*

Step 1: Doubling every second digit as shown in Table 1.

7	9	9	2	2	3	5	6	8	3	5	6	4	5	1	9
×2		×2		×2		×2		×2		×2		×2		×2	
14	9	18	2	4	3	10	6	16	3	10	6	8	5	2	9

Table 1: Doubling every second digit

Step 2: If the doubling of a number results in a two-digit number, then add the digits of the product to get a single digit number as shown in Table 2.

7	9	9	2	2	3	5	6	8	3	5	6	4	5	1	9
×2		×2		×2		×2		×2		×2		×2		×2	
14	9	18	2	4	3	10	6	16	3	10	6	8	5	2	9
5	9	9	2	4	3	1	6	7	3	1	6	8	5	2	9

Table 2: Adding the digits of the product

Step 3: Now sum all the digits.

Sum = 80. If the sum modulo 10 is equal to 0, then the number is valid according to Luhn Algorithm. Since the sum is 80 which is divisible by 10, the account number is possibly valid. Hence the validation of the credit card number. This method is primarily used to catch typographical mistakes rather than prevent fraud. By applying the algorithm, invalid card numbers can be flagged, whether they result from human error or rudimentary automated attempts. However, while the Luhn check can deter unsophisticated attackers who randomly input numbers, it does not provide strong protection against experienced fraudsters. Since the algorithm is publicly available, skilled individuals can generate valid-looking card numbers, making it an ineffective tool for preventing deliberate fraudulent activity.

Definition 2.3. *Given a function $f : U \to \mathbb{R}$ where $U \subset \mathbb{R}^{n+1}$, its level sets are the sets $f^{-1}(c)$ defined by*

$$f^{-1}(c) = \{(x_1, x_2, \ldots, x_{n+1}) \in U \mid f(x_1, x_2, \ldots, x_{n+1}) = c\},$$

for each real number c.

The value c is called the height *of the level set, and $f^{-1}(c)$ is called the level set at height c.*

The graph of the function $f : U \to \mathbb{R}$ is the subset of $\mathbb{R}^{n+2}$ defined by

$$\text{graph}(f) = \{(x_1, x_2, \ldots, x_{n+2}) \in \mathbb{R}^{n+2} \mid (x_1, x_2, \ldots, x_{n+1}) \in U \text{ and } x_{n+2} = f(x_1, x_2, \ldots, x_{n+1})\}.$$

3. Results and Discussion

The structure of credit card numbers, as well as other similar numerical sequences, exhibits a close relationship with level sets and the graph of a function in differential geometry. Credit card numbers are validated using the Luhn algorithm, which employs a checksum method to verify their authenticity. In this process, all digits except the last one—commonly known as the check digit define a function $f : U \to R$, where $U = (x_1, x_2, \ldots, x_{15}) \in \mathbb{Z}_{10}^{15}$. Check digit are selected randomly, while the check digit serves as a means of validation.

Since the method for computing check digits is well understood, it is possible to determine the check digit for any given set of numbers, thereby forming a valid credit card number. This concept aligns strongly with the mathematical framework of level sets, further reinforcing the connection between credit card number validation and differential geometry.

Consider the credit card number 7992235683564519, and suppose that we have only the first 15 digits only or the check digit number is not given.

That is, (799223568356451)$\in \mathbb{Z}_{10}^{15}$ and $x_i \in Z_{10}$ and f retains every even places from the left and doubles every odd places digits and calculates the sum of all digits and finally gives the remainder r modulus 10, This remainder r subtracted from 10 gives the check digit for the 15 tuple of numbers. By analysing the principles underlying the Luhn algorithm, we can uncover a generalized approach to how credit card numbers are assigned. Credit card companies implement a systematic method for generating these numbers, embedding it within their systems for efficient processing. However, from a mathematical perspective, particularly through the lens of differential geometry, we can derive a broader framework for their assignment.

4. Conclusions

The Luhn algorithm continues to play a vital role in validating identification numbers, ensuring accuracy and reliability in financial and technological applications. This study explores an alternative approach using level sets to verify credit card numbers and establishes a connection between the Luhn algorithm and level sets. The findings suggest that this novel approach can be extended to other identification numbers, such as IMEI and SIM card identification, highlighting the broader applicability of this relationship.

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