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Centurion Perfect Squares: A Class of Perfect Squares

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Abstract

This paper introduces a new class of perfect squares known as Centurion Perfect Squares (CPS). This is applicable to years which can be looked at as perfect squares with 4 digits as well the last two digits, ignoring the Century part. Conditions for a Perfect Square to be a Centurion Perfect Square have been derived by way of characterization. Illustrations also provided.

Keywords: Perfect Square; Centuries in a Millennium; Date Formats.

1. Introduction

It is well known that when an integer like x is multiplied by itself, it results in the perfect square x^2; for example, 25 and 81 are perfect squares, as $5 * 5 = 5^2 = 25$, $9 * 9 = 9^2 = 81$. It is interesting to note that this New Year 2025 is indeed a perfect square as $2025 = 45 * 45 = 45^2$. This particular year 2025 is much more interesting, as 25, the year without the Century part is also a Perfect square. Likewise, the years $1936 (= 44^2)$ and $2116 (= 46^2)$ are also perfect squares, in which 36 and 16, the respective years without the century part, are indeed perfect squares. This paper tries to look at the various years which have a similar property and defines such an year formally as Centurion Perfect Square (CPS). These CPSs are different from Compound Perfect Squared Squares (CPSS) [1, 2] which are a specific type of dissection where a square is tiled entirely by smaller squares of different sizes, with the added condition that at least one sub-rectangle within the dissection is also a squared rectangle. An extensive collection of information and resources on CPSS can be found at www.squaring.net. An attempt has been made in this paper to characterise these CPSs and hence lists out CPSs from the years 0 to 2500.

2. Centurion Perfect Squares as a class of Perfect Squares

It is common to denote dates in two different ways, like DD/MM/YYYY with century or DD/MM/YY without century, typically in British format.

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Definition 2.1. *A perfect square y^2 is said to be a Centurion Perfect Square (CPS), if (y^2 mod 100) is also a perfect square. In other words, $y^2 = 100c + d^2$, where y, c, and d are positive integers and $d^2 < 100$.*

Example 2.2.

1. $44^2 = 1936 = 100 * 19 + 6^2, 19 + 1$ *leading to the* 20^{th} *Century.*
2. $45^2 = 2025 = 100 * 20 + 5^2, 20 + 1$ *leading to the* 21^{st} *Century.*
3. $46^2 = 2116 = 100 * 21 + 4^2, 21 + 1$ *leading to the* 22^{nd} *Century.*

3. Characterisation of Centurion Perfect Squares

Observation 1: In the very 1st Century after A. D., the years 0000, 0001, 0004, 0009, ..., 0064, and 0081 are indeed CPSs, where $c = 0$ and $d \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$; i.e., d is a single digit.

Observation 2: The starting of year Y of a Century would be a CPS, if Y is a Perfect Square and Y mod $100 = 0$; i.e., Y is divisible by 100. For example, 0000, 0100, 0400, 0900, 1600, 2500, ..., 8100, 10000 are indeed CPSs.

Theorem 3.1. $(10n + d)^2$, *where d is a single digit, is a CPS, if either one of the following conditions are satisfied.*

1. $5|n$
2. $5|d$
3. $n \ mod \ 5 = 4.$

Proof.

1. $5|n$ and this means $n = 5k$, where k is any integer. Consider

$$\begin{aligned}(10n + d)^2 &= (10 * 5k + d)^2 = (50k + d)^2 \\ &= 2500k^2 + 100kd + d^2 = 100 * (25k^2 + kd) + d^2\end{aligned}$$

   i.e., $(10n + d)^2$ is of the form $100c + d^2$, where $c = (25k^2 + kd)$ and $d^2 < 100$, and hence, $(10n + d)^2$ is a CPS.

2. $5|d$ and this means either $d = 5$ or $d = 0$

   Case a): $d = 5$. Then, $(10n + d)^2 = (10n + 5)^2 = 100n^2 + 100n + 5^2 = 100 * (n^2 + n) + 5^2$ i.e., $(10n + d)^2$ is of the form $100c + d^2$, where $c = (n^2 + n)$ and $d^2 < 100$, and hence $(10n + d)^2$ is a CPS.

   Case b): $d = 0$. Then, $(10n + d)^2 = (10n + 0)^2 = 100n^2 + 0 + 0^2 = 100 * (n^2) + 0^2$ i.e., $(10n + d)^2$ is of the form $100c + d^2$, where $c = (n^2)$ and $d^2 < 100$, and hence $(10n + d)^2$ is a CPS.

3. $n \mod 5 = 4$ and this means $n = 5m + 4 = 5m + 5 - 1 = 5(m + 1) - 1 = 5k - 1$. In this case d is non-zero. Then, $(10n + d)^2 = (10 * (5k - 1) + d)^2 = (50k - 10 + d)^2 = (50k - (10 - d))^2 = (50k - d')^2$, where d' is the 10's Complement of d and d' is a non-zero single digit as d is a non-zero single digit.

$$= 2500k^2 - 100kd' + d'^2 = 100 * (25k^2 - kd') + d^2$$

i.e., $(10n + d)^2$ is of the form $100c + d^2$, where $c = (25k^2 - kd')$ and $d^2 < 100$, and hence, $(10n + d)^2$ is a CPS.

□

Example 3.2. *Let there be violation of the conditions in Theorem 3.1, where $n = 6$ and $d = 1$. The corresponding Perfect Square is $(10 * 6 + 1)^2 = 61^2 = 3721$ and this is not a CPS as 21 is not a Perfect Square.*

4. List of Centurion Perfect Squares

Century	Year $Y = Y_3Y_2Y_1Y_0$ as a CPS	Year Y $(Y_3Y_2Y_1Y_0)$: $(Y_2Y_1Y_0)$: (Y_1Y_0) as Perfect Squares	Century	Year $Y = Y_3Y_2Y_1Y_0$ as a CPS	Year Y $(Y_3Y_2Y_1Y_0)$: $(Y_2Y_1Y_0)$: (Y_1Y_0) as Perfect Squares
1	0000, 0001, 0004, 0009, 0016, 0025, 0036, 0049, 0064, 0081	$0^2, 1^2, 2^2, 3^3, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2$	14, 15, 16	Nil	-
2	0100	$10^2 : 10^2 : 0^2$	17	1600 1681	$40^2 : - : 0^2$ $41^2 : - : 9^2$
3	0225	$15^2 : 15^2 : 5^2$	18	1764	$42^2 : - : 8^2$
4	Nil	-	19	1849	$43^2 : - : 7^2$
5	400	$20^2 : 20^2 : 0^2$	20	1936	$44^2 : - : 6^2$
6	Nil	-	21	2025	$45^2 : 45^2 : 5^2$
7	0625	$25^2 : 25^2 : 5^2$	22	2116	$46^2 : - : 4^2$
8, 9	Nil	-	23	2209	$47^2 : - : 3^2$
10	0900	$30^2 : 30^2 : 0^2$	24	2304	$48^2 : - : 2^2$
11,12	Nil	-	25	2401	$49^2 : - : 1^2$
13	1225	$35^2 : 15^2 : 5^2$	26	2500	$50^2 : - : 0^2$

Table 1: Lists various CPSs in different centuries up to 26^{th} Century

5. Conclusion

This paper has introduced and defined a new class of Perfect Squares referred to as Centurion Perfect Squares (CPS). One such example of a CPS is the year 2025 which can be thought of as 45^2 considering the century part of it and as $5^2 = 25$ ignoring the Century part. This means that a date like 25/04/2045 in DD/MM/YYYY format has all the three components as Perfect Squares; i.e., as $5^2/2^2/45^2$. So is the case in DD/MM/YY as well; as $5^2/2^2/5^2$. The case of CPS has been analysed and characterised so as to see a Perfect Square as a Centurion Perfect Square. From Table 1, it is seen that the year 1225, corresponding to the 13^{th} Century, looks very interesting and this is a special case of CPS, possibly to be referred as Millennium Perfect Square (MPS). This means that a date like 25/04/1225 in DD/MM/YYYY format has all the three components as Perfect Squares; i.e., as $5^2/2^2/35^2$. So is the case in DD/MM/YYY format as $5^2/2^2/15^2$ and in DD/MM/YY format as $5^2/2^2/5^2$. This needs to be explored further. It would also be interesting to study further on how some of these Centurion Perfect Squares (CPSs) happen to be Compound Perfect Square (CPSS).

References

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