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Mathematical Study of Flow Over a Porous Medium Using Darcy-Brinkman Model

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Abstract: This work examines the Darcy–Brinkman flow over a stretching sheet, considering the influences of frictional heating and porous dissipation. The governing equations are obtained and streamlined via boundary layer approximations, subsequently converted into a series of self-similar equations via appropriate similarity transformations. The fourth-order Runge-Kutta method and MATLAB's built-in solver bvp4c are used to numerically solve these nonlinear equations under both velocity and thermal slip conditions. The results from certain limiting circumstances were checked against what is already known in the literature, and they matched up very well. The impact of several parameters on essential flow properties is examined, with results displayed in tables and graphical representations.

Keywords: Darcy-Brinkman porous medium, viscous dissipation, slip boundary conditions, porous dissipation, permeable stretching sheet.

1. Introduction

A porous medium is a solid matrix containing holes or spaces that are related to each other and allow fluid to pass through. Porosity is the volumetric proportion of the voids in the medium. It controls how well the medium can hold and move fluids. There are a lot of porous media in both nature and man-made things. Natural examples are sandstone, limestone, dolomite, pumice, and biological systems including the human lung, endothelium layers, and skin. Artificial examples are foamed polymers, catalyst pellets, sponges, fabrics, and filters [1]. These structures have special ways that fluids flow and heat moves through them, which makes them important in many fields, including geotechnical engineering, petroleum reservoir models, filtration systems, biomedical engineering, and energy systems [2]. One of the foundational models for flow in porous media is Darcy's law, proposed in the mid-19th century. It is based on the observation that fluid velocity is linearly proportional to the pressure gradient and gravitational force, applicable primarily at low Reynolds numbers and in tightly packed, low-permeability media. While Darcy's law has been pivotal in understanding subsurface flows and filtration processes, it neglects viscous shear within the fluid, limiting its applicability in sparsely packed or high-porosity media where such effects cannot be ignored [3]. In such cases, a more generalized model known as the Darcy-Brinkman model is utilized. It incorporates both Darcy's resistive term and a viscous shear term analogous to the Navier-Stokes formulation, allowing it to better simulate complex internal flows in porous domains [4-7].

Simultaneously, considerable attention has been given to flows over stretching sheets, which find extensive use in industrial processes involving extrusion, plastic sheet formation, glass blowing, and polymer manufacturing. In such processes, a flat

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elastic sheet is stretched in its own plane, creating a boundary layer in the adjacent fluid. The classical formulation for this kind of flow was introduced by Sakiadis for a moving flat plate and later extended by Crane for linearly stretching sheets, where self-similar solutions for velocity profiles were obtained [8, 9]. These studies laid the foundation for numerous investigations into heat and mass transfer in stretching flows, incorporating additional physical effects such as magnetic fields, thermal radiation, chemical reactions, and porous media.

Several researchers have expanded this framework by considering flows in porous environments, particularly within the scope of Darcy's law [10-14]. However, these models assume negligible viscous shear, which restricts their scope when applied to high-permeability or high-velocity scenarios. Consequently, Waqar and Pop [15] and Khan et al. [16] employed the more inclusive Darcy-Brinkman model to investigate flow over stretching surfaces. While these studies marked significant progress, they overlooked essential phenomena such as viscous dissipation and porous dissipation, which are critical to accurately modeling the energy balance in such systems.

When a fluid is stretched along a surface, it experiences an increase in kinetic energy, which is partly converted into thermal energy due to internal friction-this phenomenon is referred to as viscous dissipation. In porous media, additional heat generation occurs due to resistance imposed by the porous matrix, called porous dissipation. These effects significantly alter the temperature distribution and heat transfer rates in the system and are especially important in materials with low thermal conductivity or high porosity [17]. Neglecting these effects can lead to inaccurate modeling of thermal gradients, energy transport, and cooling behavior in real-world processes.

Moreover, conventional flow models often assume no-slip boundary conditions, meaning the fluid velocity at a solid boundary equals the velocity of the boundary itself. While this assumption holds at macroscopic scales, it fails in micro- and nano-scale flows, where the fluid exhibits velocity slip and temperature jump at the boundary. These slip conditions arise due to the rarefaction of gas molecules near walls, surface roughness, or fluid-surface interaction forces and are particularly relevant in microfluidics, nanofluidics, and MEMS (Micro-Electro-Mechanical Systems) [18].

Several studies have addressed slip effects in microchannel flow. Zhang et al. [19] analyzed heat transfer in microchannels with slip conditions under constant heat flux, while Hooman and Ejlali [20] investigated combined velocity and temperature jump conditions in gas–liquid slip flows between parallel plates. Hussanan et al. [21] studied the Newtonian heating problem with velocity slip over a vertical plate, revealing the significance of slip parameters on boundary layer development. Advanced slip models such as the second-order slip condition were considered by Liu and Guo [22] in their work on fractional Maxwell fluids, while Jing et al. [23] studied hydraulic resistance and heat transfer in elliptical microchannels. Andersson [24] provided an analytical solution for slip flow over stretching surfaces, laying groundwork for further studies by Turkyilmazoglu [25] and Yazdi et al. [26] on MHD and convective flows with slip. Additionally, Hsiao [27] examined stagnation-point nanofluid flows with slip effects under magnetic influence, revealing notable impacts on temperature and velocity profiles.

Despite these advancements, the combined effect of Darcy-Brinkman flow, viscous and porous dissipation, and slip boundary conditions has received minimal attention in the existing literature. Most existing works either neglect the dissipation mechanisms or oversimplify the slip boundary behavior. The simultaneous inclusion of all these features is essential for accurate thermal-fluid analysis in systems like porous heat exchangers, microfluidic cooling devices, and polymer processing setups.

In this context, the primary objective of the present study is to develop a comprehensive model for Darcy–Brinkman flow over a permeable stretching sheet, accounting for frictional heating (viscous dissipation), porous dissipation, and velocity and thermal slip conditions. The governing equations for momentum and energy are formulated using boundary layer theory and transformed into a set of self-similar ordinary differential equations via similarity transformations. The presence of dissipation and slip terms makes the derivation of similarity solutions particularly complex and rare in the literature.

The resulting system of nonlinear ordinary differential equations is solved numerically using the shooting method coupled with Runge-Kutta integration and validated against the results obtained using MATLAB's bvp4c routine [28]. Special-case comparisons are performed to benchmark the solutions with existing results available in the literature, ensuring the credibility and accuracy of the proposed model.

Furthermore, a parametric analysis is carried out to assess the influence of various physical parameters-such as Darcy number, Brinkman parameter, porous dissipation coefficient, velocity slip, and thermal slip coefficients important flow characteristics like dimensionless velocity, temperature profile, skin friction coefficient, and local Nusselt number. The results are graphically and tabularly represented, offering new insights into the thermofluidic behavior in porous systems with realistic boundary conditions.

This study contributes to the literature by presenting a more physically inclusive and mathematically consistent framework for modeling heat and momentum transfer in porous stretching flows, which can be extended to practical engineering systems like porous insulation materials, smart cooling surfaces, and industrial film-drawing devices.

2. Mathematical Formulation

We consider a steady, two-dimensional laminar flow of an incompressible fluid over a permeable stretching surface embedded in a Darcy-Brinkman porous medium. The surface is being stretched with a velocity that varies linearly with the horizontal coordinate, given by $U_s(x) = \alpha x$, where $\alpha > 0$ is the stretching rate constant. The flow takes place in a Cartesian coordinate system, where the x-axis is aligned along the sheet and the y-axis is perpendicular to it. The surface temperature is assumed to vary quadratically along the sheet as $T_s(x) = T_\infty + cx^2$, with $T_s > T_\infty$, where T_∞ is the ambient temperature and c is a positive constant. The sheet allows suction or injection, and velocity slip as well as thermal slip conditions are incorporated at the boundary. Under the boundary layer approximations and incorporating viscous and porous dissipation, the governing equations for mass, momentum, and energy conservation become:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum equation (Darcy-Brinkman model):

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\frac{\varepsilon^2\mu_e}{\rho}\right)\frac{\partial^2 u}{\partial y^2} - \left(\frac{\mu\varepsilon^2}{\rho K*}\right)u$$

Energy equation (including dissipation terms):

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\frac{\kappa}{\rho C_p}\right)\frac{\partial^2 T}{\partial y^2} + \left(\frac{\varepsilon^2}{\rho C_p}\right)\left[\mu_e\left(\frac{\partial u}{\partial y^2}\right) + \left(\frac{\mu u^2}{K*}\right)\right]$$

Boundary Conditions:

$$u = \alpha x + \beta_1\left(\frac{\partial u}{\partial y}\right), v = -V_0, T = T_s + \delta_1\left(\frac{\partial T}{\partial y}\right) \quad at \quad y = 0; u \to 0, T \to T_\infty \quad as \quad y \to \infty$$

To simplify the system, similarity transformations are introduced:

$$\xi = \sqrt{\left(\frac{\alpha}{\nu}\right)}y, u = \alpha x g\prime(\xi), v = -\sqrt{(\alpha\nu)}g(\xi), \theta(\xi) = \frac{(T - T_\infty)}{(T_s - T_\infty)}$$

This reduces the PDEs to the following nonlinear ODEs:

Dimensionless momentum equation:

$$\gamma g''' - g'^2 + gg'' - P_m g' = 0$$

Dimensionless energy equation:

$$\left(\frac{1}{Pr}\right)\theta'' + g\theta' - 2g'\theta + E_c(\gamma g''^2 + P_m g'^2) = 0$$

Boundary conditions:

$$g(0) = S, g'(0) = 1 + \beta g''(0), g'(\infty) = 0; \theta(0) = 1 + \delta\theta'(0), \theta(\infty) = 0$$

Skin friction and Nusselt number in dimensionless forms:

$$C_f x Re_x^{\frac{1}{2}} = g''(0), Nu_x Re_x^{\frac{1}{2}} = -\theta'(0)$$

3. Solution Methodology

To analyze the nonlinear differential system obtained from the similarity transformations, a numerical approach is employed. The transformed governing equations for momentum and energy subject to the boundary conditions are nonlinear and cannot be solved analytically due to the complexity introduced by viscous and porous dissipation, as well as slip conditions. Therefore, a numerical shooting technique integrated with the Runge-Kutta fourth-order method is adopted to obtain accurate and stable solutions.

In the shooting method, the boundary value problem is converted into an initial value problem. Since boundary conditions at infinity are involved, suitable initial guesses for $g''(0)$ and $\theta'(0)$ are made and iteratively refined using the Newton-Raphson method until convergence to the boundary conditions at $\xi \to \infty$ is achieved. A finite domain is selected for numerical integration with $\xi_{max} = 15$, which is sufficient to capture the asymptotic behavior of the solution. A step size of $\Delta\xi = 0.001$ ensures high accuracy and smooth profiles. To ensure validity, results from the shooting method are compared with MATLAB's bvp4c routine, which uses a collocation method for solving boundary value problems. The comparison confirms excellent agreement, validating the approach. In special cases, the results are benchmarked against known solutions from the literature, further verifying the correctness and robustness of the methodology. This combination of shooting and bvp4c provides a reliable and flexible tool for studying boundary layer flows in porous media with dissipation and slip effects.

4. Existing System

The investigation of fluid flow over stretching surfaces embedded in porous media has traditionally been modeled under Darcy's law, which assumes a linear momentum-pressure relationship and negligible viscous effects. The standard Darcy model is valid only at low Reynolds numbers and fails to capture the viscous shear that arises in media with high porosity or under rapid stretching conditions. In the extended Darcy–Brinkman model, an additional viscous term $\mu_e \nabla^2 u$ is introduced into the momentum equation to account for shear effects:

$$u \cdot \nabla u = -\left(\frac{1}{\rho}\right)\nabla p + \left(\frac{\mu_e}{\rho}\right)\nabla^2 u - \left(\frac{\mu}{\rho K^*}\right)u$$

Although this model is more comprehensive, many previous studies (e.g., Waqar & Pop [21], Khan et al. [22]) neglected viscous dissipation and porous heating in the energy equation. These omissions result in underestimation of internal heat generation within the porous matrix, particularly when high shear rates or permeability variations exist. Moreover, conventional boundary conditions assume no-slip at the wall: $u = U_s(x), T = T_s(x)$.

However, these assumptions are inaccurate at micro/nanoscale levels, where velocity slip and temperature jump conditions better reflect physical reality, especially in gas flows and thin films. While researchers like Zhang et al. [23], and Yazdi et al. [30] have addressed slip flows, they often exclude porous dissipation or treat viscous effects simplistically. Therefore, the current limitations in literature are:

– Incomplete energy modeling (no porous dissipation),

– Simplified momentum models (Darcy only),

– Unrealistic boundary conditions (no-slip),

– Lack of combined parametric analysis under slip and heating conditions.

5. Proposed System

This research provides an enhanced model to explore Darcy–Brinkman flow over a permeable stretching sheet, combining the effects of viscous dissipation, porous dissipation, and slip boundary conditions to overcome these restrictions. The structured computational algorithm below is used to formulate, simplify, and solve the system.

5.1. Mathematical Modeling and Algorithm

Step 1: Define the flow setup

– Stretching sheet velocity: $Us(x) = \alpha x$

– Surface temperature: $T_s(x) = T_\infty + cx^2$

– Sheet allows suction/injection: $v(x, 0) = -V_0$

Step 2: Governing Equations under Boundary Layer Approximation

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum (Darcy–Brinkman):

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\frac{\varepsilon^2 \mu_e}{\rho}\right)\frac{\partial^2 u}{\partial y^2} - \left(\frac{\mu \varepsilon^2}{\rho K^*}\right)u$$

Energy (with Dissipation):

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\frac{\kappa}{\rho C_p}\right)\frac{\partial^T}{\partial y^2} + \left(\frac{\varepsilon^2}{\rho C_p}\right)\left[\mu_e\left(\frac{\partial u}{\partial y^2}\right) + \left(\frac{\mu u^2}{K^*}\right)\right]$$

Step 3: Apply Slip Boundary Conditions

At $y = 0$:

$$u = \alpha x + \beta_1 \frac{\partial u}{\partial y}, T = T_s + \delta_1 \frac{\partial T}{\partial y}, v = -V_0$$

As $y \to \infty$: $u \to 0, T \to T_\infty$

Step 4: Similarity Transformations

$$\xi = \sqrt{\left(\frac{\alpha}{\nu}\right)}y, u = \alpha x g'(\xi), v = -\sqrt{(\alpha\nu)}g(\xi), \theta(\xi) = \frac{(T - T_\infty)}{(T_s - T_\infty)}$$

This leads to:

Momentum ODE: $\gamma g''' - g'^2 + gg'' - Pmg' = 0$

Energy ODE: $\left(\frac{1}{Pr}\right)\theta'' + g\theta' - 2g'\theta + Ec(\gamma g''^2 + Pmg'^2) = 0$

Step 5: Non-dimensional Boundary Conditions

$$g(0) = S, g'(0) = 1 + \beta g''(0), g'(\infty) = 0; \theta(0) = 1 + \delta\theta'(0), \theta(\infty) = 0$$

Step 6: Numerical Algorithm (Shooting Method + Runge-Kutta 4th Order)

1. Convert higher-order ODEs into systems of first-order equations.
2. Assume initial values for unknowns $g''(0)$, $\theta'(0)$.
3. Integrate from $\xi = 0$ to $\xi_{max} = 15.4$.
4. Use Newton–Raphson iterations to adjust guesses.
5. Check convergence using conditions at infinity.
6. Cross-verify with MATLAB's bvp4c solver.

5.2. Physical Significance and Advantages

This proposed system:

– Captures realistic thermal effects via porous and viscous dissipation.

– Models micro/nano-scale behavior using velocity and thermal slip.

– Provides a unified framework applicable to polymer extrusion, thin-film cooling, biofluidics, and microchannel flows.

– Ensures high accuracy and convergence with dual-validation numerical strategies.

6. Scientific Analysis Results

We looked at how important physical factors affected the dimensionless velocity profile, temperature distribution, and skin friction coefficient.

Table 1 shows how the velocity slip parameter beta changes for a solid medium with a surface that doesn't let water through. It is noted that an elevation in the velocity slip parameter beta results in a reduction of the skin friction coefficient. In the specific scenario of no slip Equation (6) has a precise analytical solution as defined in [22]:

Table 1 shows how this analytical solution compares to the numerical findings from the shooting approach for different values of beta and gamma.

Table 1. Comparison between analytical solution [22] and shooting method for different values of β when $\gamma = 1$ and $P_m = 0.0, S = 0.0$.

Velocity Slip Parameter β	$g'(0)$		$-g''(0)$	
	Andersson [22]	Present	Andersson [22]	Present
0.0	1.0000	1.0000	1.0000	1.0000
0.1	0.9128	0.91278	0.8721	0.87215
0.2	0.8447	0.84471	0.7764	0.77645
0.5	0.7044	0.70436	0.5912	0.59127
1.0	0.5698	0.56974	0.4302	0.43025
2.0	0.4320	0.43183	0.2840	0.28408
5.0	0.2758	0.27530	0.1448	0.14493
10.0	0.1876	0.18670	0.0812	0.08132
20.0	0.1242	0.12285	0.0438	0.04385
50.0	0.0702	0.06801	0.0186	0.01863
100.0	0.0450	0.04225	0.0095	0.00957

Table 2 shows that the numerical results obtained from the shooting method match closely with the exact analytical solution. Furthermore, the skin friction coefficient is found to increase with the Brinkman viscosity parameter gamma, while it decreases with increasing values of the suction parameter S and the porosity parameter Pm

Table 2. Skin friction coefficient $S_{fx}\mathrm{Re}_x^{1/2} = -g''(0)$ for no slip case $\beta = 0$. Comparison between exact and numerical solution.

Physical Parameters			$S_{fx}\mathrm{Re}_x^{1/2} = -g''(0)$	
γ	S	P_m	Exact (See Equation (11))	Numerical (Shooting Method)
1.0	1.0	0.5	1.82287	1.82287
2.0	1.0	0.5	1.15138	1.15140
3.0	1.0	0.5	0.89314	0.89324
0.5	**0.0**	0.3	1.61245	1.61245
0.5	**1.0**	0.3	2.89736	2.89736
0.5	**2.0**	0.3	4.56904	4.56905
2.0	0.5	**0.0**	0.84307	0.84336
2.0	0.5	**0.4**	0.97094	0.97098
2.0	0.5	**0.8**	1.08188	1.08188

Table 3 demonstrates that the skin friction coefficient is higher in the case of velocity slip beta = 1.0 as compared to the no-slip case, highlighting the influence of slip conditions on surface shear stress.

Table 3. Skin friction coefficient $S_{fx}\mathrm{Re}_x^{1/2} = -g''(0)$ for slip case $\beta = 1.0$. Comparison between Shooting method and MATLAB bvp4c.

Physical Parameters			$S_{fx}\mathrm{Re}_x^{1/2} = -g''(0)$	
γ	S	P_m	Shooting Method	bvp4c
1.0	1.0	0.5	0.610511	0.610497
2.0	1.0	0.5	0.500008	0.500006
3.0	1.0	0.5	0.439566	0.439507
0.5	**0.0**	0.3	0.550438	0.550437
0.5	**1.0**	0.3	0.712228	0.712227
0.5	**2.0**	0.3	0.808872	0.808872
2.0	0.5	**0.0**	0.406493	0.406209
2.0	0.5	**0.4**	0.452006	0.451987
2.0	0.5	**0.8**	0.485908	0.485905

Table 4 contains the local Nusselt number for various physical parameters. The results indicate that the Nusselt number increases with the Prandtl number Pr, enhancing thermal transport. In contrast, it decreases with higher values of the Eckert number Ec and the thermal slip parameter delta, indicating reduced heat transfer efficiency due to increased internal energy and lowered surface thermal gradients.

Table 4. Local Nusselt number $N_{Rx}\mathrm{Re}_x^{-1/2} = -\theta'(0)$ when $\beta = 1.0$ and $S = 0.5$ Comparison between Shooting method and MATLAB bvp4c.

Physical Parameters					$N_{Rx}\mathrm{Re}_x^{-1/2} = -\theta'(0)$	
Pr	Ec	δ	P_m	γ	Shooting Method	bvp4c
0.7	0.5	1.0	0.4	2.0	0.456141	0.456203
1.2	0.5	1.0	0.4	2.0	0.538161	0.538197
6.8	0.5	1.0	0.4	2.0	0.738928	0.738983
3.0	**0.0**	1.0	0.4	2.0	0.738078	0.738124
3.0	**0.6**	1.0	0.4	2.0	0.642319	0.642382
3.0	**1.2**	1.0	0.4	2.0	0.546560	0.546591
3.0	1.0	**0.0**	0.4	2.0	2.208602	2.208638
3.0	1.0	**0.6**	0.4	2.0	0.820808	0.820821
3.0	1.0	**1.2**	0.4	2.0	0.504071	0.504105
3.0	1.0	1.0	**0.0**	2.0	0.640207	0.640288
3.0	1.0	1.0	**0.5**	2.0	0.566207	0.566224
3.0	1.0	1.0	**1.0**	2.0	0.517044	0.517133
3.0	1.0	1.0	0.4	**1.0**	0.619665	0.619690
3.0	1.0	1.0	0.4	**2.0**	0.578480	0.578501
3.0	1.0	1.0	0.4	**3.0**	0.546450	0.546487

7. Conclusion

The mathematical analysis of fluid flow through porous media with the Darcy-Brinkman model provides a comprehensive foundation for comprehending intricate transport processes in both natural and manufactured systems. The Darcy-Brinkman model effectively reconciles pure Darcy flow with traditional Navier–Stokes dynamics by including viscous factors, hence offering a more precise depiction of flow in the vicinity of the interface between porous and free-fluid domains.

The reviewed literature substantiates that the incorporation of the Brinkman term is essential for accurately representing boundary-layer dynamics and shear phenomena in porous media. Changes in physical qualities including porosity, Brinkman viscosity, slip conditions, and thermal properties have a big effect on flow velocity, temperature distributions, skin friction, and heat transfer rates. The model works really well for simulating mixed convection, how nanofluids act, and how energy moves across complicated areas like biological tissues, geothermal systems, and industrial heat exchangers. In general, the Darcy-Brinkman model improves the ability of porous medium flow analysis to make predictions and is still an important part of making more complex multiphysics simulations. Continued improvement and testing of this model using both numerical and experimental methods will make it useful in many more areas of science and engineering.

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