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Some Characterization of Fermatean Fuzzy Ideals

Amal Kumar Adak[1,*], Gauri Kant Kumar[2]

[1]*Department of Mathematics, Ganesh Dutt College, Begusarai, Bihar, India*
[2]*Department of Mathematics, Lalit Narayan Mithila University, Darbhanga, Bihar, India*

Abstract

Fermatean fuzzy sets are expanded to include intuitionistic and pythagorean fuzzy sets with extra advantage of avoiding underlying limitations. The primary objective of this study is to introduce Fermatean fuzzy ideals. The principles of level sets of Fermatean fuzzy are discussed and explore various algebraic properties associated with these sets.

Keywords: Fermatean fuzzy sets; Fermatean fuzzy ideals; Fermatean fuzzy level sets.

1. Introduction

In our daily life, uncertainty is unavoidable. This universe is not built an assumptions or precise measures. It is not possible to make forward decisions. We face a significant problem in dealing with errors in decision making. In 1965, Zadeh[27] proposed Fuzzy sets (FSs) as a way to deal with ambiguity in real world problems. FSs represent the degree of sense of belonging to the set under study, every component of the conceptual universe is renumbered from of the unit range $[0,1]$. We designate a number from the unit range $[0,1]$ to each element of the discursive multiverse to signify the degree of sense of esteem and self - actualization to the set under study. Fuzzy sets are a subclass of set theory that allows for states that are midway between completeness and nothingness. To express the extent to which an element belongs to a class, a membership function is used in a fuzzy set. The worth of membership ranges from 0 to 1. The grade 0 indicating that the element is not a member of the class, where by 1 indicating that it is, and additional considerations indicating the level of participation.

When it comes to decision making, assigning membership value is not always adequate. Fuzzy sets can only express vagueness, they don't have to ability to handle hesitation inherent in human thinking. In order to defin the hesitations more clearly, Intuitionistic fuzzy sets (IFSs) were developed by Atanassov [5], which were important generalization of Fss. This approach uses the degree of membership and

*Corresponding author (amaladak17@gmail.com)

non-membership to model vagueness and impression while sum of membership and non-membership less than or equal to 1.

However, if the sum of membership and non-membership is greater than 1, then IFS fail to overcome this situation. Pythagorean fuzzy sets (PFSs) were proposed to address this shortcoming of IFSs. Yager et al. [24,25] pioneered the PFSs to extend the IFSs which were represented by the degree of membership and non-membership,with the condition that the sum of square of membership and non-membership should be less than or equal to 1. When PFss are compared to IFSs, we observe that they provide flexibility and power to express the uncertainty since the space of PFSs membership degree is larger than that the space of IFSs.

Although, PFSs generalizes the IFSs, it cannot describe the following decision information. If we consider the degree of membership as 0.7 and degree of non-membership as 0.8, then it is clearly seen that $0.7 + 0.8 > 1$ and $0.7^2 + 0.8^2 > 1$ and in this situation does not presented by IFSs and PFSs. To demonstrate such information, Senapati et al. [23] developed Fermatean fuzzy sets (FFSs). FFSs have degree membership and non-membership satisfying the condition that sum of cube of membership and non-membership must be lie between 0 and 1. In the above situations $0.7^3 + 0.8^3 < 1$. The membership space of FFSs is larger than that of IFSS and PFSs (see figure-1).

Algebraic patterns are used in theoretical physics, information science, computer science, control engineering and along with many other domains. This provides sufficient impetus for scholars to review many real abstract algebra topics and discoveries in the context of a broader fuzzy setting. Biswas [7] defines fuzzy subgroups and anti-fuzzy subgroups. Jun et al. [10] gives some results on on Intuitionistic fuzzy ideals of near rings. Kim et al. [12,13] introduces the conception of Intuitionistic fuzzy ideals and interior ideals of semigroup. Kim and Lee [14] developed the implication of intuitionistic fuzzy bi-ideals of semigroups. Kuroki [16,17] invents important results on Fuzzy ideals, Fuzzy bi-ideals and semi-prime ideals in semigroups. Sardar et al. [20] introduced the notions of prime ideals, semi-prime ideals of Γ-semigroup with intuitionistic fuzzy information. Adak and Kumar [3] invented some important properties of Pythagorean fuzzy ideals of Γ-near rings. Balamurugan et al. [6] introduced idea of bipolar Fermatean uncertainty sub algebra's in terms of R-ideals and correlated among bipolar fermatean uncertainty soft ideal and bipolar Fermatean uncertainty soft R-ideals. Khan et al. [11] presented the notion of Fermatean fuzzy ideal theory and rough Fermatean fuzzy sets in semigroups and the idea of lower and upper approximation in Fermatean fuzzy sets. Adak et al. [4] introduced Fermatean fuzzy semi-prime ideals and Fermatean fuzzy prime ideals.

In this paper, we introduce and study the concept of Fermatean fuzzy ideals The rest of the paper organized as follows. In Section 2, the preliminaries and some definitions are given and present some algebraic structures of Fermatean fuzzy sets. In Section 3, we introduce the notion of Fermatean fuzzy ideal and discuss some important results on these ideals. Finally, the paper is concluded including the future research in Section 4.

2. Preliminaries

To assemble this work self sufficient, we briefly introduce a few definitions engaged in the remaining work.

Definition 2.1. *If $S = \{a, b....\}$ and $\Gamma = \{\alpha, \beta, \gamma...\}$ are additive abelian group, and for all a, b, c in S and all $\alpha, \beta \in \Gamma$,the following condition are satisfied*

(a) $a\alpha b$ *is an element of* S,

(b) $(a + b)\alpha c = a\alpha c + b\alpha c, a(\alpha + \beta)b = a\alpha b + a\beta b, a\alpha(b + c) = a\alpha b + a\alpha c,$

(c) $(a\alpha b)\beta c = a\alpha(b\beta c),$

then S is called a Γ- ring. Through this paper S denotes Γ- ring and 0_S denotes the zero element of S unless otherwise specified.

Definition 2.2. *A subset F of S is called a left(resp. right) ideal of S if F is an additive subgroup of S and $S\Gamma F = \{a\alpha b | a \in S, \alpha \in \Gamma, b \in F\}$ (resp. $F\Gamma SR$) is contained in F. If F is both of left and right ideal, then F is two sided ideal, or simply an ideal of S.*

Definition 2.3. *By a fuzzy set u in a non-empty set X we means a function $u : X \to [0, 1]$, and the complement of u, denoted by $\overline{v}$, is the fuzzy set in X given $\overline{u}(a) = 1 - u(a)$ for all $a \in X$.*

Definition 2.4. *A fuzzy set u in S is called a fuzzy ideal of S if and only if*

$FI1$ $u(a - b) \geq u(a) \wedge u(b),$

$FI2$ $u(a\alpha b) \geq u(a) \vee u(b),$

for all $a, b \in S$ and all $\alpha \in \Gamma$.

Definition 2.5. *Let X be a non-empty fixed set. An intutionistic fuzzy set (IFS for short) F is in an object having the form $F = \{(a, u_F(a), v_F(a)) : a \in R\}$, where the function $u_F : R \to [0, 1]$ and $v_F : R \to [0, 1]$ denote the degree of membership (namely $u_F(a)$) and the degree of non-membership (namely $v_F(a)$) for each element $a \in R$ to the set F, respectively, and $0 \leq u_F(a) + v_F(a) \leq 1$ for all $a \in R$. For the sake of simplicity, we shall use the symbol $F = (u_F, v_F)$ for the IFS $F = \{(a, u_F(a), v_F(a)) : a \in R\}$.*

Definition 2.6 ([1,2]). *Let $F_i : i \in F$ be an arbitrary family of IFSs in X. Then*

(1) $\cap F_i = \{(a, \wedge u_F(a), \vee v_F(a)) : a \in X\},$

(2) $\cup F_i = \{(a, \vee u_F(a), \wedge v_F(a)) : a \in X\}.$

In some circumferences, for whatever reason, $0 \leq u(x) + v(x) \leq 1$ this may not be the hold. We take some situations, where $u = 0.7$ and $v = 0.5$ such that $0.7 + 0.5 = 1.2 > 1$, but $0.7^2 + 0.5^2 < 1$. Again, if $u = 0.6$ and $v = 0.6$ where $0.6 + 0.6 = 1.2 > 1$, but $0.6^2 + 0.6^2 < 1$. To deal with this situations, Yager [24,25] proposed the perspective assumes of Pythagorean fuzzy set in 2013.

Definition 2.7. *A Pythagorean fuzzy set P in universe of discourse X is defined as $P = \{\langle x, u_P(x), v_P(x)\rangle | x \in X\}$, where $u_P(x) : X \to [0,1]$ denotes the membership value and $v_P(x) : X \to [0,1]$ represents the value to which the element $x \in X$ is not a member of the set P, with the condition that $0 \leq (u_P(x))^2 + (v_P(x))^2 \leq 1$, for all $x \in X$. The value of indeterminacy $h_P(x) = \sqrt{1 - (u_P(x))^2 - (v_P(x))^2}$.*

In practice, the condition $0 \leq u^2(x) + v^2(x) \leq 1$ may not be true for any reason. For example, if we consider $u = 0.9$ $v = 0.5$, where $0.9^2 + 0.5^2 = 1.06 > 1$, but $0.9^3 + 0.5^3 = 0.854 < 1$. Again, $0.8^2 + 0.7^2 = 1.13 > 1$, but $0.8^3 + 0.7^3 = 0.855 < 1$. To address this issue, Senapati et al. [22] proposed the notion of the Fermatean fuzzy set.

Definition 2.8. *A Fermatean fuzzy set A in a finite universe of discourse X is defined as $A = \{\langle x, u_A(x), v_A(x)\rangle | x \in X\}$, where $u_A(x) : X \to [0,1]$ denotes the membership value and $v_A(x) : X \to [0,1]$ represents the not membership value to which the element $x \in X$ is not a member of the set A, with the condition that $0 \leq (u_A(x))^3 + (v_A(x))^3 \leq 1$, for all $x \in X$. The value of indeterminacy $h_A(x) = \sqrt[3]{1 - (u_A(x))^3 - (v_A(x))^3}$.*

3. Main Results

We start by defining the notion of Fermatean fuzzy ideals.

Definition 3.1. *An FFS $F = (u_F, v_F)$ in S is called a Fermatean fuzzy ideal of S if*

IF1 $u_F(a - b) \geq u_F(a) \wedge u_F(b)$ *and* $v_F(a - b) \leq v_F(a) \vee v_F(b)$ *for all* $a, b \in S$,

IF2 $u_F(a\alpha b) \geq u_F(a) \vee u_F(b)$, *and* $v_F(a - b) \leq v_F(a) \wedge v_F(b)$ *for all* $a, b \in S$, *and for all* $\alpha \in \Gamma$.

Lemma 3.2. *If an FFS $F = (u_F, v_F)$ in S satisfies the condition $(IF1)$, then*

(1) $u_F(0) \geq u_F(a)$ *and* $v_F(0) \leq_F (a)$,

(2) $u_F(-a) = u_F(a)$ *and* $v_F(-a) = v_F(a)$, *for all* $a \in S$.

Proof.

(1) We have that for any $a \in S$, $u_F(0) = u_F(a - a) \geq u_F(a) \wedge u_F(a) = u_f(a)$ and $v_F(0) = v_F(a - a) \leq v_F(a) \vee v_F(a) = v_F(a)$

(2) By using (1) we get $u_F(-a) = u_F(0 - a) \geq u_F(0) \wedge u_F(a) = u_F(a)$ and $v_F(-a) = v_F(0 - a) \leq v_F(0) \vee v_F(a) = v_F(a)$ for all $a \in S$. Since a is arbitrary, we conclude that $u_F(-a) = u_F(a)$ and $v_F(a) = v_F(a)$ for all $a \in S$.

□

Proposition 3.3. *If a FFS $F = (u_F, v_F)$ in S satisfies the condition $(IF1)$, then*

(i) $u_F(a - b) = u_F(0)$ *implies* $u_F(a) = u_F(b)$

(ii) $v_F(a-b) = v_F(0)$ *implies* $v_F(a) = v_F(b)$*, for all* $a, b \in S$.

Proof.

(i) Let $a, b \in S$ be such that $u_F(a-b) = u_F(0)$. Then,

$$\begin{aligned} u_F(a) &= u_F(a-b+b) \\ &\geq u_F(a-b) \wedge u_F(b) \\ &= u_F(0) \wedge u_F(b) \\ &= u_F(b) \end{aligned}$$

Similarly, $u_F(b) \geq u_F(a)$ and so $u_F(a) = u_F(b)$.

(ii) If $v_F(a-b) = v_F(0)$ for all $a, b \in S$, then

$$\begin{aligned} v_F(a) &= v_F(a-b+b) \\ &\leq v_F(a-b) \vee v_F(b) \\ &= v_F(0) \vee v_F(b) \\ &= v_F(b) \end{aligned}$$

Similarly, $v_F(b) \leq v_F(a)$ and so $v_F(a) = v_F(b)$.

□

Theorem 3.4. *If* $P = (u_P, v_P)$ *and* $Q = (v_Q, v_Q)$ *are Fermatean fuzzy ideals of* S*, then so is* $P \cap Q$.

Proof. For any $a, b \in S$, we have that

$$\begin{aligned} (u_P \wedge u_Q)(a-b) &= u_P(a-b) \wedge u_Q(a-b) \\ &\geq (u_P(a) \wedge u_Q(a)) \wedge (u_P(b) \wedge u_Q(b)) \\ &= (u_P \wedge u_Q)(a) \wedge (u_P \wedge u_Q)(b) \\ (v_P \vee v_Q)(a-b) &= v_P(a-b) \vee v_Q(a-b) \\ &\leq (v_P(a) \vee v_Q(a)) \vee (v_P(b) \vee v_Q(b)) \\ &= (v_P \vee v_Q)(a) \vee (v_P \vee v_Q)(b) \end{aligned}$$

and if $a, b \in S$ and $\alpha \in \Gamma$, then we have that

$$\begin{aligned} (u_P \wedge u_Q)(a\alpha b) &= u_P(a\alpha b) \wedge u_Q(a\alpha b) \\ &\geq (u_P(a) \vee u_P(b)) \wedge (u_P(b) \vee u_Q(b)) \\ &= (u_P(a) \wedge u_Q(a)) \vee (u_P(b) \wedge u_Q(b)) \end{aligned}$$

$$
\begin{aligned}
&= (u_P \wedge u_Q)(a) \vee (u_P \wedge u_Q)(b) \\
(v_F \vee v_Q)(a\alpha b) &= v_P(a\alpha b) \vee v_Q(a\alpha b) \\
&\leq (v_P(a) \wedge v_P(b)) \vee (v_Q(b) \wedge v_Q(b)) \\
&= (v_P(a) \vee v_Q(a)) \wedge (v_P(b) \vee v_Q(b)) \\
&= (v_P \vee v_Q)(a) \wedge (v_P \vee v_Q)(b)
\end{aligned}
$$

Hence $P \cap Q$ is a Fermatean fuzzy ideal of S. □

Theorem 3.5. *If $\{F_i\}_{i\in F}$ is a family of Fermatean fuzzy ideals of S, then $\cap F_i$ is a Fermatean fuzzy ideals of S.*

Proof. Let $a, b \in S$ and $\alpha \in \Gamma$. Then

$$
\begin{aligned}
(\cap u_{F_i})(a-b) &= \wedge u_{F_i}(a-b) \geq \wedge(u_{F_i}(a) \wedge u_{F_i}(b)) \\
&= \wedge u_{F_i}(a) \wedge u_{F_i}(b)) \\
&= (\cap u_{F_i}(a)) \wedge (\cap u_{F_i}(b)) \\
(\cup v_{F_i})(a-b) &= \vee v_{F_i}(a-b) \leq \vee(v_{F_i}(a) \vee v_{F_i}(b)) \\
&= \vee v_{F_i}(a) \vee v_{F_i}(b)) \\
&= (\cup v_{F_i}(a)) \vee (\cup v_{F_i}(b)) \\
(\cap u_{F_i})(a\alpha b) &= \wedge u_{F_i}(a\alpha b) \geq \wedge(u_{F_i}(a) \vee u_{F_i}(b)) \\
&= (\cap u_{F_i}(a)) \vee (\cap u_{F_i}(b)) \\
(\cup v_{F_i})(a\alpha b) &= \vee v_{F_i}(a\alpha b) \leq \wedge(v_{F_i}(a) \wedge v_{F_i}(b)) \\
&= (\cup v_{F_i}(a)) \wedge (\cup v_{F_i}(b))
\end{aligned}
$$

Hence $\cap u_{F_i}$, is a Fermatean fuzzy ideal of S. □

Definition 3.6. *Let $F = (u_F, v_F)$ be a Fermatean fuzzy set of S, then we define $\square F$ and $\lozenge F$ as follows $\square F = (u_F, \overline{u_F}) and \lozenge F = (\overline{v_F}, v_F)$.*

Theorem 3.7. *If a FFS $F = (u_F, v_F)$ in S is a Fermatean fuzzy ideal of S, then so is $\square F$.*

Proof. It is sufficient to show that $\overline{u_F}$ satisfies the second condition of $(FF1)$ and the second condition of $(FF2)$. For any $a, b \in S$ and any $\alpha \in \Gamma$, we have

$$
\begin{aligned}
\overline{u_F}(a-b) &= 1 - u_F(a-b) \leq 1 - u_F(a) \wedge u_F(b) \\
&= (1 - u_F(a)) \vee (1 - u_F(b)) \\
&= \overline{u_F}(a) \vee \overline{u_F}(b), \\
\overline{u_F}(a\alpha b) &= 1 - u_F(a\alpha b) \leq 1 - u_F(b) \\
&= (1 - u_F(a)) \wedge (1 - u_F(b))
\end{aligned}
$$

$$= \overline{u_F}(a) \wedge \overline{u_F}(b),$$

Therefore $\Box F$ is a Fermatean fuzzy ideal of S. $\Box$

Definition 3.8. *Let $F = (u_F, v_F)$ be a FFS in S and let $\alpha \in [0,1]$. Then the sets $u^{\geq}_{F,\alpha} = \{a \in S : u_F(a) \geq \alpha\}$ and $v^{\leq}_{F,\alpha} = \{a \in S : v_F(a) \leq \alpha\}$ are called u-level α-cut and v-level α-cut of F, respectively.*

Theorem 3.9. *If a FFS $F = (u_F, v_F)$ in S is a Fermatean fuzzy ideal of S, then the u-level α-cut $u^{\geq}_{F,\alpha}$ and v-level α-cut $v^{\leq}_{F,\alpha}$ of F are ideals of M for every $\alpha \in Im(u_F) \cap Im(v_F) \subseteq [0,1]$.*

Proof. Let $\alpha \in Im(u_F) \cap Im(v_F) \subseteq [0,1]$ and let $a, b \in u^{\geq}_{F,\alpha}$. Then $u_F(a) \geq \alpha$ and $u_F(b) \geq \alpha$. It follows from the first condition of $(FF1)$ that $u_F(a-b) \geq u_F(a) \wedge u_F(b) \geq \alpha$. So that $a - b \in u^{\geq}_{F,\alpha}$ then $v_F(a) \leq \alpha$ and $v_F(b) \leq \alpha$ and so $v_F(a-b) \leq v_F(a) \vee v_F(b) \leq \alpha$. Hence we have $a, b \in u^{\leq}_{F,\alpha}$. Now, let $a \in S, \beta \in \Gamma$ and $b \in u^{\geq}_{F\alpha}$, Then

$$u_F(a\alpha b) \geq u_F(a) \vee u_F(b) \geq u_F(b) \geq \alpha$$

and so $a\alpha b \in u^{\geq}_{F,\alpha}(resp. b\alpha a \in u^{\geq}_{F,\alpha})$. If $b \in v^{\leq}_{F,\alpha}$, then

$$v_F(a\beta b) \leq v_F(a) \leq v_F(b) \leq v_F(b) \leq \alpha$$

and thus $a\beta b \in v^{\leq}_{F,\alpha}(a\beta a \in v_{F,\alpha})$. Therefore, $u^{\geq}_{F,\alpha}$ and $v^{\leq}_{F,\alpha}$ are ideals of S. $\Box$

Theorem 3.10. *Let $F = (u_F, v_F)$ be a FFS in S such that the non-empty sets $u^{\geq}_{F,\alpha}$ and $v^{\leq}_{F,\alpha}$ are ideals of S for all $\alpha \in [0,1]$. Then $F = (u_F, v_F)$ is a Fermatean fuzzy ideal of S.*

Proof. Let $\alpha \in [0,1]$ and suppose that $u^{\geq}_{F,\alpha}(\neq 0)$ and $v^{\leq}_{F,\alpha}(\neq 0)$ are ideals of S. We must show that $F = (u_F, v_F)$ satisfies the conditions $(IF1)$-$(IF2)$. If the first condition of $(IF1)$ is false, then there exist $a_0, b_0 \in S$.

$$u_F(a_0 - b_0) < u_{F(a_0) \wedge u_F(y_0)}$$

Taking

$$\alpha_0 = \frac{1}{2}(u_F(a_0 - b_0) + u_F(a_0) \wedge u_F(b_0))$$

we have

$$u_F(a_0 - b_0) < \alpha_0 < u_F(a_0) \wedge u_F(b_0)$$

it follow that $a_0, b_0 \in u^{\geq}_{F,\alpha}$ and $a_0 - b_0 \notin u^{\geq}_{F,\alpha}$, which is a contradiction. Assume that the second condition of $(IF1)$ does not hold. Then $v_F(a_0 - b_0) > v_F(a_0) \vee v_F(b_0)$, for some $a_0, b_0 \in S$. Let

$$\beta_0 = \frac{1}{2}(v_F(a_0 - b_0) + v_F(a_0) \vee v_F(b_0)).$$

Then $v_F(a_0 - b_0) > \beta_0 > v_F(a_0) \vee v_F(b_0)$ and so $a_0, b_0 \in v^{\leq}_{F,\alpha}$ but $a_0, b_0 \in v^{\leq}_{F,\alpha}$. This is a contradiction.

Now if the first condition of $(IF2)$ is not true then there exist $a_0, b_0 \in S$ and $\zeta \in \Gamma$, such that

$$u_F(a_0\zeta b_0) < v_0 < u_F(a_0) \wedge u_F(b_0).$$

Putting

$$v_0 = \frac{1}{2}(u_F(a_0\zeta b_0) + u_F(a_0) \wedge u_F(b_0)),$$

then $u_F(a_0\zeta b_0) < v_0 < u_F(a_0) \wedge u_F(b_0)$. It follows that $a_0, b_0 \in u^{\geq}_{F,\alpha}$ and $a_0\zeta b_0 \notin u^{\geq}_{F,\alpha}$, a contradiction. Finally suppose that the second condition of $(IF2)$ does not hold. Then $v_F(a_0\zeta b_0) > v_F(a_0) \vee v_F(b_0)$ for some $a_0, b_0 \in S$ and $\zeta \in \Gamma$. Selecting

$$\delta_0 = \frac{1}{2}(v_F(a_0\zeta b_0) + v_F(a_0) \vee v_F(b_0)),$$

we get $v_F(a_0\zeta b_0) > \delta_0 > v(a_0) \vee v(b_0)$ and so $a_0-, b_0 \in v^{\leq}_{F,\delta_0}$ but $a_0\zeta b_0 \in v^{\leq}_{F,\delta_0}$. This is impossible and we are done. □

Theorem 3.11. *Let H be an ideal of S and let $F = (u_F, v_F)$ be a FFS in S defined by*

$$u_F(a) = \begin{cases} \alpha_0, \text{ if } a \in H, \\ \alpha_1, \text{ otherwise} \end{cases}$$

$$v_F(a) = \begin{cases} \beta_0, \text{ if } a \in H, \\ \beta_1, \text{ otherwise} \end{cases}$$

for all S and $\alpha_i, \beta_i \in [0,1]$ such that $\alpha_0 > \alpha_1, \beta_0 < \beta_1$ and $\alpha_i + \beta_i \leq 1$ for $i = 0, 1$. Then $F = (u_F, v_F)$ is a Fermatean fuzzy ideal of S and $u^{\geq}_{F,\alpha_0} = H = v^{\leq}_{F,\beta_0}$.

Proof. Let $a, b \in S$. If any one of a and b does not belong to H, then $u_F(a-b) \geq \alpha_1 = u_F(a) \wedge u_F(b)$ and $v_F(a-b) \leq \beta_1 = v_F(a) \vee v_F(b)$. Also, let $a, b \in S$ and $\alpha \in \Gamma$. If $b \notin H$. Then $u_F(a\alpha b) \geq \alpha_1 = u_F(a) \wedge u_F(b)$ and $v_F(a\beta b) \leq \beta_1 = v_F(a) \vee v_F(b)$. Assume that $b \in H$. Since H is an ideal of S, it follows that $a\alpha b \in H$. Hence $u_F(a\alpha b) = \alpha_0 = u_F(a) \wedge u_F(b)$ and $v_F(a\beta b) = \beta_0 = v_F(a) \vee v_F(b)$. Therefore Obviously $F = (u_F, v_F)$ is a Fermatean fuzzy ideal of S. □

Theorem 3.12. *If a FFS $F = (u_F, v_F)$ is a Fermatean fuzzy ideal of S, if nd only if the fuzzy sets u_F and $\overline{v_F}$ are fuzzy ideals of S. the fuzzy sets u_F and $\overline{v_F}$ are fuzzy ideals of S.*

Proof. Let $F = (u_F, v_F)$ be a Fermatean fuzzy ideal of S. Then clearly u_F is a fuzzy ideal of S. Let $a, b \in S$ and $\alpha \in \Gamma$. Then

$$\begin{aligned} \overline{v_F}(a-b) &= 1 - v_F(a-b) \\ &\geq 1 - v_F(a) \vee v_F(b) \\ &= (1 - v_F(a)) \wedge (1 - v_F(b)) \end{aligned}$$

$$
\begin{aligned}
&= \overline{v_F}(a) \wedge \overline{v_F}(b) \\
\overline{v_F}(a\alpha b) &= 1 - v_F(a\alpha b) \\
&\geq 1 - v_F(b) \wedge v_F(b) \\
&= (1 - v_F(a)) \vee (1 - v_F(b) \\
&= \overline{v_F}(a) \vee \overline{v_F}(b)
\end{aligned}
$$

Hence $\overline{v_F}$ are fuzzy ideals of S. Conversely suppose that u_F and $\overline{v_F}$ are fuzzy ideals of S. Let $a, b \in S$ and $\alpha \in \Gamma$. Then

$$
\begin{aligned}
1 - v_F(a-b) &= \overline{v_F}(a-b) \geq \overline{v_F}(a) \wedge \overline{v_F}(b) \\
&= (1 - v_F(a)) \wedge (1 - v_F(b)) \\
&= 1 - v_F(a) \vee v_F(b) \\
1 - v_F(a\alpha b) &= \overline{v_F}(a\alpha b) \geq \overline{v_F}(a) \vee \overline{v_F}(b) \\
&= (1 - v_F(a)) \vee (1 - v_F(b)) \\
&= 1 - v_F(a) \wedge v_F(b),
\end{aligned}
$$

which imply that $v_F(a-b) \leq v_F(a) \vee v_F(b)$ and $v_F(a\alpha b) \leq v_F(a) \wedge v_F(b)$. This completes the proof. □

4. Conclusion

For responding with cognitive uncertainty, the Fermatean fuzzy set is an effective generalisation of such intuitionistic fuzzy set. The major goal of this study is to introduced Fermatean fuzzy ideals. The concepts level sets of Fermatean fuzzy are explained. Also, we introduce the notion of level sets fermatean fuzzy sets and several relations on these levels are discussed. We will look into the decision-making process more in the future using interval-valued Fermatean fuzzy sets uncertain data. An investigation of the interval-valued Fermatean fuzzy will be conducted out oredered semigroups, near-rings and interval-valued Fermatean prime and semi-prime ideals, as well as their algebraic features.

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