International Journal of
Mathematics And its Applications
ISSN: 2347-1557

Int. J. Math. And Appl., **13(3)**(2025), 197–203
Available Online: http://ijmaa.in

Divided Square Difference Cordial Labeling of Various Graphs

Vimal Patel[1,*], Suresh Sorathia[1], Amit Rokad[2]

[1] *Department of Mathematics, Surendranagar University, Ambedkarnagar, Surendranagar, Gujarat, India*

[2] *Department of Mathematics, M. G. Science Institute (Affiliated to Gujarat University), Ahmedabad, Gujarat, India*

Abstract

In this paper we prove that alternate triangular belt graph, twig graph, duplication of the top vertex Alternate triangular snake graph, duplication of top vertex Pentagon snake graph, jellyfish (n, n), Spider graph with n spokes $\mathcal{S}_{n,2}$ are divided square difference cordial graphs.

Keywords: Sum divisor cordial labeling; Twig graph; Pentagon Snake graph; Spider graph.
2020 Mathematics Subject Classification: 05C78.

1. Introduction

A finite undirected graph devoid of loops or numerous edges is referred to as a graph. We recommend Harary for conventional and fundamental graph theory notations and terminology, Graph theory [2]. A graph's labeling is a map connecting the graph's nodes to a collection of numbers, typically a collection of non-negative or positive integers. Vertex labeling is the term for labeling when the domain is a set of vertices. Edge labeling is used when the domain is the set of edges. Total labeling is the labeling process where labels are applied to both vertices and edges. Gallian provides a dynamic overview of various graph labels [1].

Theorem 1.1. *The spider graph with n spokes $\mathcal{S}_{n,2}$ is divided square difference cordial graph.*

Proof. Let $\mathcal{S}_{n,2}$ be the spider graph, where n is the number of vertices of the star graph, and the vertices linking the star graph to the pendent vertices are indicated by b_i, where $i = 1, 2, \ldots, m$. Assume that the central vertex of the star graph is $a_0 = 0$, $\mathcal{S}_{n,2}$ is a graph with $2n + 1$ and $2n$ edges. The vertex set $V = \{a_i, b_i, a_0, 1 \leq i \leq n\}$. The edge set $E = \{(a_i b_i) \cup (a_0 a_i)\,;\ 1 \leq i \leq n\}$. Now let us define the function, $f : V \to \{1, 2, 3, \ldots, p\}$. Let us label the vertices as follows.

$$f(a_0) = 2,$$

*Corresponding author (vbpatel.ce@ddu.ac.in)(Research Scholar)

$$f(a_1) = 1,$$
$$f(a_{i+1}) = 2i + 2; \quad 1 \le i \le n - 1,$$
$$f(b_i) = 2i + 1; \quad 1 \le i \le n$$

The following is the induced edge labeling,

$$f^*(a_0 a_1) = 1, \quad f^*(a_0 a_{i+1}) = 0; \quad 1 \le i \le n - 1.$$
$$f^*(a_1 b_1) = 0, \quad f^*(a_{i+1} b_{i+1}) = 1; \quad 1 \le i \le n - 1.$$

therefore, $|e_f(0) - e_f(1)| \le 1$. Hence, the spider graph $\mathcal{S}_{n,2}$ is divided square difference cordial graph. $\square$

Example 1.2. *Divided square difference cordial labelling of spider graph is shown in Figure 1.*

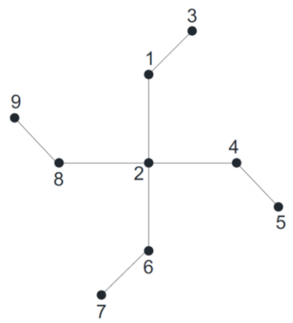

Figure 1:

Theorem 1.3. *The join spider graph with n spokes $JS_{n,2}$ is divided square difference cordial graph.*

Proof. Let $JS_{n,2}$ be the join spider graph where n is the number of vertices of the star graph, The vertices of the star graph are denoted by a_i, $i = 1, 2, \ldots, n$ and the vertices attaching the star graph to the pendent vertices are denoted by b_i, $i = 1, 2, \ldots, m$. Fix the central vertex of the star graph to $a_0 = 0$. By connecting the central vertex of two spider graphs. The vertex set $V = \{(a_0, a_i, b_i, a'_i, b'_i; 1 \le i \le n\}$. The edge set $E = \{(a_i b_i) \cup (a_0 a_i) \cup (a'_i b'_i) \cup (a'_0 a'_i)\,;\ 1 \le i \le n\}$. Now let us define the function, $f : V \to \{1, 2, 3, \ldots, p\}$. Let us label the vertices as follows:

$$f(a_0) = 1, \quad f(a_i) = 4i - 1; \quad 1 \le i \le n,$$
$$f(b_1) = 2, \quad f(b_{i+1}) = 4i + 4; \quad 1 \le i \le n - 1$$
$$f(a'_0) = 5, \quad f(a'_1) = 4, \quad f(a'_{i+1}) = 4i + 5; \quad 1 \le i \le n - 1.$$
$$f(b'_i) = 4i + 2; \quad 1 \le i \le n.$$

The following is the induced edge labeling,

$$f^*(a_0 a'_0) = 0, \quad f^*(a_0 a_i) = 0, \quad f^*(a_i b_i) = 1; \quad 1 \le i \le n.$$

$$f^*\left(a'_0a'_1\right) = 1, \quad f^*\left(a'_0a'_{i+1}\right) = 0; \qquad 1 \le i \le n-1$$
$$f^*\left(a'_1b'_1\right) = 0, \quad f^*\left(a'_{i+1}b'_{i+1}\right) = 1; \qquad 1 \le i \le n-1.$$

therefore, $|e_f(0) - e_f(1)| \le 1$. Hence, the join spider graph with n spokes $JS_{n,2}$ is divided square difference cordial graph. □

Example 1.4. *Divided square difference cordial labelling of join spider graph $JS_{n,2}$ is shown in Figure 2.*

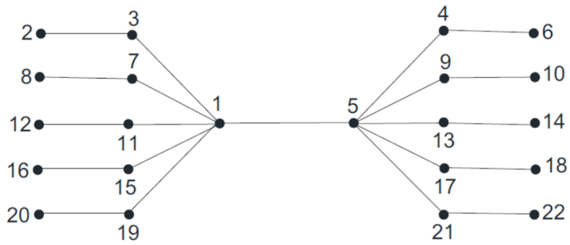

Figure 2:

Theorem 1.5. *Twig graph $T(n)$ is divided square difference cordial graph.*

Proof. Consider the graph $T(n)$ with the vertex set $V = \{a_i, b_j, c_j; 1 \le i \le n,\ 1 \le j \le n-1\}$ and the edge set $E = \{(a_ia_{i+1}),\ 1 \le i \le n-1\} \cup \{(a_ib_j),\ 2 \le i \le n,\ 1 \le j \le n-1\} \cup \{(a_ic_j),\ 2 \le i \le n,\ 1 \le j \le n-1\}$. Now let us define the function, $f : V \to \{1, 2, 3, \ldots, p\}$. Let us label the vertices as follows:

$$f(a_1) = 1, \quad f(a_{2i}) = 6i-3, \quad f(a_{2i+1}) = 6i-1; \quad 1 \le i \le n,$$
$$f(b_1) = 2, \quad f(b_{2j}) = 6j, \quad f(b_{2j+1}) = 6j-4; \quad 1 \le j \le \frac{n}{2}$$
$$f(c_{2j-1}) = 6j-2, \quad f(c_{2j}) = 6j+1; \quad 1 \le j \le \frac{n}{2}.$$

The following is the induced edge labeling,

$$f^*(a_ia_{i+1}) = 0, \quad f^*(a_{2i+1}c_{2i}) = 0; \quad i \equiv 1\ (mod\ 2).$$
$$f^*(b_ia_{i+1}) = 1, \quad f^*(a_{2i}c_{2i-1}) = 1; \quad i \equiv 0\ (mod\ 2).$$

therefore, $|e_f(0) - e_f(1)| \le 1$. Hence Twig graph $T(n)$ is divided square difference cordial graph. □

Example 1.6. *Divided square difference cordial labelling of Twig graph $T(n)$ is shown in Figure 3.*

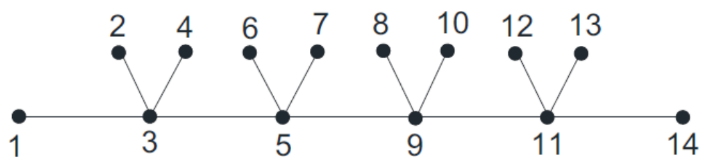

Figure 3:

Theorem 1.7. *Alternate Triangular belt graph $ATB(n)$ is divided square difference cordial graph.*

Proof. Consider the graph $G = ATB(n)$ with the vertex set $V = \{a_i, b_j; 1 \leq i \leq n, 1 \leq j \leq n\}$ and the edge set $E = \{(a_i a_{i+1}), 1 \leq i \leq n-1\} \cup \{(a_i b_i), 1 \leq i \leq n\} \cup \{(b_i b_{i+1}), 1 \leq i \leq n-1\} \cup \{(a_{i+1} b_{2i-1}), 1 \leq i \leq n\} \cup \{(a_{2i} b_{2i+1}), 1 \leq i \leq n\}$. Now let us define the function, $f : V \to \{1, 2, 3, \ldots, p\}$. Let us label the vertices as follows:

$$f(b_1) = 1, \quad f(b_{2j}) = 4j, \quad f(b_{2j+1}) = 4j + 2; \quad 1 \leq j \leq n.$$

$$f(a_i) = \begin{cases} 3; i \equiv 1(mod\ 2) \\ 2; i \equiv 0(mod\ 2) \end{cases}$$

$$f(a_{i+2}) = 2i + 3; \ 1 \leq i \leq n.$$

The following is the induced edge labeling.

$$f^*(a_i b_i) = 0, \qquad 1 \leq i \leq 2; \qquad f^*(a_i b_i) = 1, \ 3 \leq i \leq n.$$

$$f^*(a_i a_{i+1}) = 1, \qquad 1 \leq i \leq 2; \quad f^*(a_i a_{i+1}) = 0, \ 3 \leq i \leq n-1.$$

$$f^*(a_2 b_3) = 0, \ f^*(a_i b_{i+1}) = 1, 4 \leq i \leq n; \ f^*(a_{2i} b_{2i-1}) = 1, \ \ i \equiv 0 \ (mod\ 2)$$

therefore, $|e_f(0) - e_f(1)| \leq 1$. Hence, Alternate Triangular belt graph $ATB(n)$ is divided square difference cordial graph. □

Example 1.8. *Divided square difference cordial labelling of Alternate Triangular belt graph $ATB(4)$ is shown in Figure 4.*

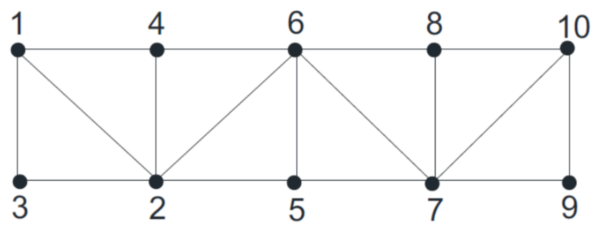

Figure 4:

Theorem 1.9. *Duplication of top vertex Alternate Triangular snake graph $DAT(n)$ is divided square difference cordial graph.*

Proof. Consider the graph G:AT(n) with the vertex set $V = \{a_i, b_j, c_j; 1 \leq i \leq n, 1 \leq j \leq \frac{n}{2}\}$ and the edge set $E = \{(a_i a_{i+1}), 1 \leq i \leq n-1\} \cup \{(a_{2i-1} c_i), 1 \leq i \leq n\} \cup \{(a_{2i} c_i), 1 \leq i \leq n\} \cup \{(a_{2i-1} b_i), 1 \leq i \leq n\} \cup \{(a_{2i} b_i), 1 \leq i \leq n\}$. Now let us define the function, $f : V \to \{1, 2, 3, \ldots, p\}$. Let us label the vertices as follows:

$$f(b_1) = 1, \ f(b_{2j}) = 8j - 2, \ f(b_{2j+1}) = 8j + 1; 1 \leq j \leq \frac{n}{2} - 1.$$

$$f(c_1) = 4,\ f(c_{2j}) = 8j - 1,\ f(c_{2j+1}) = 8j + 4; 1 \le j \le \frac{n}{2} - 1.$$
$$f(a_{4i-3}) = 8i - 6,\ f(a_{4i-2}) = 8i - 5,\ f(a_{4i-1}) = 8i - 3,\ f(a_{4i}) = 8i; 1 \le j \le n.$$

The following is the induced edge labeling.

$$f^*(a_{2i-1}b_i) = 1,\ f^*(a_{2i}b_i) = 0,\ f^*(a_{2i-1}c_i) = 0,\ f^*(a_{2i}c_i) = 1; 1 \le i \le n - 1.$$
$$f^*(a_{2i-1}a_{2i}) = 1;\ i \equiv 0\,(mod 2),\ f^*(a_{2i}a_{2i+1}) = 0; i \equiv 1\,(mod 2).$$

therefore, $|e_f(0) - e_f(1)| \le 1$. Hence, Duplication of top vertex Alternate Triangular snake graph $DAT(n)$ is divided square difference cordial graph. □

Example 1.10. *Divided square difference cordial labelling of Duplication of top vertex Alternate Triangular snake graph $AT(4)$ is shown in Figure 5.*

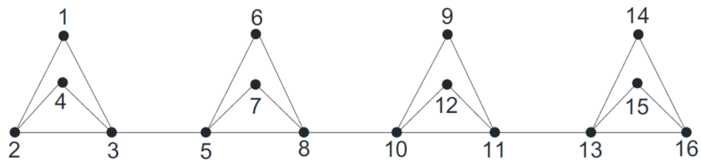

Figure 5:

Theorem 1.11. *Jelly fish $J(n, n)$ is divided square difference cordial graph.*

Proof. Consider the graph $G = J(n, n)$ with the vertex set $V = \{a_i, c_j, 1 \le i, j \le n\} \cup \{b_i, 1 \le i \le 4\}$ and the edge set $E = \{(a_i b_2), (b_2 a_i'), 1 \le i \le \frac{n}{2}\} \cup \{(b_4 c_i), (b_4 c_i'), 1 \le i \le \frac{n}{2}\} \cup \{(b_1 b_4), (b_4 b_2), (b_2 b_3), (b_3 b_4)\}$. Now let us define the function, $f : V \to \{1, 2, 3, \ldots, p\}$. Let us label the vertices as follows:

$$f(b_1) = 6,\ f(b_2) = 3, f(b_3) = 4,\ f(b_4) = 5,\ f(c_j) = 4j + 3; 1 \le j \le n.$$
$$f(c_j') = 4j + 4; \quad 1 \le j \le n.$$
$$f(a_1) = 1,\ f(a_i) = 4i + 5; 2 \le i \le n.$$
$$f(a_1') = 2,\ f(a_i') = 4i + 6; 2 \le i \le n.$$

The following is the induced edge labeling.

$$f^*(b_4 b_1) = 0, f^*(b_3 b_1) = 1, f^*(b_2 b_3) = 0, f^*(b_4 b_3) = 0, f^*(b_2 b_1) = 1,$$
$$f^*(b_1 c_i) = 1, f^*(b_1 c_i') = 0, f^*(b_2 a_i) = 0, f^*(b_2 a_i') = 1.$$

therefore, $|e_f(0) - e_f(1)| \le 1$. Hence, Jelly fish $J(n, n)$ is divided square difference cordial graph. □

Example 1.12. *Divided square difference cordial labeling of Jellyfish $J(4, 4)$ is shown in Figure 6.*

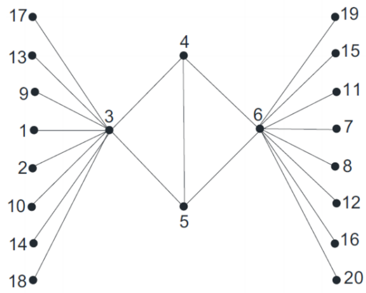

Figure 6:

Theorem 1.13. *Duplication of Top vertex Pentagonal Snake $D(P(S_n))$ is divided square difference cordial graph.*

Proof. Consider the graph $G = D(P(S_n))$ with the vertex set $V = \{a_i, b_i, a'_i, b'_i; 1 \leq i \leq n\}$ and the edge set $E = \{a'_i a_{2i-1}; 1 \leq i \leq n-1\} \cup \{a'_{i+1} a_{2i}; 1 \leq i \leq n-1\} \cup \{b_i a_{2i-1}; 1 \leq i \leq n-1\} \cup \{b_i a_{2i}; 1 \leq i \leq n-1\} \cup \{b'_i a'_{2i-1}; 1 \leq i \leq n-1\} \cup \{b'_i a'_{2i}; 1 \leq i \leq n-1\}$. Now let us define the function, $f : V \to \{1, 2, 3, \ldots, p\}$. Let us label the vertices as follows:

$$f\left(b'_i\right) = 6,\ f\left(b'_{2i}\right) = 10i + 1,\ f\left(b'_{2i+1}\right) = 10i + 5,\ 1 \leq i \leq \frac{n}{2}$$

$$f\left(b_1\right) = 1,\ f\left(b_{2i}\right) = 10i,\ f\left(b_{2i+1}\right) = 10i + 6,\ 1 \leq i \leq \frac{n}{2}$$

$$f\left(a_1\right) = 2,\ f\left(a_2\right) = 3,\ f\left(a_{4i-1}\right) = 10i - 3,\ f\left(a_{4i}\right) = 10i - 1,\ f\left(a_{4i+1}\right) = 10i + 3$$

$$f\left(a_{4i+2}\right) = 10i + 4; 1 \leq i \leq n$$

$$f\left(a'_1\right) = 4,\ f\left(a'_2\right) = 5,\ f\left(a'_{2i+1}\right) = 10i - 2,\ f\left(a'_{2i+2}\right) = 10i + 2\ ; 1 \leq i \leq n$$

The following is the induced edge labeling.

$$f^*\left(a'_i a'_{i+1}\right) = 1,\ 1 \leq i \leq 2,\ \ f^*\left(a'_i a'_{i+1}\right) = 0,\ 3 \leq i \leq n$$

$$f^*\left(a'_i a_i\right) = 0,\ 1 \leq i \leq 2,\ f^*\left(a'_2 a_3\right) = 0,\ f^*\left(a'_3 a_4\right) = 1$$

$$f^*\left(a_i b_i\right) = 1,\ 1 \leq i \leq \frac{n}{2}$$

$$f^*\left(a_{4i-2} b_{2i-1}\right) = 0,\ 1 \leq i \leq n$$

$$f^*\left(a'_{2i+1} a_{2i+3}\right) = 1$$

$$f^*\left(a_{4i} b_{2i}\right) = 1$$

$$f^*\left(a_{4i-2} b'_{2i-1}\right) = 1$$

$$f^*\left(a'_{2i+2} a_{4i+2}\right) = 0$$

$$f^*\left(a_{4i} b'_{2i}\right) = 0$$

$$f^*\left(a_i b'_i\right) = 0$$

therefore, $|e_f(0) - e_f(1)| \leq 1$. Hence, Duplication of Top vertex Pentagonal Snake $D(P(S_n))$ is divided square difference cordial graph. □

Example 1.14. *Divided square difference cordial labelling of Duplication of Top vertex Pentagonal Snake $D(P(S_6))$ is shown in Figure 7.*

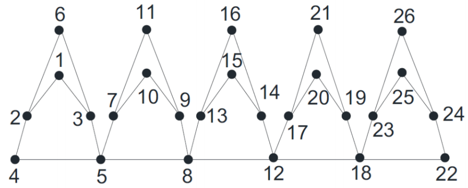

Figure 7:

References

[1] J. A. Gallian, *A Dynamic Survey of Graph labeling*, The Electronic J. Combin, 18(2015), #D56.

[2] F. Harary, *Graph Theory*, Addison -wesley, Reading, Mass, (1972).

[3] A. H. Rokad, *Fibonacci Divisor Cordial Labeling of Different Graphs*, Research & Reviews: Discrete Mathematical Structures, 8(2)(2021), 38-45.

[4] A. H. Rokad, *Fibonacci Divisor cordial Labeling of Herschel Graph in Context of Various Graph Operations*, International Journal of Advance and Innovative Research, 9(2)(XV)(2022).

[5] Shares John, Suresh Sorathia and Amit Rokad, *Lucas Divisor Cordial, Labeling of Various Graphs*, Research & Reviews: Discrete Mathematical Structures, 10(2)(2023).

[6] Savan Trivedi, Suresh Sorathia and Amit Rokad, *Divided Square Difference Cordial Labeling of Grötzsch Graph*, Journal of Emerging Technologies and Innovative Research, 10(1)(2023), 523-532.

[7] Shares John, Suresh Sorathia and Amit Rokad, *Divided Square Difference Cordial Labeling In the Context of Graphs Operations on Bistar*, Journal of Emerging Technologies and Innovative Research, 10(6)(2023), 1-8.

[8] Shares John, Suresh Sorathia and Amit Rokad, *Divided Square Difference Cordial Labeling of Herschel Graph*, International Journal of Research and Analytical Reviews, 11(2)(2024), 590-596.

[9] Vimal Patel, Suresh Sorathia and Amit Rokad, *Fibonacci Cordial Labeling of Herschel Graph In Context of Various Graph Operations*, International Journal of Applied Engineering & Technology, 5(S6)(2023), 510-518.