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Best Proximity Point Theorems for d-Rational Cyclic Contractions in Dislocated Metric Spaces

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Abstract

In this work, we define d-rational cyclic contraction in the setting of dislocated metric space. Some existence and convergence results for best proximity points have been demonstrated. Further, illustrative examples are provided in the support of results proved.

Keywords: Dislocated metric space; Best proximity point; Cyclic contraction.
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1. Introduction

The study of fixed point theory provides essential tools for analyzing nonlinear equations such as $Tx = x$, particularly when T is a self-mapping defined on a subset of a normed linear space or a metric space. The Banach contraction principle [4], established in 1922, is one of the classical results in fixed point theory.

When considering a non-self mapping $T : A \to B$, where A and B are nonempty subsets of a metric space X, the fixed point equation $Tx = x$ does not necessarily have a solution. In such situations, one seeks an element $x \in A$ that is, in some sense, closest to its image $Tx \in B$. This leads to the development of the concepts of best approximation and best proximity points. In 1969, Ky Fan [9] established the famous best approximation theorem in this direction. He proved that if A is a nonempty compact convex subset of a Hausdorff locally convex topological vector space X, and $T : A \to B$ is a continuous single-valued mapping, then there exists a point $x \in A$ such that $d(x, Tx) = d(Tx, A)$. This result initiated a broad area of research, and many generalizations and extensions of Fan's theorem have been developed by numerous researchers (one may refer [24,27,29–31]).

On the other hand, in the case of best proximity point problem, the objective is to find an element x such that $d(A, B) = \inf\{d(x, y) : x \in A, y \in B\}$. It is evident that for any point $x \in A$, we have

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$d(x,Tx) \geq d(A,B)$. A point $x \in A$ is called a best proximity point for the mapping T if $d(A,B) = d(x,Tx)$. In other words, if x is the best proximity point, then the function $d(x,Tx)$ attains its global minimum at x, and this minimum value equals $d(A,B)$. Furthermore, when $d(A,B) = 0$, then the best proximity point coincides with a fixed point of the mapping T. In 2006, Eldred and Veeramani [8] introduced the concept of cyclic contraction as follows:

Definition 1.1 ([8]). *Let A and B be two nonempty subsets of a metric space X. A mapping $T : A \cup B \to A \cup B$ is called a cyclic contraction if it is cyclic and there exists $\alpha \in (0,1)$ such that*

$$d(Tx,Ty) \leq \alpha d(x,y) + (1-\alpha)d(A,B) \quad \text{for all } x \in A, y \in B.$$

They provided sufficient conditions for the existence of a best proximity point for cyclic contractions in uniformly convex Banach spaces. Further, this result has been extended and generalized by various authors (see [2,5,16,23,26,28]).

In 1975, Khan [18] generalized Banach's contraction principle by employing a symmetric rational expression. Further, in 1977, Jaggi [15] established a fixed point theorem involving rational expressions, which initiated further studies in this direction. Later, in 2013, Yadav et al. [33] introduced the concept of rational cyclic contraction and utilized it to prove best proximity point theorems in complete metric spaces.

Subsequently, numerous extensions and generalizations of rational type contractions have been proposed in [6,7,32], and other works. Over the years, researchers have explored various generalizations of metric spaces by relaxing some of the classical axioms. One such structure is the dislocated metric space, which allows meaningful analysis even without the strict requirement of the symmetry property being bidirectional. The notion traces back to Matthews in 1986 [22] through the concept of metric domains, later refined as domain theory by Abramsky and Jung [1]. The term dislocated metric space was formally introduced in 2000 by Hitzler and Seda [14], who also extended Banach's contraction principle to this setting [13]. Subsequent contributions by Amini-Harandi [12] under the name metric-like space, as well as studies by George et al. [10], Purdhvi [25], who extended fixed point within this framework. Further developments have been made by authors such as [3,11,20,21] and others, which have significantly enriched the theory.

Inspired by these literatures, the present work defines a d-rational cyclic contraction in dislocated metric spaces and establishes best proximity point theorems along with supporting examples.

2. Preliminaries

In this section, we recall basic definitions and known results related to dislocated metric spaces, which form the foundation for the main results presented in this paper.

Definition 2.1 ([14]). *Let X be a nonempty set. A mapping $d : X \times X \to [0,\infty)$ is a dislocated metric if, for all*

$x, y, z \in X$, the following properties hold:

1. $d(x,y) \geq 0$
2. $d(x,y) = 0 \Rightarrow x = y$
3. $d(x,y) = d(y,x)$
4. $d(x,y) \leq d(x,z) + d(z,y)$.

A pair (X,d) satisfying these conditions is called a dislocated metric space, also known as a d-metric space.

Definition 2.2 ([19]). *Let A and B be nonempty subsets of a metric space (X,d). A map $T : A \cup B \to A \cup B$ is said to be cyclic map if $T(A) \subseteq B$ and $T(B) \subseteq A$.*

Definition 2.3 ([10]). *Let A and B be two nonempty closed subsets of a dislocated metric space (X,d). A mapping $T : A \cup B \to A \cup B$ is called a d-cyclic contraction if there exists $\alpha \in (0, \frac{1}{2})$ such that $d(Tx, Ty) \leq \alpha d(x,y)$ for all $x \in A$ and $y \in B$.*

Definition 2.4 ([14]). *A sequence $\{x_n\}$ in a dislocated metric space (X,d) is said to be Cauchy sequence if, for every $\epsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $m, n \geq n_0$, $d(x_m, x_n) < \epsilon$.*

Definition 2.5 ([14]). *A sequence $\{x_n\}$ in a dislocated metric space (X,d) is said to be d-convergent to a point $x \in X$ if, for every $\epsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $d(x_n, x) < \epsilon$. In this case, we say that $\{x_n\}$ d-converges to a point $x \in X$ and written as $\{x_n\} \to x$.*

Definition 2.6 ([14]). *A dislocated metric space (X,d) is said to be d-complete if every Cauchy sequence in X is d-convergent, i.e, it converges to a point in X with respect to the d-metric.*

Lemma 2.7 ([17]). *Let (X,d) be a dislocated metric space, and $\{x_n\}$ be a sequence in X that converges to x with $d(x,x) = 0$. Then $\lim_{n\to\infty} d(x_n, y) = d(x,y)$ for all $y \in X$.*

3. Main Results

In this section, we introduce the concept of a d-rational cyclic contraction and establish existence and convergence results for best proximity points.

Definition 3.1. *Let A and B be nonempty subsets of a dislocated metric space (X,d). A mapping $T : A \cup B \to A \cup B$ is said to be a d-rational cyclic contraction if there exists $\alpha, \beta, \gamma, \delta > 0$ satisfying $0 < \alpha + \beta + 4\gamma + \delta < 1$, such that*

$$d(Tx, Ty) \leq \alpha d(x,y) + \beta \frac{d(x,Tx)d(y,Ty)}{d(x,y)} + \gamma \frac{d(x,Tx)d(x,Ty) + d(y,Ty)d(y,Tx)}{d(x,y)} + \delta d(A,B) \tag{1}$$

for all $x \in A$ and $y \in B$.

Theorem 3.2. *Let A and B be two nonempty closed subsets of a complete dislocated metric space (X, d). Suppose a mapping $T : A \cup B \to A \cup B$ is a d-rational cyclic contraction in X. For $x_0 \in A \cup B$ we define $x_{n+1} = Tx_n$ for all $n \geq 0$. Then there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = d(A, B).$$

Proof. Let $x_0 \in A \cup B$ and consider an iterative sequence $\{x_n\}$ in X defined by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$. Since T is a d-rational cyclic contraction in X, by inequality (1), we have

$$\begin{aligned}
d(x_n, x_{n+1}) =& d(Tx_{n-1}, Tx_n) \\
\leq& \alpha d(x_{n-1}, x_n) + \beta \frac{d(x_{n-1}, Tx_{n-1})d(x_n, Tx_n)}{d(x_{n-1}, x_n)} \\
&+ \gamma \frac{d(x_{n-1}, Tx_{n-1})d(x_{n-1}, Tx_n) + d(x_n, Tx_n)d(x_n, Tx_{n-1})}{d(x_{n-1}, x_n)} + \delta d(A, B) \\
\leq& \alpha d(x_{n-1}, x_n) + \beta \frac{d(x_{n-1}, x_n)d(x_n, x_{n+1})}{d(x_{n-1}, x_n)} \\
&+ \gamma \frac{d(x_{n-1}, x_n)d(x_{n-1}, x_{n+1}) + d(x_n, x_{n+1})d(x_n, x_n)}{d(x_{n-1}, x_n)} + \delta d(A, B) \\
\leq& \alpha d(x_{n-1}, x_n) + \beta d(x_n, x_{n+1}) \\
&+ \gamma \frac{d(x_{n-1}, x_n)d(x_{n-1}, x_{n+1}) + d(x_n, x_{n+1})[d(x_n, x_{n-1}) + d(x_{n-1}, x_n)]}{d(x_{n-1}, x_n)} + \delta d(A, B) \\
\leq& \alpha d(x_{n-1}, x_n) + \beta d(x_n, x_{n+1}) \\
&+ \gamma \frac{(x_{n-1}, x_n)[d(x_{n-1}, x_{n+1}) + 2d(x_n, x_{n+1})]}{d(x_{n-1}, x_n)} + \delta d(A, B) \\
\leq& \alpha d(x_{n-1}, x_n) + \beta d(x_n, x_{n+1}) + \gamma[d(x_{n-1}, x_n) + 3d(x_n, x_{n+1})] + \delta d(A, B).
\end{aligned}$$

It follows that

$$d(x_n, x_{n+1}) \leq (\alpha + \gamma)d(x_{n-1}, x_n) + (\beta + 3\gamma)d(x_n, x_{n+1}) + \delta d(A, B),$$

and hence

$$(1 - \beta - 3\gamma)d(x_n, T_{n+1}) \leq (\alpha + \gamma)d(x_{n-1}, x_n) + \delta d(A, B).$$

Thus

$$d(x_n, x_{n+1}) \leq \frac{(\alpha + \gamma)}{1 - \beta - 3\gamma} d(x_{n-1}, x_n) + \frac{\delta}{1 - \beta - 3\gamma} d(A, B),$$

which implies that,

$$d(x_n, x_{n+1}) \leq kd(x_{n-1}, x_n) + (1 - k)d(A, B),$$

where $\frac{(\alpha+\gamma+\delta)}{1-\beta-3\gamma} < 1$ and hence $k = \frac{(\alpha+\gamma)}{1-\beta-3\gamma} < 1$.
Repeating this process continuously, we get

$$\begin{aligned}
d(x_n, x_{n+1}) &\leq kd(x_{n-1}, x_n) + (1-k)d(A, B) \\
&\leq k^2 d(x_{n-2}, x_{n-1}) + (1-k^2)d(A, B) \\
&\vdots \\
&\leq k^n d(x_0, x_1) + (1-k^n)d(A, B).
\end{aligned}$$

Since $k \in (0, 1)$, we have $\lim_{n\to\infty} k^n = 0$, which implies

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = d(A, B).$$

This completes the proof. □

Theorem 3.3. *Let A and B be nonempty closed subsets of a dislocated metric space (X, d). Suppose $T : A \cup B \to A \cup B$ is a d-rational cyclic contraction in X. Let $x_0 \in A \cup B$ and define the sequence $\{x_n\}$ by $x_{n+1} = Tx_n$. If $\{x_{2n}\}$ is a convergent subsequence of $\{x_n\}$ that converges to x in A with $d(x, x) = 0$. Then the sequence $\{x_n\}$ is bounded.*

Proof. By Theorem 3.2, we have

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = d(A, B).$$

Suppose $\{x_{2n}\}$ is a convergent subsequence of $\{x_n\}$ that converges to $x \in A$ with $d(x, x) = 0$.
Since T is a d-rational cyclic contraction in X, from inequality (1) and the lemma 2.7, we have

$$\begin{aligned}
d(x_{2n}, Tx) =& d(Tx_{2n-1}, Tx) \\
\leq& \alpha d(x_{2n-1}, x) + \beta \frac{d(x_{2n-1}, Tx_{2n-1})d(x, Tx)}{d(x_{2n-1}, x)} \\
&+ \gamma \frac{d(x_{2n-1}, Tx_{2n-1})d(x_{2n-1}, Tx) + d(x, Tx)d(x, Tx_{2n-1})}{d(x_{2n-1}, x)} + \delta d(A, B) \\
\leq& \alpha d(x_{2n-1}, x) + \beta \frac{d(x_{2n-1}, x_{2n})d(x, Tx)}{d(x_{2n-1}, x)} \\
&+ \gamma \frac{d(x_{2n-1}, x_{2n})d(x_{2n-1}, Tx) + d(x, Tx)d(x, x_{2n})}{d(x_{2n-1}, x)} + \delta d(A, B) \\
\leq& \alpha d(x_{2n-1}, x) + \beta \frac{[d(x_{2n-1}, x) + d(x, x_{2n})]d(x, Tx)}{d(x_{2n-1)}, x)} \\
&+ \gamma \frac{[d(x_{2n-1}, x) + d(x, x_{2n})]d(x_{2n-1}, Tx) + d(x, Tx)d(x, x_{2n})}{d(x_{2n-1}, x)} + \delta d(A, B) \\
\leq& \alpha d(x_{2n-1}, x) + \beta d(x, Tx) + \gamma d(x_{2n-1}, Tx) + \delta d(A, B) \\
\leq& \alpha[d(x_{2n-1}, x_{2n}) + d(x_{2n}, x)] + \beta[d(x, x_{2n}) + d(x_{2n}, Tx)] \\
&+ \gamma[d(x_{2n-1}, x_{2n}) + d(x_{2n}, Tx)] + \delta d(A, B)
\end{aligned}$$

$$\leq (\alpha+\gamma)d(x_{2n-1},x_{2n}) + \beta d(x_{2n},Tx) + \gamma d(x_{2n},Tx) + \delta d(A,B).$$

It follows that

$$(1-\beta-\gamma)d(x_{2n},Tx) \leq (\alpha+\gamma)d(x_{2n-1},x_{2n}) + \delta d(A,B).$$

Therefore

$$d(x_{2n},Tx) \leq \frac{\alpha+\gamma}{(1-\beta-\gamma)}d(x_{2n-1},x_{2n}) + \frac{\delta}{(1-\beta-\gamma)}d(A,B).$$

Let

$$\mu = \frac{\alpha+\gamma}{(1-\beta-\gamma)}d(x_{2n-1},x_{2n}) + \frac{\delta}{(1-\beta-\gamma)}d(A,B).$$

Thus, we have

$$d(x_{2n},Tx) = \mu,$$

which implies

$$x_{2n} \in \overline{B}\big(T(x,\mu)\big) \text{ for all } n \in \mathbb{N}.$$

Now, consider

$$d(x_{2n+1},Tx) \leq d(x_{2n},x_{2n+1}) + d(x_{2n},Tx) \leq L+\mu,$$

for some constant L, which gives

$$x_{2n+1} \in \overline{B}\big(Tx,\mu+L\big) \text{ for all } n \in \mathbb{N}.$$

Since

$$x_{2n} \in \overline{B}\big(Tx,\mu\big) \subset \overline{B}\big(Tx,\mu+L\big) \text{ for all } n \in \mathbb{N},$$

it follows that

$$x_n \in \overline{B}\big(Tx,\mu+L\big) \text{ for all } n \in \mathbb{N}.$$

So the sequence $\{x_n\}$ is bounded. This completes the proof. □

Theorem 3.4. *Let A and B be nonempty closed subsets of a dislocated metric space (X,d). Let $T : A \cup B \to A \cup B$ is a d-rational cyclic contraction in X, and let $x_0 \in A \cup B$ with the sequence $\{x_n\}$ defined by $x_{n+1} = Tx_n$ for all $n \geq 0$. Then we have the following:*

(i). If $x_0 \in A$ and the subsequence $\{x_{2n_k}\}$ of $\{x_{2n}\}$ converges to $x \in A$ such that $d(x,x) = 0$, then $x \in A$ is a best proximity point of T, that is,

$$d(x,Tx) = d(A,B).$$

(ii). If $x_0 \in B$ and the subsequence $\{x_{2n_k-1}\}$ of $\{x_{2n-1}\}$ converges to $y \in B$ such that $d(y,y) = 0$, then $y \in B$

is a best proximity point of T, that is,

$$d(y, Ty) = d(A, B).$$

Proof. Suppose $\{x_{2n_k}\}$ is convergent subsequence of the sequence $\{x_{2n}\}$, converging to a point $x \in A$ such that $d(x, x) = 0$. Since T is a d-rational cyclic contraction in X, so by applying lemma 2.7, together with Theorem 3.2, implies

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = d(A, B).$$

By inequality (1), we have

$$\begin{aligned}
d(x_{2n_k}, Tx) =& d(Tx_{2n_k-1}, Tx)\\
\leq& \alpha d(x_{2n_k-1}, x) + \beta \frac{d(x_{2n_k-1}, Tx_{2n_k-1})d(x, Tx)}{d(x_{2n_k-1}, x)}\\
&+ \gamma \frac{d(x_{2n_k-1}, Tx_{2n_k-1})d(x_{2n_k-1}, Tx) + d(x, Tx)d(x, Tx_{2n_k-1})}{d(x_{2n_k-1}, x)} + \delta d(A, B)\\
\leq& \alpha d(x_{2n_k-1}, x) + \beta \frac{d(x_{2n_k-1}, x_{2n_k})d(x, Tx)}{d(x_{2n_k-1}, x)}\\
&+ \gamma \frac{d(x_{2n_k-1}, x_{2n_k})d(x_{2n_k-1}, Tx) + d(x, Tx)d(x, x_{2n_k})}{d(x_{2n_k-1}, x)} + \delta d(A, B)\\
\leq& \alpha d(x_{2n_k-1}, x) + \beta \frac{[d(x_{2n_k-1}, x) + d(x, x_{2n_k})]d(x, Tx)}{d(x_{2n_k-1)}, x)}\\
&+ \gamma \frac{[d(x_{2n_k-1}, x) + d(x, x_{2n_k})]d(x_{2n_k-1}, Tx) + d(x, Tx)d(x, x_{2n_k})}{d(x_{2n_k-1}, x)} + \delta d(A, B)\\
\leq& \alpha d(x_{2n_k-1}, x) + \beta d(x, Tx) + \gamma d(x_{2n_k-1}, Tx) + \delta d(A, B)\\
\leq& \alpha [d(x_{2n_k-1}, x_{2n_k}) + d(x_{2n_k}, x)] + \beta [d(x, x_{2n_k}) + d(x_{2n_k}, Tx)]\\
&+ \gamma [d(x_{2n_k-1}, x_{2n_k}) + d(x_{2n_k}, Tx)] + \delta d(A, B)\\
\leq& (\alpha + \gamma)d(x_{2n_k-1}, x_{2n_k}) + \beta d(x_{2n_k}, Tx) + \gamma d(x_{2n_k}, Tx) + \delta d(A, B)\\
\leq& (\alpha + \gamma)d(x_{2n_k-1}, x_{2n_k}) + \beta d(x_{2n_k}, Tx) + 3\gamma d(x_{2n_k}, Tx) + \delta d(A, B).
\end{aligned}$$

It follows that

$$(1 - \beta - 3\gamma)d(x_{2n_k}, Tx) \leq (\alpha + \gamma)d(x_{2n_k-1}, x_{2n_k}) + \delta d(A, B).$$

Hence,

$$\begin{aligned}
d(x_{2n_k}, Tx) &\leq \frac{\alpha + \gamma}{(1 - \beta - 3\gamma)} d(x_{2n_k-1}, x_{2n_k}) + \frac{\delta}{(1 - \beta - 3\gamma)} d(A, B).\\
d(x_{2n_k}, Tx) &\leq kd(x_{2n_k-1}, x_{2n_k}) + (1 - k)d(A, B).
\end{aligned}$$

where $\frac{(\alpha+\gamma+\delta)}{1-\beta-3\gamma} < 1$, and hence $k = \frac{(\alpha+\gamma)}{1-\beta-3\gamma} < 1$.

Letting $n \to \infty$, we conclude that

$$\begin{aligned} d(x, Tx) &\leq kd(A, B) + (1-k)d(A, B) \\ &\leq d(A, B). \end{aligned}$$

Hence, $d(x, Tx) = d(A, B)$. Therefore, x is the best proximity point of T. In a similar manner, we can prove case (ii). □

Let us demonstrate the following example.

Example 3.5. *Consider $X = \mathbb{R}^+ \cup \{0\}$ equipped with $d(x, y) = x + y$ for all $x, y \in X$. Clearly, (X, d) is a dislocated metric space. Let $A = [0, \frac{1}{2}]$, $B = [\frac{1}{2}, 1]$ and define a mapping $T : A \cup B \to A \cup B$ by*

$$T(x) = \begin{cases} 1, & for\ x \in [0, \frac{1}{2}) \\ 0, & for\ x \in [\frac{1}{2}, 1]. \end{cases}$$

In this case, we have $d(A, B) = \frac{1}{2}$, $T(A) \subset B$, $T(B) \subset A$. Moreover, for every $x \in A$, $y \in B$, there exists $\alpha, \beta, \gamma, \delta > 0$ such that the cyclic map T satisfies the d-rational cyclic contraction. Also, it is observed that $\frac{1}{2} \in B$ is the best proximity point for T.

Corollary 3.6. *Let A and B be two nonempty closed subsets of a complete dislocated metric space (X, d). Suppose that $T : A \cup B \to A \cup B$ is a cyclic mapping satisfying*

$$d(Tx, Ty) \leq \alpha d(x, y) \quad \textit{for all } x \in A,\ y \in B,$$

where $0 < \alpha < 1$. Then, there is a unique fixed point in $A \cap B$.

Proof. By taking $\beta = \gamma = \delta = 0$ in Theorem 3.2, and following the same argument as in that theorem, the result follows immediately. This result also corresponds to Theorem 3.3 in [10]. □

Corollary 3.7. *Let A and B be two nonempty closed subsets of a complete dislocated metric space (X, d). Suppose that $T : A \cup B \to A \cup B$ is a cyclic mapping satisfying*

$$d(Tx, Ty) \leq \alpha d(x, y) + (1-\alpha)d(A, B) \quad \textit{for all} x \in A,\ y \in B,$$

where $0 < \alpha < 1$. Then there is a sequence $\{x_n\}$ such that

$$\lim_{n \to \infty} d(x_n, x_{n+1}) = d(A, B).$$

Proof. By taking $\gamma = \beta = 0$ in Theorem 3.2, and following the same argument as in that theorem, the result follows immediately. □

Corollary 3.8. *Let A and B be two nonempty closed subsets of a complete dislocated metric space (X, d). Suppose that $T : A \cup B \to A \cup B$ is a cyclic mapping, and there exists $\alpha,\ \gamma,\ \delta > 0$ such that $\alpha + 4\gamma + \delta < 1$, satisfying*

$$d(Tx, Ty) \leq \alpha d(x, y) + \gamma \frac{d(x, Tx)d(x, Ty) + d(y, Ty)d(y, Tx)}{d(x, y)} + \delta d(A, B)$$

for all $x \in A,\ y \in B$. Then there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = d(A, B).$$

Proof. By taking $\beta = 0$ in Theorem 3.2, and following the same argument as in that theorem, the result follows immediately. □

Corollary 3.9. *Let A and B be two non-empty, closed subsets of a complete dislocated metric space (X, d). Suppose that $T : A \cup B \to A \cup B$ is a cyclic mapping, and there exists $\alpha,\ \beta,\ \delta > 0$ such that $\alpha + \beta + \delta < 1$, satisfying*

$$d(Tx, Ty) \leq \alpha d(x, y) + \beta \frac{d(x, Tx)d(y, Ty)}{d(x, y)} + \delta d(A, B)$$

for all $x \in A$, $y \in B$. Then there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = d(A, B).$$

Proof. By taking $\gamma = 0$ in Theorem (3.2) and following the same argument as in that theorem, the result follows immediately. □

Corollary 3.10. *Let A and B be two nonempty closed subsets of a complete dislocated metric space (X, d). Suppose that $T : A \cup B \to A \cup B$ is a cyclic mapping, and there exists $\gamma \in (0, \frac{1}{4})$, satisfying*

$$d(Tx, Ty) \leq k \frac{d(x, Tx)d(x, Ty) + d(y, Ty)d(y, Tx)}{d(x, y)}$$

for all $x \in A$, $y \in B$. Then T has a unique fixed point in $A \cap B$.

Proof. By taking $\alpha = \beta = \delta = 0$ in Theorem (3.2) and following the same reasoning as in that theorem, the result follows immediately. □

References

[1] S. Abramsky and A. Jung, *Domain theory, Handbook of logic in computer science*, Claendon Press, Oxford, 6(4)(1994), 1-167.

[2] M. A. Al-Thagafi and N. Shahzad, *Convergence and existence results for best proximity points*, Nonlinear Anal., 70(10)(2009), 3665-3671.

[3] H. Aydi and A. Felhi, *Best proximity points for cyclic Kannan-Chatterjea-Ciric type contractions on metric-like spaces*, J. Nonlinear Sci. Appl., 9(5)(2016), 2458-2466.

[4] S. Banach, *Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales*, Fundamenta Mathematicae, 3(1)(1922), 133-181.

[5] M. A. Baseri, H. Mazaheri and T. D. Narang, *Some results on convergence and existence of best proximity points*, J. Mahani Math. Res. Cent., 7(1)(2018), 13-24.

[6] S. Chaipornjareansri and J. Nantadilok, *Some best proximity point results for MT-rational cyclic contractions*, J. Math. Anal., 10(6)(2019), 9-22.

[7] V. Dewangan and A. Banerjee, *Best proximity point for new symmetric rational cyclic contraction in metric spaces*, Int. J. Math. Trends Technol., 56(3)(2018), 201-207.

[8] A. A. Eldred and P. Veeramani, *Existence and convergence of best proximity points*, J. Math. Anal. Appl., 323(2)(2006), 1001-1006.

[9] K. Fan, *Extensions of two fixed point theorems of* F.E. B*rowder*, Math. Z, 112(3)(1969), 234-240.

[10] R. George, R. Rajagopalan and S. Vinayagam, *Cyclic contractions and fixed points in dislocated metric spaces*, Int. J. Math. Anal., 7(9)(2013), 403-411.

[11] L. Gholizadeh and E. Karapınar, *Best proximity point results in dislocated metric spaces via* R-*functions*, Rev. Real Acad. Cienc. Exactas Fís. Nat. Ser. A Mat., 112(4)(2018), 1391-1407.

[12] A. A. Harandi, *Metric-like spaces, partial metric spaces and fixed points*, Fixed Point Theory Appl., 2012(1)(2012), 1-10.

[13] P. Hitzler, *Generalized metrics and topology in logic programming semantics*, Ph. D. thesis, National University of Ireland (University College, Cork), (2001).

[14] P. Hitzler and A. K. Seda, *Dislocated topologies*, J. Electr. Electron. Eng., 51(12)(2000), 3-7.

[15] D. Jaggi, *Some unique fixed point theorems*, Indian J. Pure Appl. Math., 8(2)(1977), 223-230.

[16] E. Karapınar and I. M. Erhan, *Best proximity point on different type contractions*, Appl. Math. Inf. Sci., 3(3)(2011), 342-353.

[17] E. Karapınar and P. Salimi, *Dislocated metric space to metric spaces with some fixed point theorems*, Fixed Point Theory Appl.,(2013), 1-19.

[18] M. S. Khan, *A fixed point theorem in metric spaces*, Riv. Mat. Univ. Parma., 1(1975), 69-72.

[19] W. A. Kirk, P. S. Srinivasan and P. Veeramani, *Fixed points for mappings satisfying cyclical contractive conditions*, Fixed Point Theory, 4(1)(2003), 79-89.

[20] P. S. Kumari and D. Panthi, *Cyclic contractions and fixed point theorems on various generating spaces*, Fixed Point Theory Appl., 2015(1)(2015), 1-17.

[21] P. S. Kumari, K. Zoto and D. Panthi, *d-Neighborhood system and generalized* F*-contraction in dislocated metric space*, SpringerPlus, 4 (2015), 1-10.

[22] S. G. Matthews, *Metric domains for completeness*, Ph. D. thesis, University of Warwick. Department of Computer Science, (1985).

[23] M. Petric, *Best proximity point theorems for weak cyclic Kannan contractions*, Filomat, 25(1)(2011), 145-154.

[24] J. B. Prolla, *Fixed-point theorems for set-valued mappings and existence of best approximants*, Numer. Funct. Anal. Optim., 5(4)(1983), 449-455.

[25] K. Prudhvi, *A unique fixed point theorem on a generalized d-cyclic contraction mapping in d-metric spaces*, Am. J. Appl. Math. Stat., 6(3)(2018), 107-108.

[26] V. S. Raj, *A best proximity point theorem for weakly contractive non-self-mappings*, Nonlinear Anal. Theory Methods Appl., 74(14)(2011), 4804-4808.

[27] S. Reich, *Approximate selections, best approximations, fixed points, and invariant sets*, J. Math. Anal. Appl., 62(1)(1978), 104-113.

[28] S. Rezapour, M. Derafshpour and N. Shahzad, *Best proximity points of cyclic ϕ-contractions on reflexive Banach spaces*, Fixed Point Theory Appl., (2010), 1-7.

[29] V. M. Sehgal and S. P. Singh, *A generalization to multifunctions of Fan's best approximation theorem*, Proc. Amer. Math. Soc., 102(3)(1988), 534-537.

[30] V. M. Sehgal and S. P. Singh, *A theorem on best approximations*, Numer. Funct. Anal. Optim., 10(1)(1989), 181-184.

[31] V. Vetrivel and P. Veeramani and P. Bhattacharyya, *Some extensions of Fan's best approximation theorem*, Numer. Funct. Anal. Optim., 13(3-4)(1992), 397-402.

[32] M. R. Yadav, A. Sharma and B. S. Thakur, *Best proximity point theorems for* MT-K *and* MT-C *rational cyclic contractions in metric spaces*, Int. J. Math. Comput. Sci, 7(8)(2014), 1384-1389.

[33] M. R. Yadav, B. S. Thakur and A. Sharma, *Best proximity point theorems for rational expression in complete metric spaces*, J. Comp. Math. Sci., 4(3)(2013), 153-159.