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Analyzing Complexities in Decision-Making using Rough Intuitionistic Fuzzy Diagraphs

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Abstract

This research article introduces a hybrid approach which integrates a rough set with an intuitionistic fuzzy set. A rough intuitionistic fuzzy architecture is formed to study the vagueness, incompleteness, and ambiguity based information systems in some real life problems by using this hybridization. This study basically extends the existing rough fuzzy hybrid approach by generalizing some of its definitions and theorems. Finally, application of the proposed hybrid mechanism to decision-making problems underscores the potential of our method. Present article provides an efficient and effective tool for dealing complexities in decision-making. In particular, an efficient algorithm is developed to solve decision-making problem. Time complexity of proposed algorithm is also computed.

Keywords: Rough intuitionistic fuzzy set; Rough intuitionistic fuzzy digraph; Decision making; Algorithms.
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1. Introduction

The intuitive concept of fuzzy set introduced in [1], was generalized by Atanassov's intuitionistic fuzzy set [2]. It considers favourable and unfavourable association both simultaneously in dealing of problematic scenario. These theories have been successfully applied in many fields for decision-makings covering area medical, engineering, graph theory, pattern recognition, etc.

Related Works

Gupta et al. surveyed the fuzzy logic-based systems used in medical diagnosis [3]. Kumar and Pandey introduced fuzzy linear programming to find the patients' waiting time by satisfying some defined satisfaction targets of patients at an OPD of a healthcare unit [4]. Vague sets have been proposed a generalized fuzzy technique and its assessment in multicriteria decision-making in medical

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diagnosis [5]. Fuzzy independent sets, domination fuzzy sets, and fuzzy chromatic sets are invariants concerning the isomorphism transformations of the fuzzy graphs, it was discussed in [6]. In reference to Atanassov's intuitionistic fuzzy sets, it has been successfully applied in many fields for making decisions [7,8]. A new order function in an intuitionistic fuzzy environment was utilized in group decision-making [9]. Ding et al. studied hybridization of rough set and generalized intuitionistic fuzzy sets in decision-making problems [10]. Moreover, some prominent and relevant applications of fuzzy graphs and intuitionistic fuzzy graphs discussed in [11–16].

Related Works on Rough Set and Various Hybridized Techniques using Rough Set

The rough set theory proposed by Pawlak is an excellent and elegant mathematical tool for the analysis of uncertainty, inconsistency, and incompleteness in an information space. The fundamental idea of this theory relies on approximation of sets by a pair of sets called lower approximation and upper approximation [17]. Bourahla applied rough set theory for reasoning on vague ontologies [18]. Kumar developed a rule base for the disease pneumonia through rough set-based data analysis [19]. Due to the limitation of human knowledge to understand the complex problems, it is very difficult to apply and succeed only a single type of uncertainty method to deal with such problems. Therefore, it is necessary to develop hybrid models by incorporating the advantages of many other different mathematical intuitions dealing with uncertainty [20]. It provides the way to merge rough set theory with different theories, namely fuzzy set, intuitionistic fuzzy set, and soft set theory. Dubois and Prade initiated and mixed fuzzy set and rough set to generate fuzzy rough set and rough fuzzy set. Chakraborty et al. studied fuzziness in rough sets [21]. Cornelis et al. defined intuitionistic fuzzy rough sets: at the crossroads of imperfect knowledge [23]. Rough fuzzy sets have applications in decision making [22]. A hybrid approach consisting of rough set theory and intuitionistic fuzzy set theory was proposed by Mazarbhuiya and Shenify for the detection of anomalies [24]. Mareay et al. introduced some properties on rough intuitionistic fuzzy sets [25]. Bhattacharya proposed a novel similarity measure on intuitionistic fuzzy rough sets [26]. Zhou and Wu generalized some approximation operators on intuitionistic fuzzy rough sets [27]. Malik and Akram created a new approach based on intuitionistic fuzzy rough graphs [28]. Some other recent approaches using intuitionistic fuzzy rough sets have been studied in [29–32].

Motivation for the study

Fariha and Akram put forward a hybrid technique to solve decision-making problems mixing theories of rough set and fuzzy set (Fariha and Akram, 2018). In this article, authors choose rough fuzzy diagraphs and introduce some relevant concepts like tensor product, strong product, lexicographic product and symmetric difference, etc. to assist the decision-making. Although the theory offers a broad range of applications, there are some areas where it could be improved. Because of considering the membership function only, this theory is not capable for dealing the situations in which decision-maker (DMs) face 'neither this nor that' situation to evaluate their preferences. This kind of uncertainty is usually known as the uncertainty with hesitation. The theory of intuitionistic fuzzy sets could be very useful for handling such types of uncertainty and vagueness in the data of decision making. Our motivation in this paper is to improve upon this decision-making approach and preserving its genuine nature. To achieve this, we propose the concept of rough intuitionistic fuzzy hybridization over the approach given in [22]. Our contributions are as follows:

a) Generalize the concept of a rough fuzzy diagraph to rough intuitionistic fuzzy diagraph.

b) Generalize the definitions: tensor product, strong product, lexicographic product and symmetric difference.

c) Generalize the decision-making approach in rough intuitionistic fuzzy environment.

d) Illustration of the current technique to show its validity and effectiveness of solving real-life issues. For this purpose, we will discuss the model of selection of best university for the applicants and best location for industry set up. We do comparative analysis of the developed model and some existing techniques and show our preference for the mentioned approach.

The rest of this article is distributed as follows: Section 2 contains a brief review of rough sets, rough fuzzy sets, rough intuitionistic fuzzy sets, and rough intuitionistic fuzzy diagraphs, consists of the establishment of generalized definitions, generalized tensor product, generalized strong product, generalized lexicographic products, and generalized symmetric difference. In Section 3, the practical and real-life use of suggested tools are prepared based on the proposed approach for decision-making. Section 4 shows the validity and confirmation of results. Finally, Section 5 concluded the comments and future plans of the author.

2. Rough Intuitionistic Fuzzy Diagraphs

Definition 2.1 (Pawlak 1982)**.** *Let $(\Omega^*, \mathcal{R})$ be a crisp approximation space. A crisp subset, $\widetilde{A}$ of Ω^* is called a rough set if $\left[\underline{\mathcal{R}}\widetilde{A}\right] \neq \left[\overline{\mathcal{R}}\widetilde{A}\right]$ where, $\left[\underline{\mathcal{R}}\widetilde{A}\right] = \left\{u \in \Omega^* \mid [u]^{\mathcal{R}} \subseteq \widetilde{A}\right\}$, $\left[\overline{\mathcal{R}}\widetilde{A}\right] = \left\{u \in \Omega^* \mid [u]^{\mathcal{R}} \cap \widetilde{A} \neq \phi\right\}$ and $[u]^{\mathcal{R}}$ is the equivalence class of u. A rough set is denoted as $\left(\left[\underline{\mathcal{R}}\widetilde{A}\right], \left[\overline{\mathcal{R}}\widetilde{A}\right]\right)$.*

Definition 2.2 (Zafar and Akram 2018)**.** *Let $(\Omega^*, \mathcal{R})$ be a crisp approximation space. Let $\widetilde{A} \in \mathcal{F}(\Omega^*)$, where $\mathcal{F}(\Omega^*)$ represents the fuzzy power set defined on Ω^*. Lower and upper approximations of the fuzzy set $\widetilde{A}$, denoted as $\left[\underline{\mathcal{R}}\widetilde{A}\right]$ and $\left[\overline{\mathcal{R}}\widetilde{A}\right]$, respectively, defined by fuzzy sets as follows: $\forall\ u \in \Omega^*$,*

$$\left[\underline{\mathcal{R}}\widetilde{A}\right](u) = \bigwedge_{v\in\Omega^*} \left((1 - \mathcal{R}(u,v)) \vee \widetilde{A}(v)\right),$$

$$\left[\overline{\mathcal{R}}\widetilde{A}\right](u) = \bigvee_{v\in\Omega^*} \left(\mathcal{R}(u,v) \vee \widetilde{A}(v)\right).$$

The ordered pair, $\left(\left[\underline{\mathcal{R}}\widetilde{A}\right], \left[\overline{\mathcal{R}}\widetilde{A}\right]\right)$ *is called a rough fuzzy set.*

Definition 2.3 (Zafar and Akram 2018)**.** *Let* Ω^* *be a universe of discourse,* $\mathcal{R}$ *be an equivalence relation on* Ω^**. Let* $\widetilde{A}$ *be a fuzzy set on* Ω^* *and* $\left(\left[\underline{\mathcal{R}}\widetilde{A}\right], \left[\overline{\mathcal{R}}\widetilde{A}\right]\right)$ *is a rough fuzzy set. Let* $B^* \subseteq \Omega^* \times \Omega^*$*. Let* $\mathcal{S}$ *be an equivalence relation on* B^* *such that* $((u_1, u_2), (v_1, v_2)) \in \mathcal{S} \iff (u_1, v_1), (u_2, v_2) \in \mathcal{R}$*. Now again let* $\widetilde{B}$ *be a fuzzy set on* $B^* \subseteq \Omega^* \times \Omega^*$ *such that*

$$\widetilde{B}(uv) \preccurlyeq \min\left\{\left[\underline{\mathcal{R}}\widetilde{A}\right](u), \left[\underline{\mathcal{R}}\widetilde{A}\right](v)\right\}, \ \forall\, u, v \in \Omega^*,$$

where uv *is same as* (u, v) *and* $\widetilde{A}$ *is a fuzzy set on* Ω^**. The lower and upper approximations of* $\widetilde{B}$*, represented by* $\left[\underline{\mathcal{S}}\widetilde{B}\right], \left[\overline{\mathcal{S}}\widetilde{B}\right]$*, are fuzzy sets, defined by* $\forall\ uv \in B^*$*,*

$$\left[\underline{\mathcal{S}}\widetilde{B}\right](uv) = \bigwedge_{wx \in B^*} \left((1 - \mathcal{S}(uv, wx)) \vee \widetilde{B}(wx)\right),$$

$$\left[\overline{\mathcal{S}}\widetilde{B}\right](uv) = \bigvee_{wx \in B^*} \left(\mathcal{S}(uv, wx) \wedge \widetilde{B}(wx)\right).$$

The ordered pair $\left(\left[\underline{\mathcal{S}}\widetilde{B}\right], \left[\overline{\mathcal{S}}\widetilde{B}\right]\right)$ *is called rough fuzzy relation.*

Definition 2.4. *Let* $(\Omega^*, \mathcal{R})$ *be a crisp approximation space. Let* $\widetilde{A} \in \mathcal{IF}(\Omega^*)$*, where* $\mathcal{IF}(\Omega^*)$ *represents the Atanassov's intuitionistic fuzzy power set. The lower and upper approximations of the intuitionistic fuzzy set* $\widetilde{A}$*, denoted as* $\left[\underline{\mathcal{R}}\widetilde{A}\right]$ *and* $\left[\overline{\mathcal{R}}\widetilde{A}\right]$*, respectively, defined by intuitionistic fuzzy sets as follows:* $\forall\ u \in \Omega^*$*,*

$$\left[\underline{\mathcal{R}}\widetilde{A}\right] = \left\{\left\langle u, \mu_{\left[\underline{\mathcal{R}}\widetilde{A}\right]}(u), v_{\left[\underline{\mathcal{R}}\widetilde{A}\right]}(u)\right\rangle \mid u \in \Omega^*\right\},$$

$$\left[\overline{\mathcal{R}}\widetilde{A}\right] = \left\{\left\langle u, \mu_{\left[\overline{\mathcal{R}}\widetilde{A}\right]}(u), v_{\left[\overline{\mathcal{R}}\widetilde{A}\right]}(u)\right\rangle \mid u \in \Omega^*\right\}.$$

Analogous to [20], following compositions are employed to calculate $\left[\underline{\mathcal{R}}\widetilde{A}\right]$ *and* $\left[\overline{\mathcal{R}}\widetilde{A}\right]$*:*

$$\mu_{\left[\underline{\mathcal{R}}\widetilde{A}\right]}(u) = \bigwedge_{v \in \Omega^*} \left((1 - \mathcal{R}(u, v)) \vee \mu_{\widetilde{A}}(v)\right),$$

$$v_{\left[\underline{\mathcal{R}}\widetilde{A}\right]}(u) = \bigvee_{v \in \Omega^*} \left(\mathcal{R}(u, v) \wedge v_{\widetilde{A}}(v)\right),$$

$$\mu_{\left[\overline{\mathcal{R}}\widetilde{A}\right]}(u) = \bigvee_{v \in \Omega^*} \left(\mathcal{R}(u, v) \wedge \mu_{\widetilde{A}}(v)\right),$$

$$v_{\left[\overline{\mathcal{R}}\widetilde{A}\right]}(u) = \bigwedge_{v \in \Omega^*} \left((1 - \mathcal{R}(u, v)) \vee v_{\widetilde{A}}(v)\right).$$

The ordered pair $\left(\left[\underline{\mathcal{R}}\widetilde{A}\right], \left[\overline{\mathcal{R}}\widetilde{A}\right]\right)$ *is called a rough intuitionistic fuzzy set.*

Definition 2.5. *Let* $(\Omega^*, \mathcal{R})$ *be the crisp approximation space and* $\widetilde{A}$ *be an intuitionistic fuzzy set on* Ω^* *and* $\left(\left[\underline{\mathcal{R}}\widetilde{A}\right], \left[\overline{\mathcal{R}}\widetilde{A}\right]\right)$ *is a rough intuitionistic fuzzy set. Let* $B^* \subseteq \Omega^* \times \Omega^*$*,* $\mathcal{S}$ *be a crisp equivalence relation on* B^* *such that* $((u_1, u_2), (v_1, v_2)) \in \mathcal{S} \iff (u_1, v_1), (u_2, v_2) \in \mathcal{R}$*. Now again let* $\widetilde{B}$ *be an intuitionistic fuzzy set on*

$B^* \subseteq \Omega^* \times \Omega^*$ *such that*

$$\widetilde{B}(uv) \preccurlyeq \sqcap \left\{ \left[\underline{\mathcal{R}}\widetilde{A}\right](u), \left[\underline{\mathcal{R}}\widetilde{A}\right](v) \right\}, \forall\, u,v \in \Omega^*. \tag{1}$$

where $\sqcap = MIN$ *(Intuitionistic fuzzy t-norm). Then the lower and upper approximations of* $\widetilde{B}$*, denoted by* $\left[\underline{\mathcal{S}}\widetilde{B}\right], \left[\overline{\mathcal{S}}\widetilde{B}\right]$*, are intuitionistic fuzzy sets, defined as* $\forall\, wx \in B^*$,

$$\left[\underline{\mathcal{S}}\widetilde{B}\right] = \left\{ \left\langle uv, \mu_{\left[\underline{\mathcal{S}}\widetilde{B}\right]}(uv), v_{\left[\underline{\mathcal{S}}\widetilde{B}\right]}(uv) \right\rangle \mid uv \in \Omega^* \times \Omega^* \right\},$$
$$\left[\overline{\mathcal{S}}\widetilde{B}\right] = \left\{ \left\langle uv, \mu_{\left[\overline{\mathcal{S}}\widetilde{B}\right]}(uv), v_{\left[\overline{\mathcal{S}}\widetilde{B}\right]}(uv) \right\rangle \mid uv \in \Omega^* \times \Omega^* \right\},$$

where

$$\mu_{\left[\underline{\mathcal{S}}\widetilde{B}\right]}(uv) = \bigwedge_{wx \in B^*} \left((1 - \mathcal{S}(uv, wx)) \vee \mu_{\widetilde{B}}(wx) \right),$$
$$v_{\left[\underline{\mathcal{S}}\widetilde{B}\right]}(uv) = \bigvee_{wx \in B^*} \left(\mathcal{S}(uv, wx) \wedge v_{\widetilde{B}}(wx) \right),$$
$$\mu_{\left[\overline{\mathcal{S}}\widetilde{B}\right]}(uv) = \bigvee_{wx \in B^*} \left(\mathcal{S}(uv, wx) \wedge \mu_{\widetilde{B}}(wx) \right),$$
$$v_{\left[\overline{\mathcal{S}}\widetilde{B}\right]}(uv) = \bigwedge_{yz \in B^*} \left((1 - \mathcal{S}(uv, wx)) \vee v_{\widetilde{B}}(wx) \right).$$

The ordered pair $\left(\left[\underline{\mathcal{S}}\widetilde{B}\right], \left[\overline{\mathcal{S}}\widetilde{B}\right]\right)$ *is called rough intuitionistic fuzzy relation.*

Definition 2.6. *A rough intuitionistic fuzzy diagraph on a set* Ω^* *is a 4-tuple,* $(\mathcal{R}, \mathcal{R}\widetilde{A}, \mathcal{S}, \mathcal{S}\widetilde{B})$ *such that* $\mathcal{R}$ *- a crisp equivalence relation on* Ω^**;* $\mathcal{S}$ *- be a crisp equivalence relation on* B^**;* $\mathcal{R}\widetilde{A} = \left(\left[\underline{\mathcal{R}}\widetilde{A}\right], \left[\overline{\mathcal{R}}\widetilde{A}\right]\right)$ *be a rough intuitionistic fuzzy set;* $\mathcal{S}\widetilde{B} = \left(\left[\underline{\mathcal{S}}\widetilde{B}\right], \left[\overline{\mathcal{S}}\widetilde{B}\right]\right)$ *be a rough intuitionistic fuzzy relation defined on* $\Omega^* \times \Omega^*$*. Now,* $\widetilde{D} = ([\mathcal{R}\widetilde{A}], [\mathcal{S}\widetilde{B}])$ *is defined as a rough intuitionistic fuzzy diagraph.* $\underline{\widetilde{D}} = \left(\left[\underline{\mathcal{R}}\widetilde{A}\right], \left[\underline{\mathcal{S}}\widetilde{B}\right]\right)$, $\overline{\widetilde{D}} = \left(\left[\overline{\mathcal{R}}\widetilde{A}\right], \left[\overline{\mathcal{S}}\widetilde{B}\right]\right)$ *are lower and upper approximations of rough intuitionistic fuzzy diagraph* $\widetilde{D}$ *such that*

$$\left[\underline{\mathcal{S}}\widetilde{B}\right](uv) \preccurlyeq \sqcap \left\{ \left[\underline{\mathcal{R}}\widetilde{A}\right](u), \left[\underline{\mathcal{R}}\widetilde{A}\right](v) \right\}, \forall\, u,v \in \Omega^*,$$
$$\left[\overline{\mathcal{S}}\widetilde{B}\right](uv) \preccurlyeq \sqcap \left\{ \left[\overline{\mathcal{R}}\widetilde{A}\right](u), \left[\overline{\mathcal{R}}\widetilde{A}\right](v) \right\}, \forall\, u,v \in \Omega^*,$$

where $\preccurlyeq$ *denotes Intuitionistic fuzzy inequality.*

Example 2.7. *Let* $\Omega^* = \{p, q, r, s\}$ *be a set and* $\mathcal{R}$ *an equivalence relation on* Ω^* *defined as:*

$\mathcal{R}$	p	q	r	s
p	1	0	1	0
q	0	1	0	1
r	1	0	1	0
s	0	1	0	1

Table 1: An equivalence relation $\mathcal{R}$

Let $\widetilde{A} = \{(p, 0.8, 0.2), (q, 0.6, 0.3), (r, 0.7, 0.1), (s, 0.5, 0.3)\}$ *be an intuitionistic fuzzy set on* Ω^* *and* $\mathcal{R}\widetilde{A} = \left(\left[\underline{\mathcal{R}}\widetilde{A}\right], \left[\overline{\mathcal{R}}\widetilde{A}\right]\right)$*, a rough intuitionistic fuzzy set is obtained using composition formulae given in the definition. So,*

$$\left[\underline{\mathcal{R}}\widetilde{A}\right] = \{(p, 0.7, 0.2), (q, 0.5, 0.3), (r, 0.7, 0.2), (s, 0.5, 0.3)\},$$
$$\left[\overline{\mathcal{R}}\widetilde{A}\right] = \{(p, 0.8, 0.1), (q, 0.6, 0.3), (r, 0.8, 0.1), (s, 0.6, 0.3)\}.$$

Let $B^* = \{pq, qr, qs, rs\}$ *and* $\mathcal{S}$*, an equivalence relation on* B^* *defined as:*

$\mathcal{S}$	pq	qr	qs	rs
pq	1	0	0	1
qr	0	1	0	0
qs	0	0	1	0
rs	1	0	0	1

Table 2: An equivalence relation $\mathcal{S}$

Now an intuitionistic fuzzy set $\widetilde{B}$ *defined on* B^* *is*

$$\widetilde{B} = \{(pq, 0.4, 0.4), (qr, 0.5, 0.4), (qs, 0.2, 0.7), (rs, 0.3, 0.7)\}.$$

Using definition, $\left[\underline{\mathcal{S}}\widetilde{B}\right]$ *and* $\left[\overline{\mathcal{S}}\widetilde{B}\right]$ *are illustrated as follows:*

$$\left[\underline{\mathcal{S}}\widetilde{B}\right] = \{(pq, 0.3, 0.4), (qr, 0.5, 0.4), (qs, 0.2, 0.7), (rs, 0.3, 0.7)\},$$
$$\left[\overline{\mathcal{S}}\widetilde{B}\right] = \{(pq, 0.4, 0.4), (qr, 0.5, 0.4), (qs, 0.2, 0.7), (rs, 0.4, 0.4)\}.$$

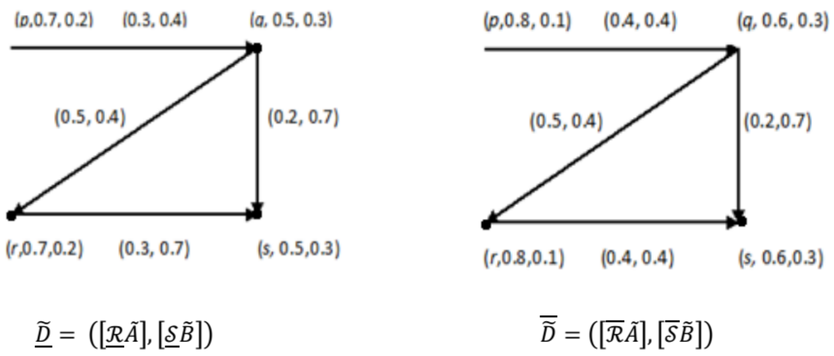


Figure 1: Lower and upper approximations of rough intuitionistic fuzzy diagraph.

Now there are some generalized intuitions to construct rough intuitionistic fuzzy diagraphs:

Definition 2.8. *Let* $\widetilde{D}_1$ *and* $\widetilde{D}_2$ *are two rough intuitionistic fuzzy diagraphs. The generalized tensor product of* $\widetilde{D}_1$ *and* $\widetilde{D}_2$ *is a rough intuitionistic fuzzy diagraph* $\widetilde{D} = \widetilde{D}_1 \circledcirc \widetilde{D}_2 = \left(\underline{\widetilde{D}}_1 \circledcirc \underline{\widetilde{D}}_2, \overline{\widetilde{D}}_1 \circledcirc \overline{\widetilde{D}}_2\right)$ *where* $\underline{\widetilde{D}}_1 \circledcirc \underline{\widetilde{D}}_2 = \left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\underline{\mathcal{R}}\widetilde{A}_2\right], \left[\underline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)$ *and* $\overline{\widetilde{D}}_1 \circledcirc \overline{\widetilde{D}}_2 = \left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\overline{\mathcal{R}}\widetilde{A}_2\right], \left[\overline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)$ *are intuitionistic fuzzy diagraphs with the following definitions:*

For lower approximation:

a) $\left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\underline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_1,u_2) = \sqcap\left\{\left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right]\right)(u_1), \left(\left[\underline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_2)\right\}, \ \forall\ (u_1,u_2) \in \left[\underline{\mathcal{R}}\widetilde{A}_1\right] \times \left[\underline{\mathcal{R}}\widetilde{A}_2\right]$

b) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1,u_2)(v_1,v_2)) = \sqcap\left\{\left[\underline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\underline{\mathcal{S}}\widetilde{B}_2\right](u_2,v_2)\right\}, \ \forall\ u_1v_1 \in \left[\underline{\mathcal{S}}\widetilde{B}_1\right],\ u_2v_2 \in \left[\underline{\mathcal{S}}\widetilde{B}_2\right]$

For upper approximation:

c) $\left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\overline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_1,u_2) = \sqcap\left\{\left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right]\right)(u_1), \left(\left[\overline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_2)\right\}, \ \forall\ (u_1,u_2) \in \left[\overline{\mathcal{R}}\widetilde{A}_1\right] \times \left[\overline{\mathcal{R}}\widetilde{A}_2\right]$

c) $\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1,u_2)(v_1,v_2)) = \sqcap\left\{\left[\overline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\overline{\mathcal{S}}\widetilde{B}_2\right](u_2v_2)\right\}, \ \forall\ u_1v_1 \in \left[\overline{\mathcal{S}}\widetilde{B}_1\right],\ u_2v_2 \in \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$

Theorem 2.9. *The generalized tensor product of two rough intuitionistic fuzzy diagraphs is also a rough intuitionistic fuzzy diagraph.*

Proof. Let $\widetilde{D}_1 = \left(\underline{\widetilde{D}}_1, \overline{\widetilde{D}}_1\right)$ and $\widetilde{D}_2 = \left(\underline{\widetilde{D}}_2, \overline{\widetilde{D}}_2\right)$ be two rough intuitionistic fuzzy diagraphs. Let $\widetilde{D} = \widetilde{D}_1 \circledcirc \widetilde{D}_2 = \left(\underline{\widetilde{D}}_1 \circledcirc \underline{\widetilde{D}}_2, \overline{\widetilde{D}}_1 \circledcirc \overline{\widetilde{D}}_2\right)$ be the generalized tensor product of $\widetilde{D}_1$ and $\widetilde{D}_2$ where $\underline{\widetilde{D}}_1 \circledcirc \underline{\widetilde{D}}_2 = \left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\underline{\mathcal{R}}\widetilde{A}_2\right], \left[\underline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)$ and $\overline{\widetilde{D}}_1 \circledcirc \overline{\widetilde{D}}_2 = \left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\overline{\mathcal{R}}\widetilde{A}_2\right], \left[\overline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)$. Our claim is that $D_1 \circledcirc D_2$ is a rough intuitionistic fuzzy diagraph. To show the claim, it is enough to prove that $\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\underline{\mathcal{S}}\widetilde{B}_2\right]$ and $\left[\overline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$ are intuitionistic fuzzy relations on $\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\underline{\mathcal{R}}\widetilde{A}_2\right]$ and $\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\overline{\mathcal{R}}\widetilde{A}_2\right]$, respectively. Firstly, to show the assertion $\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\underline{\mathcal{S}}\widetilde{B}_2\right]$ is an intuitionistic fuzzy relation on $\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\underline{\mathcal{R}}\widetilde{A}_2\right]$. If $u_1v_1 \in \left[\underline{\mathcal{S}}\widetilde{B}_1\right]$, $u_2v_2 \in \left[\underline{\mathcal{S}}\widetilde{B}_2\right]$, then

$$\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)(u_1,u_2)(v_1,v_2) = \sqcap\left\{\left[\underline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\underline{\mathcal{S}}\widetilde{B}_2\right](u_2v_2)\right\}. \tag{2}$$

Using the condition of the (1) as given below:

$$\left[\underline{\mathcal{S}}\widetilde{B}\right](u_1v_1) \preccurlyeq \sqcap\left\{\left[\underline{\mathcal{R}}\widetilde{A}\right](u_1), \left[\underline{\mathcal{R}}\widetilde{A}\right](v_1)\right\},\ \forall\ u_1, v_1 \in \Omega^*.$$

This can be interpreted as follows:

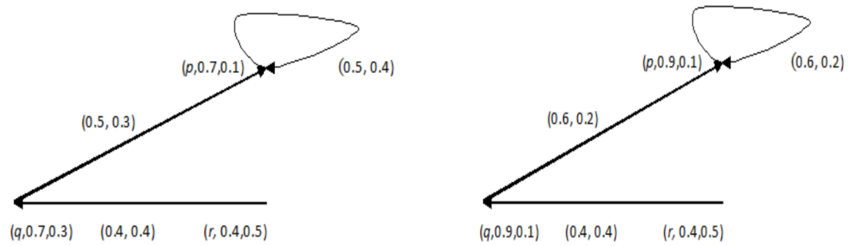
$$\mu_{\underline{\mathcal{S}}\widetilde{B}}(u_1v_1) \leq \min\left(\mu_{\left[\underline{\mathcal{R}}\widetilde{A}\right]}(u_1), \mu_{\left[\underline{\mathcal{R}}\widetilde{A}\right]}(v_1)\right),$$
$$\nu_{\underline{\mathcal{S}}\widetilde{B}}(u_1v_1) \leq \max\left(\nu_{\left[\underline{\mathcal{R}}\widetilde{A}\right]}(u_1), \nu_{\left[\underline{\mathcal{R}}\widetilde{A}\right]}(v_1)\right).$$

Therefore, (2) can be written as

$$\begin{aligned}
&\preccurlyeq \sqcap\left\{\left[\underline{\mathcal{R}}\widetilde{A}_1\right](u_1), \left[\underline{\mathcal{R}}\widetilde{A}_1\right](v_1), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](u_2), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](v_2)\right\} \\
&= \sqcap\left\{\left[\underline{\mathcal{R}}\widetilde{A}_1\right](u_1), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](u_2), \left[\underline{\mathcal{R}}\widetilde{A}_1\right](v_1),, \left[\underline{\mathcal{R}}\widetilde{A}_2\right](v_2)\right\} \\
&= \sqcap\left\{\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\underline{\mathcal{R}}\widetilde{A}_2\right](u_1,u_2), \left[\underline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\underline{\mathcal{R}}\widetilde{A}_2\right](v_1,v_2)\right\}.
\end{aligned}$$

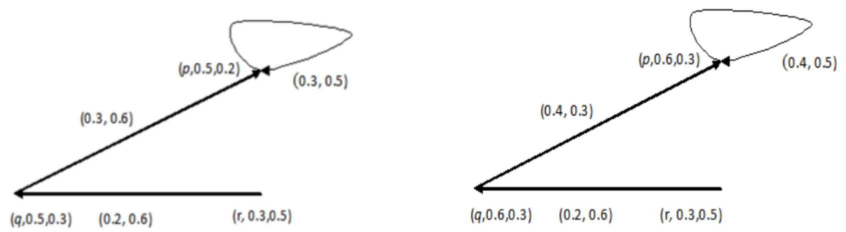
Thus $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)$ is an intuitionistic fuzzy relation on $\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\underline{\mathcal{R}}\widetilde{A}_2\right]$. Similarly it can also be shown that $\left[\overline{\mathcal{S}}\widetilde{B}_1\right] \circledcirc \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$ is an intuitionistic fuzzy relation on $\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \circledcirc \left[\overline{\mathcal{R}}\widetilde{A}_2\right]$. Hence; $\widetilde{D}$ is a rough intuitionistic fuzzy diagraph. □

Example 2.10. *Let $\Omega^* = \{p, q, r\}$ be a crisp set. Let $\widetilde{D}_1 = \left(\underline{\widetilde{D}}_1, \overline{\widetilde{D}}_2\right)$ and $\widetilde{D}_2 = \left(\underline{\widetilde{D}}_2, \overline{\widetilde{D}}_2\right)$ be two rough intuitionistic fuzzy digraphs on Ω^*, these diagraphs are shown as follows in Fig. 2 and Fig. 3, respectively.*



$\underline{\widetilde{D}}_1 = ([\underline{\mathcal{R}}\tilde{A}_1], [\underline{\mathcal{S}}\tilde{B}_1])$ $\overline{\widetilde{D}}_1 = ([\overline{\mathcal{R}}A_1], [\overline{\mathcal{S}}\tilde{B}_1])$

Figure 2: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}_1$



$\underline{\widetilde{D}}_2 = ([\underline{\mathcal{R}}\tilde{A}_2], [\underline{\mathcal{S}}\tilde{B}_2])$ $\overline{\widetilde{D}}_2 = ([\overline{\mathcal{R}}A_2], [\overline{\mathcal{S}}\tilde{B}_2])$

Figure 3: Lower and upper approximations of a rough intuitionistic fuzzy diagraph $\widetilde{D}_2$

The generalized tensor product of rough intuitionistic fuzzy diagraphs is depicted in Figure 2 and Figure 3, is a rough intuitionistic fuzzy diagraph shown in the Figure 4 by its lower and upper approximations as follows:

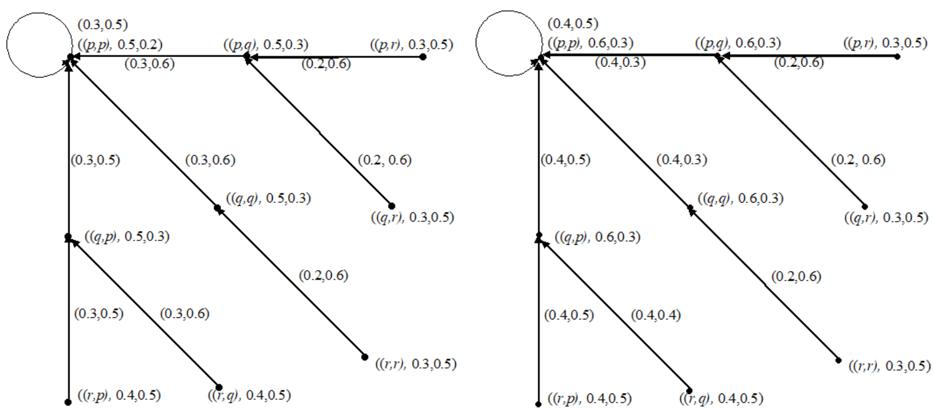


Figure 4: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}_1 \circledcirc \widetilde{D}_2$

Definition 2.11. *Let $\widetilde{D}_1$ and $\widetilde{D}_2$ are two rough intuitionistic fuzzy diagraphs. The generalized lexicographic product of $\widetilde{D}_1$ and $\widetilde{D}_2$ is a rough intuitionistic fuzzy diagraph $\widetilde{D} = \widetilde{D}_1 \odot \widetilde{D}_2 = \left(\underline{\widetilde{D}}_1 \odot \underline{\widetilde{D}}_2, \overline{\widetilde{D}}_1 \odot \overline{\widetilde{D}}_2\right)$ where*

$\underline{\widetilde{D}}_1 \odot \underline{\widetilde{D}}_2 = \left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \odot \left[\underline{\mathcal{R}}\widetilde{A}_2\right], \left[\underline{\mathcal{S}}\widetilde{B}_1\right] \odot \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)$ *and* $\overline{\widetilde{D}}_1 \odot \overline{\widetilde{D}}_2 = \left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \odot \left[\overline{\mathcal{R}}\widetilde{A}_2\right], \left[\overline{\mathcal{S}}\widetilde{B}_1\right] \odot \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)$ *are intuitionistic fuzzy diagraphs with the following definitions:*

For lower approximation:

a) $\left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \odot \left[\underline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_1,u_2) = \sqcap\left\{\left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right]\right)(u_1), \left(\left[\underline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_2)\right\}, \ \forall\ (u_1,u_2) \in \left[\underline{\mathcal{R}}\widetilde{A}_1\right] \times \left[\underline{\mathcal{R}}\widetilde{A}_2\right]$

b) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \odot \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u,u_2)(v,v_2)) = \sqcap\left\{\left[\underline{\mathcal{R}}\widetilde{A}_1\right](u), \left[\underline{\mathcal{S}}\widetilde{B}_2\right](u_2,v_2)\right\}, \ \forall\ u \in [\underline{\mathcal{R}}A_1],\ u_2v_2 \in \left[\underline{\mathcal{S}}\widetilde{B}_2\right]$

c) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \odot \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1,u_2)(v_1,v_2)) = \sqcap\left\{\left[\underline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\underline{\mathcal{S}}\widetilde{B}_2\right](u_2,v_2)\right\}, \ \forall\ u_1v_1 \in \left[\underline{\mathcal{S}}\widetilde{B}_1\right],\ u_2v_2 \in \left[\underline{\mathcal{S}}\widetilde{B}_2\right]$

For upper approximation:

d) $\left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \odot \left[\overline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_1,u_2) = \sqcap\left\{\left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right]\right)(u_1), \left(\left[\overline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_2)\right\}, \ \forall\ (u_1,u_2) \in \left[\overline{\mathcal{R}}\widetilde{A}_1\right] \times \left[\overline{\mathcal{R}}\widetilde{A}_2\right]$

d) $\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] \odot \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)((u,u_2)(v,v_2)) = \sqcap\left\{\left[\overline{\mathcal{R}}\widetilde{A}_1\right](u), \left[\overline{\mathcal{S}}\widetilde{B}_2\right](u_2v_2)\right\}, \ \forall\ u \in \left[\overline{\mathcal{R}}\widetilde{A}_1\right],\ u_2v_2 \in \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$

d) $\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] \odot \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1,u_2)(v_1,v_2)) = \sqcap\left\{\left[\overline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\overline{\mathcal{S}}\widetilde{B}_2\right](u_2v_2)\right\}, \ \forall\ u_1v_1 \in \left[\overline{\mathcal{S}}\widetilde{B}_1\right],\ u_2v_2 \in \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$

where $\sqcap$ *- intuitionistic fuzzy t-norm, for simplicity MIN can be chosen for analysis.*

Theorem 2.12. *The generalized lexicographic product of two rough intuitionistic fuzzy diagraphs is also a rough intuitionistic fuzzy diagraph.*

Proof. The proof follows similar structure as Theorem 2.1 and is omitted for brevity. □

Example 2.13. *Let* $\Omega^* = \{p,q,r\}$ *be a set and* $\widetilde{D}_1 = \left(\underline{\widetilde{D}}_1, \overline{\widetilde{D}}_2\right)$, $\widetilde{D}_2 = \left(\underline{\widetilde{D}}_2, \overline{\widetilde{D}}_2\right)$ *be two rough intuitionistic fuzzy digraphs on* Ω^*. *The generalized lexicographic product of* $\widetilde{D}_1$, $\widetilde{D}_2$ *is characterized by* $\underline{\widetilde{D}}_1 \odot \underline{\widetilde{D}}_2 = \left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \odot \left[\underline{\mathcal{R}}\widetilde{A}_2\right], \left[\underline{\mathcal{S}}\widetilde{B}_1\right] \odot \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)$ *and* $\overline{\widetilde{D}}_1 \odot \overline{\widetilde{D}}_2 = \left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \odot \left[\overline{\mathcal{R}}\widetilde{A}_2\right], \left[\overline{\mathcal{S}}\widetilde{B}_1\right] \odot \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)$ *is a rough intuitionistic fuzzy diagraph depicted in the Figure 5.*

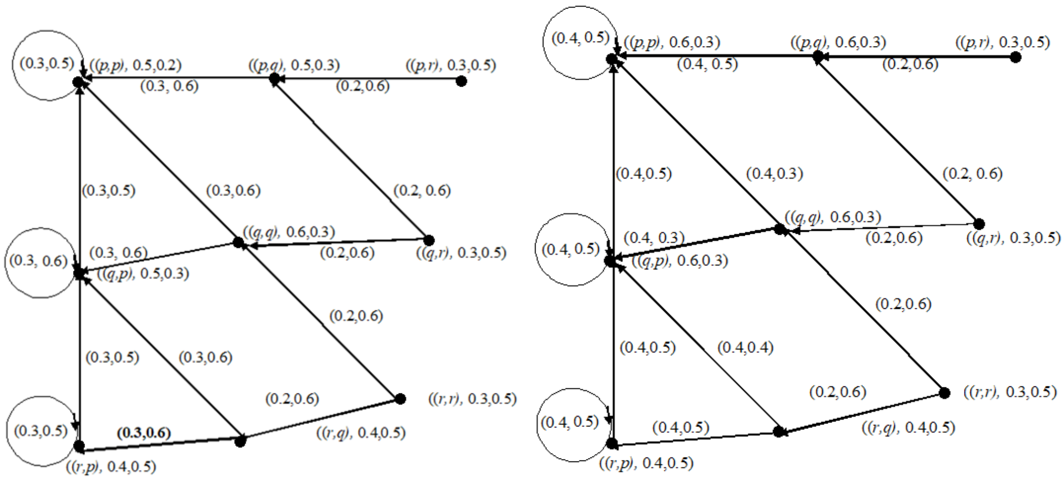


Figure 5: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}_1 \odot \widetilde{D}_2$

Definition 2.14. *Let $\widetilde{D}_1$ and $\widetilde{D}_2$ are two rough intuitionistic fuzzy diagraphs. The generalized strong product of $\widetilde{D}_1$ and $\widetilde{D}_2$ is a rough intuitionistic fuzzy diagraph $\widetilde{D} = \widetilde{D}_1 \otimes \widetilde{D}_2 = \left(\underline{\widetilde{D}}_1 \otimes \underline{\widetilde{D}}_2, \overline{\widetilde{D}}_1 \otimes \overline{\widetilde{D}}_2\right)$ where $\underline{\widetilde{D}}_1 \otimes \underline{\widetilde{D}}_2 = \left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \otimes \left[\underline{\mathcal{R}}\widetilde{A}_2\right], \left[\underline{\mathcal{S}}\widetilde{B}_1\right] \otimes \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)$ and $\overline{\widetilde{D}}_1 \otimes \overline{\widetilde{D}}_2 = \left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \otimes \left[\overline{\mathcal{R}}\widetilde{A}_2\right], \left[\overline{\mathcal{S}}\widetilde{B}_1\right] \otimes \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)$ are intuitionistic fuzzy diagraphs with the following definitions:*

For lower approximation:

a) $\left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \otimes \left[\underline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_1, u_2) = \sqcap\left\{\left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right]\right)(u_1), \left(\left[\underline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_2)\right\}, \forall\ (u_1, u_2) \in \left[\underline{\mathcal{R}}\widetilde{A}_1\right] \times \left[\underline{\mathcal{R}}\widetilde{A}_2\right]$

b) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \otimes \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u, u_2)(v, v_2)) = \sqcap\left\{\left[\underline{\mathcal{R}}\widetilde{A}_1\right](u), \left[\underline{\mathcal{S}}\widetilde{B}_2\right](u_2, v_2)\right\}, \forall\, u \in \left[\underline{\mathcal{R}}\widetilde{A}_1\right],\ u_2v_2 \in \left[\underline{\mathcal{S}}\widetilde{B}_2\right]$

c) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \otimes \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1, w)(v_1, w)) = \sqcap\left\{\left[\underline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](w)\right\}, \forall\, w \in \left[\underline{\mathcal{R}}\widetilde{A}_2\right],\ u_1v_1 \in \left[\underline{\mathcal{S}}\widetilde{B}_1\right]$

d) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \otimes \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1, u_2)(v_1, v_2)) = \sqcap\left\{\left[\underline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\underline{\mathcal{S}}\widetilde{B}_2\right](u_2, v_2)\right\}, \forall\ u_1v_1 \in \left[\underline{\mathcal{S}}\widetilde{B}_1\right],\ u_2v_2 \in \left[\underline{\mathcal{S}}\widetilde{B}_2\right]$

For upper approximation:

e) $\left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \otimes \left[\overline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_1, u_2) = \sqcap\left\{\left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right]\right)(u_1), \left(\left[\overline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_2)\right\}, \forall\ (u_1, u_2) \in \left[\overline{\mathcal{R}}\widetilde{A}_1\right] \times \left[\overline{\mathcal{R}}\widetilde{A}_2\right]$

e) $\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] \otimes \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)((u, u_2)(v, v_2)) = \sqcap\left\{\left[\overline{\mathcal{R}}\widetilde{A}_1\right](u), \left[\overline{\mathcal{S}}\widetilde{B}_2\right](u_2v_2)\right\}, \forall\, u \in \left[\overline{\mathcal{R}}\widetilde{A}_1\right],\ u_2v_2 \in \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$

e) $\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] \otimes \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1, w)(v_1, w)) = \sqcap\left\{\left[\overline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\overline{\mathcal{R}}\widetilde{A}_2\right](w)\right\}, \forall\, u_1v_1 \in \left[\overline{\mathcal{S}}\widetilde{B}_1\right],\ w \in \left[\overline{\mathcal{R}}\widetilde{A}_2\right]$

e) $([\overline{S}B_1] \otimes [\overline{S}B_2])((u_1, u_2)(v_1, v_2)) = \sqcap\left\{\left[\overline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\overline{\mathcal{S}}\widetilde{B}_2\right](u_2v_2)\right\}\quad \forall\ u_1v_1 \in \left[\overline{\mathcal{S}}\widetilde{B}_1\right],\ u_2v_2 \in \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$

Example 2.15. *Let $\Omega^* = \{p, q, r\}$ be a crisp set, $\widetilde{D}_1 = \left(\underline{\widetilde{D}}_1, \overline{\widetilde{D}}_2\right)$, $\widetilde{D}_2 = \left(\underline{\widetilde{D}}_2, \overline{\widetilde{D}}_2\right)$ be two rough intuitionistic fuzzy digraphs on Ω^* shown in Fig. 2 and Fig. 3.The generalized strong product of $\widetilde{D}_1 = \left(\underline{\widetilde{D}}_1, \overline{\widetilde{D}}_2\right)$, $\widetilde{D}_2 = \left(\underline{\widetilde{D}}_2, \overline{\widetilde{D}}_2\right)$ be rough intuitionistic fuzzy digraphs on Ω^* characterized by $\underline{\widetilde{D}}_1 \otimes \underline{\widetilde{D}}_2 = \left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \otimes \left[\underline{\mathcal{R}}\widetilde{A}_2\right], \left[\underline{\mathcal{S}}\widetilde{B}_1\right] \otimes \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)$ and $\overline{\widetilde{D}}_1 \otimes \overline{\widetilde{D}}_2 = \left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \otimes \left[\overline{\mathcal{R}}\widetilde{A}_2\right], \left[\overline{\mathcal{S}}\widetilde{B}_1\right] \otimes \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)$, is shown in the Figure 6.*

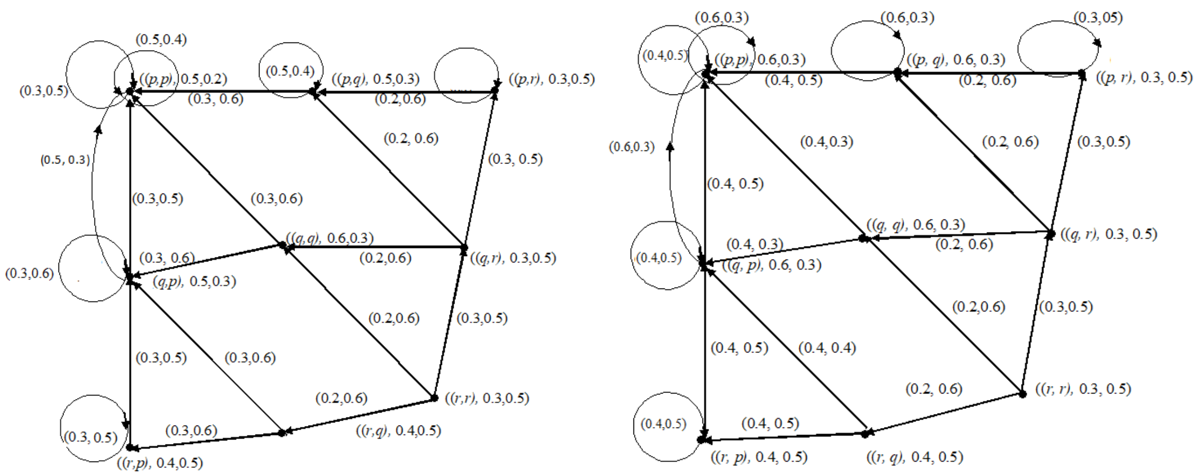


Figure 6: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}_1 \otimes \widetilde{D}_2$

Theorem 2.16. *The generalized strong product of two rough intuitionistic fuzzy diagraphs is also a rough intuitionistic fuzzy diagraph.*

Proof. The proof follows similar structure as previous theorems and is omitted for brevity. □

Definition 2.17. *The rejection of* $\widetilde{D}_1$ *and* $\widetilde{D}_2$ *is a rough intuitionistic fuzzy diagraphs* $\widetilde{D} = \widetilde{D}_1|\widetilde{D}_2 = \left(\underline{\widetilde{D}}_1|\underline{\widetilde{D}}_2, \underline{\widetilde{D}}_1|\underline{\widetilde{D}}_2\right)$ *where* $\underline{\widetilde{D}}_1|\underline{\widetilde{D}}_2 = \left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] | \left[\underline{\mathcal{R}}\widetilde{A}_2\right], \left[\underline{\mathcal{S}}\widetilde{B}_1\right] | \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)$ *and* $\overline{\widetilde{D}}_1|\overline{\widetilde{D}}_2 = \left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] | \left[\overline{\mathcal{R}}\widetilde{A}_2\right], \left[\overline{\mathcal{S}}\widetilde{B}_1\right] | \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)$:

For lower approximation:

a) $\left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] | \left[\underline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_1, u_2) = MIN\left\{\left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right]\right)(u_1), \left(\left[\underline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_2)\right\}, \forall\ (u_1, u_2) \in \left[\underline{\mathcal{R}}\widetilde{A}_1\right] \times \left[\underline{\mathcal{R}}\widetilde{A}_2\right]$

b) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] | \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u, u_2)(v, v_2)) = MIN\left\{\left[\underline{\mathcal{R}}\widetilde{A}_1\right](u), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](u_2), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](v_2)\right\}, \forall\ u \in \left[\underline{\mathcal{R}}\widetilde{A}_1\right], u_2v_2 \notin \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$

c) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] | \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1, u_2)(v_1, v_2)) = MIN\left\{\left[\underline{\mathcal{R}}\widetilde{A}_1\right](u_1), \left[\underline{\mathcal{R}}\widetilde{A}_1\right](v_1), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](u_2), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](v_2)\right\}, \forall\ u_1v_1 \notin \left[\underline{\mathcal{S}}\widetilde{B}_1\right], u_2v_2 \notin [\underline{\mathcal{S}}B_2]$

d) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] | \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1, w)(v_1, w)) = MIN\left\{\left[\underline{\mathcal{R}}\widetilde{A}_1\right](u_1), \left[\underline{\mathcal{R}}\widetilde{A}_1\right](v_1), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](w)\right\}, \forall\ u_1v_1 \notin \left[\underline{\mathcal{S}}\widetilde{B}_1\right], w \in \left[\underline{\mathcal{R}}\widetilde{A}_2\right]$

For upper approximation:

e) $\left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] | \left[\overline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_1, u_2) = MIN\left\{\left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right]\right)(u_1), \left(\left[\overline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_2)\right\}, \forall\ (u_1, u_2) \in \left[\overline{\mathcal{R}}\widetilde{A}_1\right] \times \left[\overline{\mathcal{R}}\widetilde{A}_2\right]$

e) $\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] | \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)((u, u_2)(v, v_2)) = MIN\left\{\left[\overline{\mathcal{R}}\widetilde{A}_1\right](u), \left[\overline{\mathcal{R}}\widetilde{A}_2\right](u_2), \left[\overline{\mathcal{R}}\widetilde{A}_2\right](v_2)\right\}, \forall\ u \in \left[\overline{\mathcal{R}}\widetilde{A}_1\right], u_2v_2 \notin \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$

e) $\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] | \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1, u_2)(v_1, v_2)) = MIN\left\{\left[\overline{\mathcal{R}}\widetilde{A}_1\right](u_1), \left[\overline{\mathcal{R}}\widetilde{A}_1\right](v_1), \left[\overline{\mathcal{R}}\widetilde{A}_2\right](u_2), \left[\overline{\mathcal{R}}\widetilde{A}_2\right](v_2)\right\}, \forall\ u_1v_1 \notin [\overline{\mathcal{S}}B_1], u_2v_2 \notin [\overline{\mathcal{S}}B_2]$

e) $\left(\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] | [\overline{\mathcal{S}}B_2]\right)\right)((u_1, w)(v_1, w)) = MIN\left\{\left[\overline{\mathcal{R}}\widetilde{A}_1\right](u_1), \left[\overline{\mathcal{R}}\widetilde{A}_1\right](v_1), \left[\overline{\mathcal{R}}\widetilde{A}_2\right](w)\right\}, \forall\ u_1v_1 \notin \left[\overline{\mathcal{S}}\widetilde{B}_1\right], w \in \left[\overline{\mathcal{R}}\widetilde{A}_2\right]$

Example 2.18. *Let* $\Omega^* = \{p, q, r\}$ *be a set.* $\widetilde{D}_1 = \left(\underline{\widetilde{D}}_1, \overline{\widetilde{D}}_1\right)$, $\widetilde{D}_2 = \left(\underline{\widetilde{D}}_2, \overline{\widetilde{D}}_2\right)$ *be two rough intuitionistic fuzzy digraphs on* Ω^* *given in the Figure 7.*

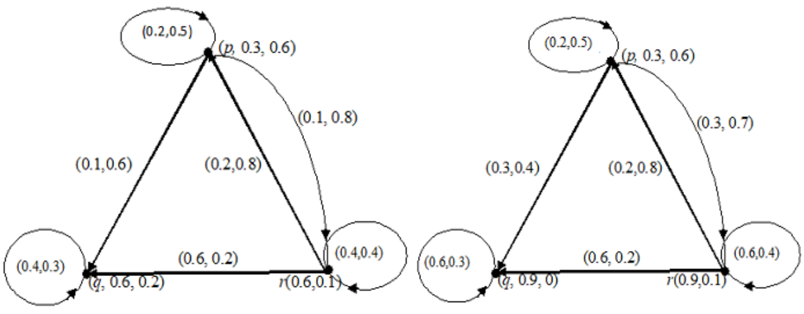


Figure 7: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}_1$

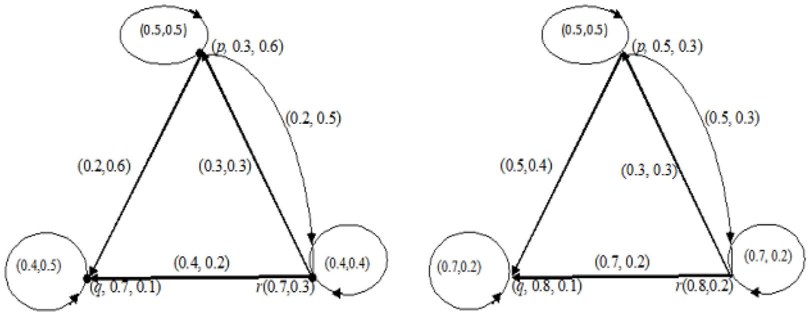


Figure 8: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}_2$

The rejection $\widetilde{D}_1|\widetilde{D}_2 = \left(\underline{\widetilde{D}}_1|\underline{\widetilde{D}}_2, \overline{\widetilde{D}}_1|\overline{\widetilde{D}}_2\right)$ *is shown in the Figure 9.*

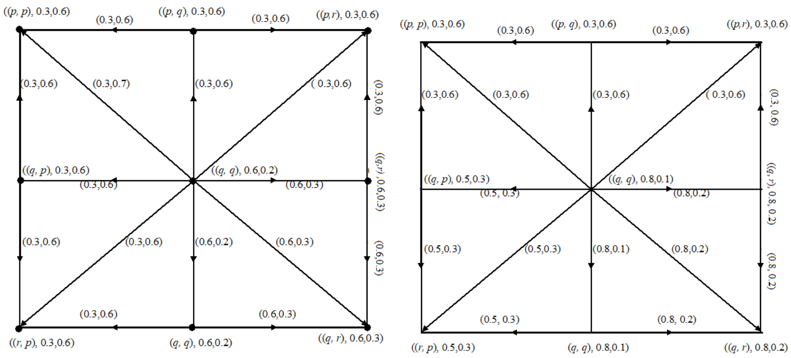


Figure 9: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}_1|\widetilde{D}_2$

Theorem 2.19. *The rejection of two rough intuitionistic fuzzy digraphs is also a rough intuitionistic fuzzy digraph.*

Proof. The proof follows similar structure as previous theorems and is omitted for brevity. □

Definition 2.20. *Let* $\widetilde{D}_1$ *and* $\widetilde{D}_2$ *are two rough intuitionistic fuzzy diagraphs. The symmetric difference of* $\widetilde{D}_1$ *and* $\widetilde{D}_2$ *is a rough intuitionistic fuzzy diagraph* $\widetilde{D} = \widetilde{D}_1 \oplus \widetilde{D}_2 = \left(\underline{\widetilde{D}}_1 \oplus \underline{\widetilde{D}}_2, \overline{\widetilde{D}}_1 \oplus \overline{\widetilde{D}}_2\right)$ *is an intuitionistic fuzzy diagraph with the following definitions:*

For lower approximation:

(i) $\left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \oplus \left[\underline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_1, u_2) = \sqcap\left\{\left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right]\right)(u_1), \left(\left[\underline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_2)\right\}, \forall\ (u_1, u_2) \in \left[\underline{\mathcal{R}}\widetilde{A}_1\right] \times \left[\underline{\mathcal{R}}\widetilde{A}_2\right]$

(ii) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \oplus \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u, u_2)(v, v_2)) = \sqcap\left\{\left[\underline{\mathcal{R}}\widetilde{A}_1\right](u), \left[\underline{\mathcal{S}}\widetilde{B}_2\right](u_2, v_2)\right\}, \forall\, u \in \left[\underline{\mathcal{R}}\widetilde{A}_1\right], u_2v_2 \in \left[\underline{\mathcal{S}}\widetilde{B}_2\right]$

(iii) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \oplus \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1, w)(v_1, w)) = \sqcap\left\{\left[\underline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](w)\right\}, \forall\ u_1v_1 \in \left[\underline{\mathcal{S}}\widetilde{B}_1\right], w \in \left[\underline{\mathcal{R}}\widetilde{A}_2\right]$

(iv) $\left(\left[\underline{\mathcal{S}}\widetilde{B}_1\right] \oplus \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1, u_2)(v_1, v_2))$

$$= \begin{cases} \sqcap\left\{\left[\underline{\mathcal{R}}\widetilde{A}_1\right](u_1), \left[\underline{\mathcal{R}}\widetilde{A}_1\right](v_1), \left[\underline{\mathcal{S}}\widetilde{B}_2\right](u_2v_2)\right\} & \forall u_1v_1 \notin \left[\underline{\mathcal{S}}\widetilde{B}_1\right], u_2v_2 \in \left[\underline{\mathcal{S}}\widetilde{B}_2\right] \\ \sqcap\left\{\left[\underline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](u_2), \left[\underline{\mathcal{R}}\widetilde{A}_2\right](v_2)\right\} & \forall\ u_1v_1 \in \left[\underline{\mathcal{S}}\widetilde{B}_1\right], u_2v_2 \notin \left[\underline{\mathcal{S}}\widetilde{B}_2\right] \end{cases}$$

For upper approximation:

(v) $\left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \oplus \left[\overline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_1,u_2) = \sqcap\left\{\left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right]\right)(u_1), \left(\left[\overline{\mathcal{R}}\widetilde{A}_2\right]\right)(u_2)\right\}, \ \forall\ (u_1,u_2) \in \left[\overline{\mathcal{R}}\widetilde{A}_1\right] \times \left[\overline{\mathcal{R}}\widetilde{A}_2\right]$

(v) $\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] \oplus \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)((u,u_2)(v,v_2)) = \sqcap\left\{\left[\overline{\mathcal{R}}\widetilde{A}_1\right](u), \left[\overline{\mathcal{S}}\widetilde{B}_2\right](u_2v_2)\right\}, \ \forall\ u \in \left[\overline{\mathcal{R}}\widetilde{A}_1\right],\ u_2v_2 \in \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$

(v) $\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] \oplus \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1,w)(v_1,w)) = \sqcap\left\{\left[\overline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\overline{\mathcal{R}}\widetilde{A}_2\right](w)\right\}, \ \forall\ u_1v_1 \in \left[\overline{\mathcal{S}}\widetilde{B}_1\right],\ w \in \left[\overline{\mathcal{S}}\widetilde{B}_2\right]$

(v) $\left(\left[\overline{\mathcal{S}}\widetilde{B}_1\right] \oplus \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)((u_1,u_2)(v_1,v_2))$

$$= \begin{cases} \sqcap\left\{\left[\overline{\mathcal{R}}\widetilde{A}_1\right](u_1), \left[\overline{\mathcal{R}}\widetilde{A}_1\right](v_1), \left[\overline{\mathcal{S}}\widetilde{B}_2\right](u_2v_2)\right\} & \forall u_1v_1 \notin \left[\overline{\mathcal{S}}\widetilde{B}_1\right],\ u_2v_2 \in \left[\overline{\mathcal{S}}\widetilde{B}_2\right] \\ \sqcap\left\{\left[\overline{\mathcal{S}}\widetilde{B}_1\right](u_1v_1), \left[\overline{\mathcal{R}}\widetilde{A}_2\right](u_2), \left[\overline{\mathcal{R}}\widetilde{A}_2\right](v_2)\right\} & \forall\ u_1v_1 \in \left[\overline{\mathcal{S}}\widetilde{B}_1\right],\ u_2v_2 \notin \left[\overline{\mathcal{S}}\widetilde{B}_2\right] \end{cases}$$

Example 2.21. *Let* $\Omega^* = \{p,q,r\}$ *be a crisp set. Let* $\widetilde{D}_1 = \left(\underline{\widetilde{D}}_1, \overline{\widetilde{D}}_1\right)$ *and* $\widetilde{D}_2 = \left(\underline{\widetilde{D}}_2, \overline{\widetilde{D}}_2\right)$ *be two rough intuitionistic fuzzy digraphs presented in Figure 10 and Figure 11, respectively.*

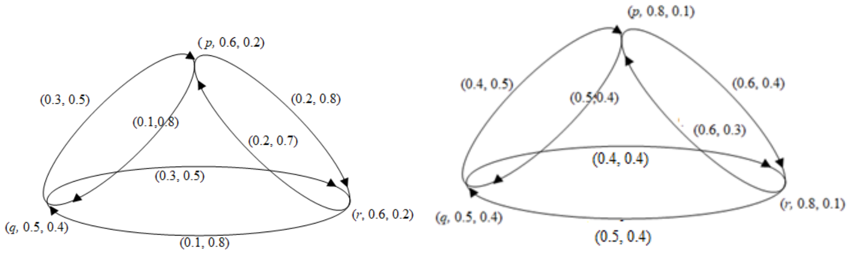


Figure 10: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}_1$

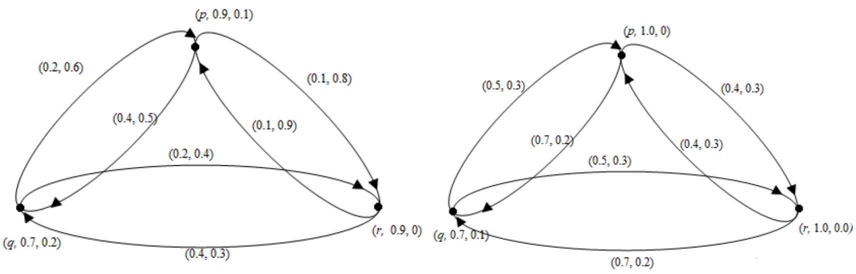


Figure 11: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}_2$

The symmetric difference of $\widetilde{D}_1$ *and* $\widetilde{D}_2$ *is* $\widetilde{D} = \widetilde{D}_1 \oplus \widetilde{D}_2 = \left(\underline{\widetilde{D}}_1 \oplus \underline{\widetilde{D}}_2, \overline{\widetilde{D}}_1 \oplus \overline{\widetilde{D}}_2\right)$ *where* $\underline{\widetilde{D}}_1 \oplus \underline{\widetilde{D}}_2 = \left(\left[\underline{\mathcal{R}}\widetilde{A}_1\right] \oplus \left[\underline{\mathcal{R}}\widetilde{A}_2\right], \left[\underline{\mathcal{S}}\widetilde{B}_1\right] \oplus \left[\underline{\mathcal{S}}\widetilde{B}_2\right]\right)$ *and* $\overline{\widetilde{D}}_1 \oplus \overline{\widetilde{D}}_2 = \left(\left[\overline{\mathcal{R}}\widetilde{A}_1\right] \oplus \left[\overline{\mathcal{R}}\widetilde{A}_2\right], \left[\overline{\mathcal{S}}\widetilde{B}_1\right] \oplus \left[\overline{\mathcal{S}}\widetilde{B}_2\right]\right)$, *is a rough intuitionistic fuzzy diagraphs shown in the Figure 12.*

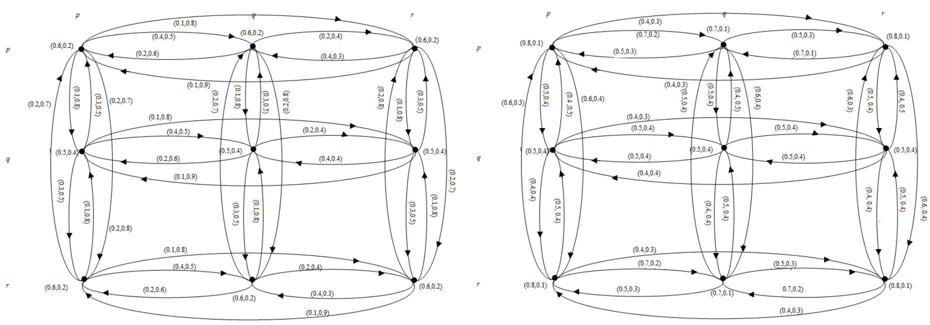


Figure 12: Lower and upper approximations of rough intuitionistic fuzzy diagraph of symmetric difference

Theorem 2.22. *The symmetric difference of two rough intuitionistic fuzzy digraphs is also a rough intuitionistic fuzzy digraph.*

Proof. The proof follows similar structure as previous theorems and is omitted for brevity. □

3. Decision Making based on Rough Intuitionistic Fuzzy Diagraphs

Decision-making based on rough fuzzy diagraphs are being extended in this section through two examples of [22] by rough intuitionistic diagraphs so that viability and validity of the current approach can be investigated.

Example 3.1 (First application). *Suppose* $\Omega^* = \{C_1, C_2, C_3, C_4, C_5\}$ *be a set of five universities which are to be examined for best choice. An equivalence relation initially showing same features is defined in the following Table 3:*

$\mathcal{R}$	C_1	C_2	C_3	C_4	C_5
C_1	1	0	1	0	0
C_2	0	1	0	1	0
C_3	1	0	1	0	0
C_4	0	1	0	1	0
C_5	0	0	0	0	1

Table 3: Crisp relation showing same feature

DMs object is to choose the best university to get admission. On behalf of past experience, it is viable to consider positive and negative factors simultaneously. Because not everyone has the exact source of knowledge addressing the performance of any system. So keeping this thought in mind, let $\widetilde{A}$ *be an intuitionistic fuzzy knowledge describing the labels for each criterion:*

$$\widetilde{A} = \{\langle C_1, 0.7,\ 0.1\rangle, \langle C_2, 0.6,\ 0.4\rangle, \langle C_3, 0.4, 0.4\rangle, \langle C_4, 0.8, 0.0\rangle, \langle C_5, 0.3, 0.5\rangle\}.$$

Using the concept defined earlier. The rough intuitionistic fuzzy set associated to the intuitionistic fuzzy set $\widetilde{A}$ *is given as follows in terms of lower approximation and upper approximation, respectively.*

$$\left[\underline{\mathcal{R}}\widetilde{A}\right] = \{\langle C_1, 0.4,\ 0.4\rangle, \langle C_2, 0.6,\ 0.4\rangle, \langle C_3, 0.4, 0.4\rangle, \langle C_4, 0.6, 0.4\rangle, \langle C_5, 0.3, 0.5\rangle\},$$
$$\left[\overline{\mathcal{R}}\widetilde{A}\right] = \{\langle C_1, 0.7,\ 0.1\rangle, \langle C_2, 0.8, 0.0\ \rangle, \langle C_3, 0.7, 0.1\rangle, \langle C_4, 0.8, 0.0\rangle, \langle C_5, 0.3, 0.5\rangle\}.$$

Let $B^* = \{C_1C_2, C_1C_4, C_2C_5, C_3C_1, C_3C_2, C_3C_5, C_4C_2, C_4C_3, C_5C_2, C_5C_4\} \subseteq \Omega^* \times \Omega^*$ *and* $\mathcal{S}$ *- an equivalence relation on* B^* *defined as shown in Table 4. This relation* $\mathcal{S}$ *represents the equivalence classes of relationships among different universities. The relation between* C_1C_2 *and* C_3C_2 *represents that relation between* C_1 *&* C_2 *is same as* C_3 *&* C_2*. Now let us define an intuitionistic fuzzy set* $\widetilde{B}$ *on* B^**:*

$$\widetilde{B} = \{\ \langle C_1C_2, 0.35,\ 0.50\rangle, \langle C_1C_4, 0.40,\ 0.40\rangle, \langle C_2C_5, 0.22, 0.60\rangle, \langle C_3C_1, 0.25, 0.75\rangle, \langle C_3C_2, 0.30, 0.40\rangle,$$

$\langle C_3C_5, 0.28, 0.40\rangle, \langle C_4C_2, 0.50, 0.40\rangle, \langle C_4C_3, 0.15, 0.80\rangle, \langle C_5C_2, 0.18, 0.68\rangle, \langle C_5C_4, 0.24, 0.65\rangle\}.$

In the intuitionistic fuzzy set $\widetilde{B}$, the element, $\langle C_1C_2, 0.35, 0.50\rangle$ shows that C_1 has 35% better than C_2 with some aspect and on behalf of some other virtues C_1 is in worse condition than C_2 with degree 50% i.e. there may be some evidence which shows such non-association and 15% part has been hidden now for this association, it is possible because some information about facility may be suppressed by the service providers that creates abstention part. Now our aim is to find the lower approximation $[\underline{S}\widetilde{B}]$ and upper approximation$[\overline{S}\widetilde{B}]$, respectively.

$$[\underline{S}\widetilde{B}] = \{(C_1C_2, 0.3, 0.5), (C_1C_4, 0.3, 0.5), (C_2C_5, 0.22, 0.60), (C_3C_1, 0.25, 0.75), (C_3C_2, 0.40, 0.40), (C_3C_5, 0.28, 0.40), (C_4C_2, 0.50, 0.40), (C_4C_3, 0.15, 0.80), (C_5C_2, 0.18, 0.68), (C_5C_4, 0.18, 0.68)\},$$

$$[\overline{S}\widetilde{B}] = \{(C_1C_2, 0.4, 0.4), (C_1C_4, 0.4, 0.4), (C_2C_5, 0.6, 0.22), (C_3C_1, 0.75, 0.25), (C_3C_2, 0.40, 0.40), (C_3C_5, 0.40, 0.28), (C_4C_2, 0.4, 0.5), (C_4C_3, 0.80, 0.15), (C_5C_2, 0.65, 0.24), (C_5C_4, 0.65, 0.24)\}.$$

Thus, a rough intuitionistic fuzzy diagraph $\widetilde{D}$ having lower and upper approximations is depicted below.

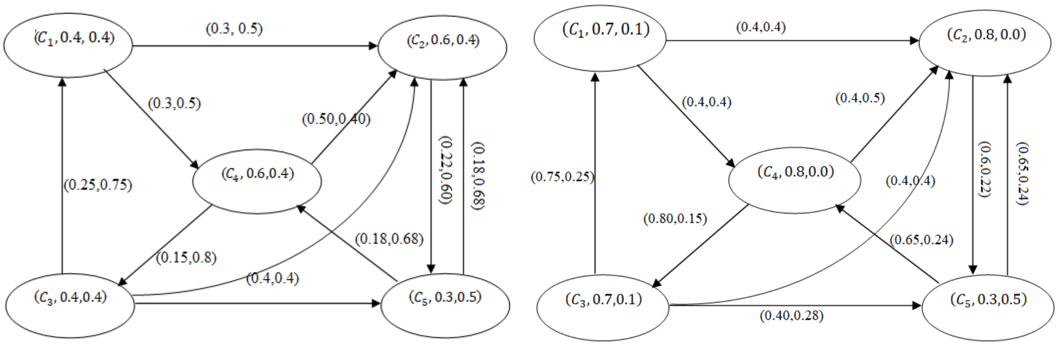


Figure 13: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}$

To get the final decision $[\underline{S}\widetilde{B}]$ and $[\overline{S}\widetilde{B}]$ are dissolved by suitable choice of aggregation. Using ring sum operation:

$$Dm = \left\{\left\langle C_iC_j, \mu_{[\underline{S}\widetilde{B}]}(C_iC_j) + \mu_{[\overline{S}\widetilde{B}]}(C_iC_j) - \mu_{[\underline{S}\widetilde{B}]}(C_iC_j) \cdot \mu_{[\overline{S}\widetilde{B}]}(C_iC_j), v_{[\underline{S}\widetilde{B}]}(C_iC_j) \cdot v_{[\overline{S}\widetilde{B}]}(C_iC_j)\right\rangle C_iC_j \in \Omega^* \times \Omega^*\right\}.$$ *Therefore*

$$Dm = \{(C_1C_2, 0.58, 0.2), (C_1C_4, 0.58, 0.2), (C_2C_5, 0.69, 0.13), (C_3C_1, 0.81, 0.18), (C_3C_2, 0.64, 0.16), (C_3C_5, 0.57, 0.11), (C_4C_2, 0.7, 0.2), (C_4C_3, 0.83, 0.12), (C_5C_2, 0.71, 0.16), (C_5C_4, 0.71, 0.16)\}.$$

$$scor(C_iC_j) = \text{ Score function } \mu_{Dm}(C_iC_j) - v_{Dm}(C_iC_j) + \mu_{Dm}(C_iC_j) \cdot \pi(C_iC_j)$$

$$scorDm = \{(C_1C_2, 0.51), (C_1C_4, 0.51), (C_2C_5, 0.70), (C_3C_1, 0.63), (C_3C_2, 0.61), (C_3C_5, 0.64), (C_4C_2, 0.57), (C_4C_3, 0.75), (C_5C_2, 0.64), (C_5C_4, 0.64)\}.$$

D_{ifr}	C_1	C_2	C_3	C_4	C_5	Sum Scor(R)
C_1	1	0.51		0.51		2.02
C_2		1			0.70	1.7
C_3	0.63	0.61	1		0.64	2.88
C_4		0.61	0.75	1		2.36
C_5		0.64		0.64	1	2.28
Sum Score(C)	1.63	3.37	1.75	2.15	2.34	

Table 4: Similarity relation between criteria based on their scores

Average of lower and upper approximation:

$$\left[\mathcal{R}\widetilde{A}\right] = \{\langle C_1, 0.55,\ 0.25\rangle, \langle C_2, 0.7, 0.2\ \rangle, \langle C_3, 0.55, 0.25\rangle, \langle C_4, 0.7, 0.2\rangle, \langle C_5, 0.3, 0.5\rangle\}.$$

$$scor\left[\mathcal{R}\widetilde{A}\right](C_i) = \mu_{[\mathcal{R}\widetilde{A}]}(C_i) - v_{[\mathcal{R}\widetilde{A}]}(C_i) + \mu_{[\mathcal{R}\widetilde{A}]}(C_i)\,\pi_{[\mathcal{R}\widetilde{A}]}(C_i);$$

$$scor\left(\left[\mathcal{R}\widetilde{A}\right]\right) = \{\langle C_1, 0.41\rangle, \langle C_2, 0.57\ \rangle, \langle C_3,\ 0.41\rangle, \langle C_4, 0.57\rangle, \langle C_5, -\ 0.14\rangle\}.$$

Preference value is defined as

$Pre(C_i) = SumScor(R)\ (C_i) + SumScore(C)\ (C_i) + scor\left(\left[\mathcal{R}\widetilde{A}\right]\right)\ (C_i)$

$Pre(C_i) = \{\langle C_1, 4.06\rangle, \langle C_2, 5.64\ \rangle, \langle C_3,\ 5.04\rangle, \langle C_4, 5.08\rangle, \langle C_5, 4.48\rangle\}$, *$C_2$ is the best option for DM.*

Ranking Order	Name of the Approach
$C_2 \succ C_4 \succ C_3 \succ C_5 \succ C_1$	Proposed rough intuitionistic fuzzy diagraph approach
$C_4 \succ C_1 \succ C_3 \succ C_2 \succ C_5$	Existing rough fuzzy approach

Table 5: Final preference among criteria and comparative analysis

Table 7 shows the algorithm for the proposed method and this can be implemented on MATLAB.

Example 3.2 (Second application)**.** *In this example a decision maker aims to address the best location for industry set up. $\Omega^* = \{F_1, F_2, F_3, F_4, F_5, F_6\}$ be the set of possible locations and $\mathcal{R}$ is the crisp relation showing the same demographic conditions:*

$\mathcal{R}$	F_1	F_2	F_3	F_4	F_5	F_6
F_1	1	0	0	1	0	0
F_2	0	1	0	0	0	1
F_3	0	0	1	0	1	0
F_4	1	0	0	1	0	0
F_5	0	0	1	0	1	0
F_6	0	1	0	0	0	1

Table 6: Crisp relation showing same features

Let the following intuitionistic fuzzy set, showing the attractiveness or features of each location:

$$\widetilde{A} = \{(F_1, 0.6,\ 0.3), (F_2, 0.3,\ 0.5), (F_3, 0.9, 0.0), (F_4, 0.5, 0.4), (F_5, 1.0, 0.0), (F_6, 0.4, 0.5)\}.$$

The lower and upper approximations of $\widetilde{A}$ are found as follows:

$$\left[\underline{\mathcal{R}}\widetilde{A}\right] = \{(F_1, 0.5,\ 0.4), (F_2, 0.3,\ 0.5), (F_3, 0.9, 0.0), (F_4, 0.5, 0.4), (F_5, 0.9, 0.0), (F_6, 0.3,\ 0.5)\},$$
$$\left[\overline{\mathcal{R}}\widetilde{A}\right] = \{(F_1, 0.6,\ 0.3), (F_2, 0.4, 0.5\ ), (F_3, 1, 0.0), (F_4, 0.6, 0.3), (F_5, 1.0,\ 0.0), (F_6, 0.4,\ 0.5)\}.$$

Let $B^* = \{F_1F_2, F_1F_4, F_2F_3, F_2F_6, F_3F_2, F_3F_6, F_4F_2, F_4F_5, F_5F_2, F_5F_3, F_6F_1, F_6F_5\} \subseteq \Omega^* \times \Omega^*$ *and* $\mathcal{S}$ *an equivalence relation on* B^* *defined as shown in Table 9. Now let us define an intuitionistic fuzzy set on* B^**:*

$$\widetilde{B} = \{\ (F_1F_2, 0.30,\ 0.55), (F_1F_4, 0.40,\ 0.40), (F_2F_3, 0.20, 0.60), (F_2F_6, 0.25, 0.75), (F_3F_2, 0.30, 0.50), (F_3F_6, 0.28, 0.40), (F_4F_2, 0.18, 0.40), (F_4F_5, 0.50, 0.30), (F_5F_2, 0.22, 0.68), (F_5F_3, 0.70, 0.15), (F_6F_1, 0.24, 0.45), (F_6F_5, 0.30, 0.55)\}.$$

Then,

$$\left[\underline{\mathcal{S}}\widetilde{B}\right] = \{(F_1F_2, 0.18,\ 0.55), (F_1F_4, 0.40,\ 0.40), (F_2F_3, 0.20, 0.60), (F_2F_6, 0.25, 0.75), (F_3F_2, 0.22, 0.68), (F_3F_6, 0.22, 0.68), (F_4F_2, 0.18, 0.55), (F_4F_5, 0.50, 0.30), (F_5F_2, 0.22, 0.68), (F_5F_3, 0.70, 0.15), (F_6F_1, 0.24, 0.45), (F_6F_5, 0.20, 0.6)\},$$
$$\left[\overline{\mathcal{S}}\widetilde{B}\right] = \{(F_1F_2, 0.30,\ 0.40), (F_1F_4, 0.40,\ 0.40), (F_2F_3, 0.70, 0.15), (F_2F_6, 0.25, 0.75), (F_3F_2, 0.40, 0.40), (F_3F_6, 0.30, 0.40), (F_4F_2, 0.30, 0.40), (F_4F_5, 0.50, 0.30), (F_5F_2, 0.30, 0.40), (F_5F_3, 0.70, 0.15), (F_6F_1, 0.24, 0.45), (F_6F_5, 0.30, 0.55)\}.$$

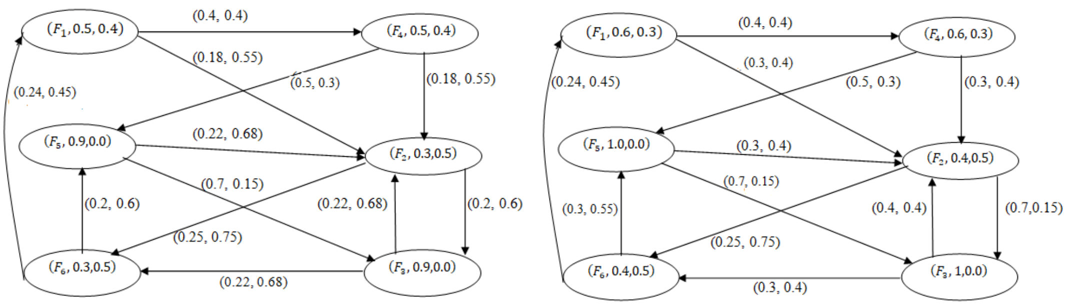


Figure 14: Lower and upper approximations of rough intuitionistic fuzzy diagraph $\widetilde{D}$

$$Dm = \{(F_1F_2, 0.43,\ 0.22), (F_1F_4, 0.64,\ 0.16), (F_2F_3, 0.76, 0.09), (F_2F_6, 0.44, 0.56), (F_3F_2, 0.53, 0.27), (F_3F_6, 0.45, 0.27), (F_4F_2, 0.43, 0.22), (F_4F_5, 0.75, 0.09), (F_5F_2, 0.45, 0.27), (F_5F_3, 0.91, 0.02), (F_6F_1, 0.42, 0.20), (F_6F_5, 0.44, 0.33)\}.$$

$$scor(F_iF_j) = Score\ \ function\ \ \mu_{Dm}\left(F_iF_j\right) - \upsilon_{Dm}\left(F_iF_j\right) + \mu_{Dm}\left(F_iF_j\right) \cdot \pi\left(F_iF_j\right)$$

$$scorDm = \{(F_1F_2, 0.35), (F_1F_4, 0.61), (F_2F_3, 0.78), (F_2F_6, -0.13), (F_3F_2, 0.36), (F_3F_6, 0.31),$$

$(F_4F_2, 0.36), (F_4F_5, 0.78), (F_5F_2, 0.31), (F_5F_3, 0.95), (F_6F_1, 0.38), (F_6F_5, 0.21)\}.$

Table 7: Similarity relation between criteria based on their scores

$\mathcal{R}$	F_1	F_2	F_3	F_4	F_5	F_6	Sum Scor(R)
F_1	1	0.35		0.61			1.96
F_2		1	0.78			-0.13	1.65
F_3		0.36	1			0.31	1.67
F_4		0.36		1	0.78		2.14
F_5		0.31	0.95		1		2.26
F_6	0.38				0.61	1	1.99
Sum Score(C)	1.38	2.38	2.73	1.61	2.39	1.18	

The lower and upper approximations of $\widetilde{A}$ are found as follows:

Average of lower and upper approximation:

$$\left[\mathcal{R}\widetilde{A}\right] = \{\langle F_1, 0.55,\ 0.35\rangle, \langle F_2, 0.35, 0.5\ \rangle, \langle F_3, 0.95, 0.0\rangle, \langle F_4, 0.55, 0.35\rangle, \langle F_5, 0.95, 0.0\rangle, \langle F_6, 0.35, 0.5\rangle\}.$$

$$scor(F_i) = \mu_{[\mathcal{R}\widetilde{A}]}(F_i) - v_{[\mathcal{R}\widetilde{A}]}(F_i) + \mu_{[\mathcal{R}\widetilde{A}]}(F_i) * \pi_{[\mathcal{R}\widetilde{A}]}(F_i);$$

$$scor\left(\left[\mathcal{R}\widetilde{A}\right]\right) = \{\langle F_1, 0.26\rangle, \langle F_2, -0.10\ \rangle, \langle F_3,\ 1\rangle, \langle F_4, 0.26\rangle, \langle F_5, 1\rangle, \langle F_6,\ -0.10\rangle\}.$$

Preference value is defined as

$$Pre(F_i) = Sum\ \ Scor(R)\ \ (F_i) + Sum\ \ Score(C)\ \ (F_i) + scor\left(\left[\mathcal{R}\widetilde{A}\right]\right)\ (F_i)$$

$$Pre(F_i) = \langle F_1, 4.18\rangle, \langle F_2, 3.2\ \rangle, \langle F_3,\ 4.34\rangle, \langle F_4, 4.54\rangle, \langle F_5, 5.52\rangle, \langle F_6,\ 3.88\rangle.$$

F_5 is the best choice for the decision maker.

Table 8: Final preference among criteria and comparative analysis

Ranking Order	Name of the Approach
$F_5 \succ F_4 \succ F_3 \succ F_1 \succ F_6 \succ F_2$	Proposed rough intuitionistic fuzzy diagraph approach
$F_5 \succ F_4 \succ F_1 \succ F_3 \succ F_2 = F_6$	Existing rough fuzzy approach

4. Conclusion

Rough set theory is a mathematical tool to deal with incomplete and vague information. Intuitionistic fuzzy set theory studies the vague, imperfect, and ambiguous in decision-making problems impressively. We have applied these theories by hybridizing them in this work. In this article, we choose different composition rules dealing with lower approximation and upper approximations for rough intuitionistic fuzzy sets, define rough intuitionistic fuzzy diagraphs, propose generalized strong product, generalized strong product, generalized lexicographic product and symmetric differences, etc. Moreover, two theories, rough set and intuitionistic fuzzy set, have been combined, and a framework for modeling and processing incomplete information in information systems has been proposed. This framework is tested for two decision-making problems in different contexts. The numerical computations provide practical viability and visualization to assess potential outcomes of the proposed technique. For future perspective, the proposed approach can be extended by hybridizing interval-valued intuitionistic fuzzy sets and rough sets.

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