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Improvement of Exponential Chain-Type Estimators of Population Mean Under Double Sampling

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Abstract

Three exponential chain-type regressions, ratios, and population-average regression estimators utilising information from two auxiliary variables under double sampling are proposed in this study. Up to the first approximation order, details of behaviours are performed with bias and average square errors in mind. We compared it to other popular chain type estimating systems of the same kind and offered survey professionals some suggestions on how to improve their own methods.

Keywords: Two-phase; auxiliary information; bias; mean square error.
2020 Mathematics Subject Classification: 62D05.

1. Introduction

It is well-established in survey sampling to use auxiliary information to improve performance (gains) compared to an estimator that does not obtain such information. The ratio, product, and regression estimators can be used to estimate the mean of the search variable if you know the population mean of the auxiliary variable. "Whenever the information of an auxiliary variable is unknown, it is common practice to use a two-phase random sampling technique. This technique delivers a cost-effective approximation of the unknown population mean in the first step of sampling. Chand [1] proposed a way to combine the accessible data from secondary variables with the main study variable by using known population averages of another auxiliary variable in the first phase sample. Among the writers to be mentioned are Kiregyera [2, 3], Mukherjee et al. [4], Srivastava et al. [5], Upadhyaya et al. [6], Singh [7], Singh and Singh [8], and many more. Pradhan [9], Singh and Majhi [10], Singh and Khalid [11], Khan and Hossian [12], and Singh and Upadhyaya [13] all suggested regression and chain-type estimates that relied on two auxiliary variables in Singh et al. [14], Khare and Kumar [15] used chain ratio in regression estimators to estimate the population mean in two-phase sampling

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with non-reactions. Majhi and Singh [16] developed chain regression specifically for use in regression and to estimate population averages for research variables. In order to estimate population averages more accurately than state-of-the-art estimates of the same kind, this work seeks to develop efficient chain-type estimators for two-stage sampling.

2. Two-Phase Sampling Scheme

The two-stage sampling scheme considers the finite population of $N = 1, 2, 3, \ldots, N$, $U = (U_1, U_2, U_3, \ldots, U_N)$ of size of N and $U_i (i = 1, 2, 3, \ldots, N)$ is a simulation variable, and i is an observation variable for the population y unit, i is the observation variable for the population unit i Also let x_i and z_i be the observations on the auxiliary variables x and z respectively for the i$^{\text{th}}$ unit of the population $U_i (i = 1, 2, 3, \ldots, N)$. Here x and z be called as first and second auxiliary variables respectively such that y is highly correlated with x while in compare to x, it is remotely correlated with z (i.e. $\rho_{yx} > \rho_{yz}$). Further let $\overline{Y} = \sum\limits_{i=1}^{N} y_i, \overline{X} = \sum\limits_{i=1}^{N} x_i$ and $\overline{Z} = \sum\limits_{i=1}^{N} z_i$ be the population means of y, x and z respectively. When the population mean $\overline{X}$ of x is unknown but information on z is available on all the units of the population, we use the two phase sampling be such that (a) the first phase sample $\mathrm{s}' \, (\mathrm{s}' \subset U)$ of size $n' \, (n' < N)$ is drawn from the population by SRSWOR to observe auxiliary variables x and z only. (b) From the first phase sample s', we select a second phase sample $s \, (s \subset s')$ of size $n \, (n < n')$ by SRSWOR to the characteristic y under study.

3. Estimators Using One Auxiliary Variable

Equation (1) shows the two-phase sample ratio estimator for the population mean Y proposed by Sukhatme [17] in cases when the initial knowledge of the population mean of the auxiliary variable X is lacking. Just below this equation, up to the first approximation, you can find an expression for the mean square error of the two-phase sampling ratio estimator (double sampling) in (2).

$$\overline{y}_{rd} = \frac{\overline{y}}{\overline{x}} \overline{x}' \tag{1}$$

$$M\left(\overline{y}_{rd}\right) = \overline{Y}^2 \left[f_1 C_y^2 + f_3 \left(C_x^2 - 2\rho_{yx} C_y C_x\right)\right] \tag{2}$$

In two-phase sampling schemes, linear regression estimators are defined by (3) and the expressions of their average square errors up to the first approximation order are defined by (4).

$$\overline{y}_{lrd} = \overline{y} + b_{yx}(n) \left(\overline{x}' - \overline{x}\right) \tag{3}$$

$$M\left(\overline{y}_{lrd}\right) = S_y^2 \left[f_1 \left(1 - \rho_{yx}^2\right) + f_2 \rho_{yx}^2\right] \tag{4}$$

where, $b_{yx}(n)$ is the sample regression coefficient y on x is calculated based on s and calculated from data, $\overline{y} = \frac{1}{n} \sum\limits_{i \in s} y_i, \bar{x} = \frac{1}{n} \sum\limits_{i \in s} x_i, \bar{x}' = \frac{1}{n'} \sum\limits_{i \in s} x_i, f_1 = \left(\frac{1}{n} - \frac{1}{N}\right), f_2 = \left(\frac{1}{n'} - \frac{1}{N}\right), f_3 = \left(f_1 - f_2\right) =$

$\left(\frac{1}{n}-\frac{1}{n'}\right) S_x^2 = \frac{1}{N-1}\sum\limits_{i=1}^{N}\left(x_i-\overline{X}\right)^2, S_y^2 = \frac{1}{N-1}\sum\limits_{i=1}^{N}\left(y_i-\overline{Y}\right)^2, C_x = \frac{S_x}{\overline{X}}, C_y = \frac{S_y}{\overline{Y}}$ and ρ_{yx} be the correlation coefficient between variables y and x.

4. Estimators based on Two Auxiliary Variables

Chand [1] proposed chain ratio-type estimator of population mean $\overline{Y}$ defined by

$$\overline{y}_{rc} = \frac{\overline{y}}{\overline{x}}\frac{\overline{x}'}{\overline{z}'}\overline{Z} \tag{5}$$

The mean square error of the estimator $\bar{y}_{rc}$ up to the first order of approximations is expressed as

$$M\left(\overline{y}_{rc}\right) = \overline{Y}^2\left[f_1C_y^2 + f_3\left(C_x^2 - 2\rho_{yx}C_yC_x\right) + f_2\left(C_z^2 - 2\rho_{yz}C_yC_z\right)\right] \tag{6}$$

where, $C_z = \frac{S_z}{\overline{Z}}, S_z^2 = \frac{1}{N-1}\sum\limits_{i=1}^{N}\left(z_i-\overline{Z}\right)^2$ and ρ_{yz} be correlation coefficient between variables y and z. Kiregyera [2,3] extended the work of Chand [1] and proposed chain-type ratio to regression, regression to ratio and regression to regression estimators of population mean of $\overline{Y}$, which are defined by

$$\overline{y}_{k1} = \frac{\overline{y}}{\overline{x}}\left[\overline{x}' + b_{xz}\left(n'\right)\left(\overline{Z}-\overline{z}'\right)\right] \tag{7}$$

$$\overline{y}_{k2} = \overline{y} + b_{yx}(n)\left(\overline{x}'_{rd}-\overline{x}\right); \overline{x}'_{rd} = \frac{\overline{x}'}{\overline{z}'}\overline{Z} \tag{8}$$

$$\overline{y}_{k3} = \overline{y} + b_{yx}(n)\left(\overline{x}'_{ld}-\overline{x}\right); \overline{x}'_{ld} = \left[\overline{x}' + b_{xz}\left(n'\right)\left(\overline{Z}-\overline{z}'\right)\right] \tag{9}$$

The mean square error of the estimators $\overline{y}_{k1}, \overline{y}_{k2}$ and $\overline{y}_{k3}$ up to first order of approximations are derived as

$$M\left(\overline{y}_{k1}\right) = \overline{Y}^2\left[f_3\left(C_x^2 + C_y^2 - 2\rho_{yx}C_yC_x\right) + f_2C_y^2 + f_2\rho_{xz}C_x\left(\rho_{xz}C_x - 2\rho_{yz}C_y\right)\right] \tag{10}$$

$$M\left(\bar{y}_{k2}\right) = \bar{Y}^2C_y^2\left[f_1\left(1-\rho_{yx}^2\right) + f_2\left(\rho_{yx}^2 + \rho_{yx}^2\frac{C_z^2}{C_x^2} - 2\rho_{yx}\rho_{yz}\frac{C_z}{C_x}\right)\right] \tag{11}$$

$$M\left(\bar{y}_{k3}\right) = \bar{Y}^2C_y^2\left[f_3\left(1-\rho_{yx}^2\right) + f_2\left(1 + \rho_{xz}^2\rho_{yx}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}\right)\right] \tag{12}$$

where, $b_{xz}\left(n'\right)$ is the sample regression coefficient of the variable x on z calculated from the data based on s', $\overline{z}' = \frac{1}{n'}\sum\limits_{i\in s'} z_i$ and ρ_{xz} be correlation coefficient between variables x and z.

5. Suggested Estimators

Motivated with the work of chain-type estimators in two-phase sampling schemes, we suggest three different exponential chain-type estimators of population mean $\bar{Y}$ of the study variable y. The suggested estimators are defined as

$$T_d^{(1)} = \bar{y} + b_{yx}(n)\left[\bar{x}'\exp\left(\frac{\bar{Z}-\bar{z}}{\bar{Z}+\bar{z}}\right) - \bar{x}\exp\left(\frac{\bar{Z}-\bar{z}}{\bar{Z}+\bar{z}}\right)\right] \tag{13}$$

$$T_d^{(2)} = \bar{y}\exp\left(\frac{\bar{x}'-\bar{x}}{\bar{x}'+\bar{x}}\right)\exp\left(\frac{\bar{Z}-\bar{z}'}{\bar{Z}+\bar{z}'}\right) \tag{14}$$

$$T_d^{(3)} = \bar{y}\exp\left(\frac{\bar{Z}-\bar{z}}{\bar{Z}+\bar{z}}\right) + b_{yx}(n)\left[\bar{x}'\exp\left(\frac{\bar{Z}-\bar{z}'}{\bar{Z}+\bar{z}'}\right) - \bar{x}\exp\left(\frac{\bar{Z}-\bar{z}}{\bar{Z}+\bar{z}}\right)\right] \tag{15}$$

where, $b_{yx}(n)$ is the sample regression coefficients between the variables y on x and based on the sample size n.

6. Properties of the Estimators $T_d^{(i)}(i = 1,2,3)$

Theorem 6.1. *The bias of the estimators $T_d^{(i)}(i = 1,2,3)$ up to the first order of approximations are derived as*

$$B\left\{T_d^{(1)}\right\} = f_3\left[-\frac{3}{8}\frac{\bar{X}}{\bar{Z}^2}\frac{\alpha_{002}\alpha_{110}}{\alpha_{200}} + \frac{1}{2}\left\{\frac{1}{\bar{z}}\frac{\alpha_{101}\alpha_{110}}{\alpha_{200}} + \frac{\bar{X}}{\bar{z}}\frac{\alpha_{111}}{\alpha_{200}} - \frac{\bar{X}}{\bar{z}}\frac{\alpha_{201}\alpha_{110}}{\alpha_{200}^2}\right\} + \left(\frac{\alpha_{300}\alpha_{110}}{\alpha_{200}^2} - \frac{\alpha_{201}}{\alpha_{200}}\right)\right] \tag{16}$$

$$B\left\{T_d^{(2)}\right\} = \left[\frac{3}{8}f_2\frac{\overline{y}}{\overline{z}^2}\alpha_{002} - \frac{1}{2}\left\{f_3\frac{\alpha_{110}}{\overline{X}} + f_2\frac{\alpha_{011}}{\overline{Z}}\right\} - \frac{1}{8}\left\{f_1\frac{\overline{Y}}{\overline{X}^2}\alpha_{200} + 3f_2\frac{\overline{Y}}{\overline{X}^2}\alpha_{200}\right\}\right] \tag{17}$$

and

$$\begin{aligned} B\left\{T_d^{(3)}\right\} = \Bigg[&f_1\left(\frac{3}{8}\frac{\overline{Y}}{\overline{Z}^2}\alpha_{002} - \frac{1}{2}\frac{\alpha_{011}}{\overline{Z}}\right) \\ &+f_3\beta_{yx}\left(\frac{\alpha_{300}}{S_x^2} + \frac{1}{2}\frac{\alpha_{101}}{\overline{Z}} + \frac{1}{2}\frac{\alpha_{111}}{\overline{Z}S_{yx}} - \frac{3}{8}\frac{\overline{X}}{\overline{Z}^2}\alpha_{002} - \frac{\alpha_{210}}{S_{yx}} - \frac{1}{2}\frac{\overline{X}}{\overline{Z}}\frac{\alpha_{201}}{S_x^2}\right)\Bigg] \end{aligned} \tag{18}$$

where, $\alpha_{rst} = E\left[\left(x_i - \overline{X}\right)^r\left(y_i - \overline{Y}\right)^s\left(z_i - \overline{Z}\right)^t\right]; (r,s,t) \geq 0$ are integers.

Theorem 6.2. *The mean square errors of the estimators $T_d^{(i)}(i = 1,2,3)$ up to $o\left(n^{-1}\right)$ are derived as*

$$M\left\{T_d^{(1)}\right\} = \left[f_1\left(1 + \frac{1}{2}\rho_{yx}^2\right) + f_3\left\{\rho_{yx}\rho_{yz} - \rho_{yx}^2\left(1 + \rho_{xz}^2\right)\right\}\right]S_y^2 \tag{19}$$

$$M\left\{T_d^{(2)}\right\} = \overline{Y}^2\left[f_1C_y^2 + f_3\left(\frac{C_x^2}{4} - \rho_{yx}C_yC_x\right) + f_2\left(\frac{C_z^2}{4} - \rho_{yz}C_yC_z\right)\right] \tag{20}$$

and

$$\begin{aligned} M\left\{T_d^{(3)}\right\} = \overline{Y}^2\Bigg[&f_1\left(C_y^2 + \frac{1}{4}C_z^2 - \rho_{yz}C_yC_z\right) \\ &+f_3\left\{\rho_{yx}^2c_y^2\left(\frac{1}{4}\frac{c_z^2}{c_x^2} - 1\right) + c_y^2\frac{c_z}{c_x}\left(\rho_{yx}\rho_{yz} - \rho_{yx}^2\rho_{xz}\right) + \rho_{yx}\rho_{xz}c_yc_z\right\}\Bigg] \end{aligned} \tag{21}$$

7. Comparison of the Proposed Estimators $T_d^{(i)} (i = 1, 2, 3)$

In this section we compare the proposed estimators $T_d^{(i)} (i = 1, 2, 3)$ with respect to the estimators $\overline{y}_{rd}, \overline{y}_{lrd}, \overline{y}_{rc}, \overline{y}_{k1}, \overline{y}_{k2}$ and $\overline{y}_{k3}$. Preference zones of the estimators $T_d^{(i)} (i = 1, 2, 3)$ are explored and shown below:

(i) $T_d^{(i)} (i = 1, 2, 3)$ are better than $\overline{y}_{rd}$ if $M\left\{T_d^{(i)}\right\} \leq M\left(\overline{y}_{rd}\right)$, which give

$$\left[f_1\rho_{yx}^2 + 2f_3\left(\rho_{yx}\rho_{yz} - \rho_{yx}^2 - \rho_{yx}^2\rho_{xz} + 2\rho_{yx} - 1\right)\right] \leq 0; \quad (\text{for } i = 1) \tag{22}$$

$$f_2\left(\frac{c_z^2}{4} - \rho_{yz}c_yc_z\right) \leq f_3\left(\frac{3}{4}c_x^2 - \rho_{yx}c_yc_x\right); \quad (\text{for } i = 2) \tag{23}$$

and

$$f_1\left(\frac{c_z^2}{4} - \rho_{yz}c_yc_z\right) \leq f_3\left(c_x^2 - 2\rho_{yx}c_yc_x - A\right); \quad (\text{for } i = 3) \tag{24}$$

where, $A = \left\{\rho_{yx}^2c_y^2\left(\frac{1}{4}\frac{c_z^2}{c_x^2} - 1\right) + c_y^2\frac{c_z}{c_x}\left(\rho_{yx}\rho_{yz} - \rho_{yx}^2\rho_{xz}\right) + \rho_{yx}\rho_{xz}c_yc_z\right\}$.

(ii) $T_d^{(i)} (i = 1, 2, 3)$ are preferable over $\overline{y}_{lrd}$ if $M\left\{T_d^{(i)}\right\} \leq M\left(\overline{y}_{lrd}\right)$, which converge to

$$f_1\left(\frac{3}{2}\rho_{yx}^2\right) + f_3\left\{\rho_{yx}\rho_{yz} - \rho_{yx}^2\left(1 + \rho_{xz}\right)\right\} \leq f_2\rho_{yx}^2; \quad (\text{for } i = 1) \tag{25}$$

$$f_3\left(\frac{c_x^2}{4} - \rho_{yx}c_yc_x + c_y^2\rho_{yx}^2\right) \leq f_2\left(\rho_{yz}c_yc_z - \frac{c_y^2}{4}\right); \quad (\text{for } i = 2) \tag{26}$$

and

$$f_3\left(A + c_y^2\rho_{yx}^2\right) \leq f_1\left(\rho_{yz}c_yc_z - \frac{c_y^2}{4}\right); \quad (\text{for } i = 3) \tag{27}$$

(iii) $T_d^{(i)} (i = 1, 2, 3)$ will dominate $\overline{y}_{rc}$ if $M\left\{T_d^{(i)}\right\} \leq M\left(\overline{y}_{rc}\right)$, subsequently we get

$$f_1\left(\frac{1}{2}\rho_{yx}^2\right) + f_3\left\{\rho_{yx}\left(\rho_{yz} - \rho_{yx} - \rho_{yx}\rho_{xz} + 2\right) - 1\right\} \leq f_2\left(1 - 2\rho_{yz}\right); \quad (\text{for } i = 1) \tag{28}$$

$$f_3\left(\frac{3c_x^2}{4} - \rho_{yx}c_yc_x\right) \geq f_2\left(\rho_{yz}c_yc_z - \frac{3c_z^2}{4}\right); \quad (\text{for } i = 2) \tag{29}$$

and

$$f_2\left(\rho_{yz}c_yc_z - \frac{3c_z^2}{4}\right) \leq f_3\left\{c_x^2 + \rho_{yz}c_yc_z - \rho_{yx}c_yc_x - \frac{c_z^2}{4} - A\right\}; \quad (\text{for } i = 3) \tag{30}$$

(iv) $T_d^{(i)} (i = 1, 2, 3)$ are more efficient than $\overline{y}_{k1}$ if $M\left\{T_d^{(i)}\right\} \leq M\left(\overline{y}_{k1}\right)$, which gives

$$\begin{aligned} &f_1\left(1 + \frac{1}{2}\rho_{yx}^2\right) + f_3\left\{\rho_{yx}\rho_{yz} - \rho_{yx}^2\left(1 + \rho_{xz}\right)\right\} \\ &\qquad \leq \left\{2f_3\left(1 - \rho_{yx}\right) + f_2\left(1 + \rho_{xz}^2 - 2\rho_{xz}\rho_{yz}\right)\right\}; \quad (\text{for } i = 1) \end{aligned} \tag{31}$$

$$f_3\left(\rho_{yx}c_yc_x-\frac{3c_x^2}{4}\right)\leq f_2\left\{\rho_{xz}c_x\left(\rho_{xz}c_x-2\rho_{yz}c_y\right)-\frac{c_z^2}{4}+\rho_{yz}c_yc_z\right\};\quad (\text{for } i=2) \tag{32}$$

and

$$f_2\left(\frac{c_z^2}{4}+2\rho_{xz}\rho_{yz}c_yc_x-\rho_{yz}c_yc_z-\rho_{xz}^2c_x^2\right)\leq f_3\left\{c_x^2+\rho_{yz}c_yc_z-\frac{c_z^2}{4}-2\rho_{yx}c_yc_x-A\right\};\quad (\text{for } i=3) \tag{33}$$

(v) $T_d^{(i)}(i=1,2,3)$ are preferable over $\overline{y}_{k2}$ if $M\left\{T_d^{(i)}\right\}\leq M\left(\overline{y}_{k2}\right)$, which satisfies the condition

$$f_3\left[\rho_{yx}\left\{\frac{3}{2}\rho_{yx}+\rho_{yz}-\rho_{yx}\left(1+\rho_{xz}\right)\right\}\right]\leq f_2\left(\rho_{yx}^2\left(\frac{c_z^2}{c_x^2}-\frac{1}{2}\right)-\rho_{yx}\rho_{yz}\frac{c_z}{c_x}\right);\quad (\text{for } i=1) \tag{34}$$

$$f_3\left(\frac{c_x}{2}-\rho_{yx}c_y\right)^2\leq f_2\left\{c_y^2\left(\rho_{yx}^2\frac{c_z^2}{c_x^2}-\rho_{yx}\rho_{yz}\frac{c_z}{c_x}\right)+\rho_{yz}c_yc_y+\frac{c_z^2}{4}\right\};\quad (\text{for } i=2) \tag{35}$$

and

$$f_3\left[\frac{c_z^2}{4}-\rho_{yz}c_yc_z+A+\rho_{yx}^2c_y^2\right]\leq f_2\left[c_y^2\left(\rho_{yx}^2\frac{c_z^2}{c_x^2}-2\rho_{yx}\rho_{yz}\frac{c_z}{c_x}\right)-\left(\frac{c_z^2}{4}-\rho_{yz}c_yc_z\right)\right];\quad (\text{for } i=3) \tag{36}$$

(vi) $T_d^{(i)}(i=1,2,3)$ will dominate $\overline{y}_{k3}$ if $M\left\{T_d^{(i)}\right\}\leq M\left(\overline{y}_{k3}\right)$, which subsequently holds

$$f_1\left(1+\frac{1}{2}\rho_{yx}^2\right)+f_2\left(2\rho_{yx}\rho_{yz}\rho_{xz}-\rho_{yx}^2\rho_{xz}^2-1\right)\leq f_3\left[1+\rho_{yx}\left(\rho_{yx}\rho_{xz}-\rho_{yz}\right)\right];\quad (\text{for } i=1) \tag{37}$$

$$\begin{aligned}f_1c_y^2+f_2&\left[\frac{c_z^2}{4}-\rho_{yx}c_yc_z-\left(1+\rho_{yx}^2\rho_{xz}^2-2\rho_{yx}\rho_{yz}\rho_{zx}\right)c_y^2\right]\\&\leq f_3\left[\left(1-\rho_{yx}^2\right)c_y^2+\left(\rho_{yx}c_yc_x-\frac{c_x^2}{4}\right)\right];\quad (\text{for } i=2)\end{aligned} \tag{38}$$

and

$$f_1\left(c_y^2+\frac{c_z^2}{4}-\rho_{yz}c_yc_z\right)-f_2c_y^2\left(1+\rho_{yx}^2\rho_{xz}^2-2\rho_{yx}\rho_{yz}\rho_{zx}\right)\leq f_3\left[c_y^2\left(1-\rho_{yx}^2\right)-A\right] \tag{39}$$

8. Analysis of Empirical Results

To examine the performances of the proposed estimators $T_d^{(i)}(i=1,2,3)$, we have computed the percent relative efficiencies of $T_d^{(i)}(i=1,2,3)$ with respect to $\overline{y}_{rd}, \overline{y}_{lrd}, \overline{y}_{rc}$ and $\overline{y}_{k1}$ based on three natural populations and presented in Table 1. The percent relative efficiencies of the estimators $T_d^{(i)}$ with respect to an estimator δ are defined as $PRE=\left[\frac{MSE(\delta)}{MSE\left\{T_d^{(i)}\right\}}\right]\times 100;\ (i=1,2,3)$.

Data I- Source: Srivastava et al. [5]

The variables are y: the measurement of weight of children, x : mid-arm circumference of children and z : skull circumference of children.

$N = 55, n = 18, n' = 30, \overline{Y} = 17.08 \text{ kg}, \overline{X} = 16.92 \text{ cm}, \overline{Z} = 50.44 \text{ cm}, C_y^2 = 0.0161, C_x^2 = 0.0049, C^2_{z=0.0007}, \rho_{yx} = 0.54, \rho_{yz} = 0.51$ and $\rho_{xz} = -0.08$.

Data II- Source: Anderson [18]

The variables are y : head length of second son, x : head length of first son,z : head breadth of first son.

$N = 25, n = 7, n' = 10, \overline{Y} = 183.72, \overline{X} = 185.72, \overline{Z} = 151.12, C_y = 0.0546, C_x = 0.0526, C_{z=0.0488}, \rho_{yx} = 0.7108, \rho_{yz} = 0.6932$ and $\rho_{xz} = 0.7346$.

Data III- Source: Murthy [19]

The variables are y : area under wheat in 1964, x : area under wheat in 1963 and z : cultivated area in 1961.

$N = 34, n = 7, n' = 10, \overline{Y} = 199.44$ acre, $\overline{X} = 208.89$ acre, $\overline{Z} = 747.59$ acre, $C_y^2 = 0.5673, C_x^2 = 0.5191, C_z^2 = 0.3527$, $\rho_{yx} = 0.9801, \rho_{yz} = 0.9043$ and $\rho_{xz} = 0.9097$.

9. Conclusions

From Table 1 it clear that the suggested estimators $T_d^{(i)} (i = 1, 2, 3)$ are more efficient over the estimators $\overline{y}_{rd}, \overline{y}_{lrd}, \overline{y}_{rc}$ and $\overline{y}_{k1}$ for all the data set. Therefore, the suggested estimators may be recommended for its real life applications to survey statistician.

	PRE FOR POPULATION I			
Estimators	$\overline{y}_{\text{rd}}$	$\overline{y}_{\text{lrd}}$	$\overline{y}_{\text{rc}}$	$\overline{y}_{k1}$
$T_d^{(1)}$	131.45	113.56	112.06	101.07
$T_d^{(2)}$	138.27	125.46	123.31	111.49
$T_d^{(3)}$	135.23	118.06	115.41	103.41
	PRE FOR POPULATION II			
$T_d^{(1)}$	133.10	112.50	109.08	101.57
$T_d^{(2)}$	145.05	126.18	124.10	113.34
$T_d^{(3)}$	134.28	117.26	115.51	103.23
	PRE FOR POPULATION III			
$T_d^{(1)}$	112.1981	108.6203	109.2922	100.03
$T_d^{(2)}$	115.45	113.07	112.73	101.23
$T_d^{(3)}$	103.24	102.08	101.75	92.32

Table 1: The percent relative efficiencies (PRE) based on the above three data sets of the proposed estimators $T_d^{(i)} (i = 1, 2, 3)$ with respect to the estimators $\overline{y}_{rd}, \overline{y}_{lrd}, \overline{y}_{rc}$ and $\overline{y}_{k1}$.

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