International Journal of
Mathematics And its Applications
ISSN: 2347-1557

Int. J. Math. And Appl., **14(2)**(2026), 153–165
Available Online: http://ijmaa.in

Influence of Crack Depth on Stress Distribution in a Semi-Infinite Elastic Medium Under a Moving Load

Biru Mandal[1,*], Rajesh Kumar Tiwary[1]

[1] *Department of Mathematics, Binod Bihari Mahto Koyalanchal University, Dhanbad, Jharkhand, India*

Abstract

This study investigates the influence of crack depth on stress distribution in a semi-infinite elastic medium under a moving load. A buried Mode-III crack is considered at varying depths (0.1 m to 3.0 m), and the load moves with constant velocity ranging from 10 m/s to 50 m/s. Using Fourier transform techniques, the displacement and stress fields are derived. The results show that increasing crack depth significantly reduces surface stress and alters the stress distribution pattern. Validation with existing literature confirms model accuracy. This model is applicable to fault-line analysis and underground rail/road evaluations. A time-dependent stress analysis is also performed to highlight the dynamic interaction between moving loads and buried cracks, offering practical insights for subsurface infrastructure monitoring and failure prediction.

Keywords: Crack Depth; Moving Load; Semi-Infinite Elastic Medium; Mode-III Crack; Fourier Transform; Stress Distribution; Buried crack.

1. Introduction

The study of wave propagation and stress concentration in elastic media due to dynamic loading has significant implications in engineering and geophysical applications. In particular, the interaction of moving loads with internal defects such as cracks is critical in understanding failure mechanisms in structures like pavements, tunnels, rock layers, and buried pipelines. Subsurface cracks, often invisible from the surface, can lead to localized stress amplification and long-term structural degradation when subjected to moving vehicular or seismic loads. Classical elasticity theory, especially under Mode-III (anti-plane shear) deformation, provides a tractable framework to analyze such problems. The use of semi-infinite models allows for simplified yet realistic representation of ground or structural media. Foundational works such as those by Timoshenko and Goodier [1], Sneddon [2], and Achenbach [3] provide the theoretical base for elastic wave modeling and stress field formulation using analytical techniques like Fourier transforms.

*Corresponding author (mandalbiru7@gmail.com)

Previous studies have explored the influence of moving loads on irregular surfaces [4, 5], and some have examined crack-wave interactions under anti-plane shear deformation [6, 7]. However, most of these models assume the crack to be located at a fixed depth or on the surface. The depth variation of internal cracks - a physically realistic and engineering-relevant feature - has not been incorporated into these classical models. In real-world conditions, cracks may exist at various depths due to material heterogeneity, pre-existing flaws, or excavation effects. The depth at which a crack is located plays a key role in controlling the magnitude and distribution of dynamic stresses generated by surface loads. A shallow crack may cause high surface stress concentration, while a deeper crack may interact with shear waves differently due to propagation delay and energy dispersion [8]. To the best of the authors' knowledge, no prior analytical model has incorporated variable crack depth in a semi-infinite elastic medium subjected to a moving point load under Mode-III deformation. Addressing this gap, the present study develops a mathematical model to evaluate the effect of crack depth variation on shear stress distribution in a semi-infinite homogeneous elastic medium. A buried Mode-III crack is considered at different depths ranging from 0.1 m to 3.0 m, and a point load moves at constant velocity (10–50 m/s) along the surface. The resulting out-of-plane displacement and shear stress are computed using Fourier transform methods [2, 6]. This analysis offers important insights into depth-dependent stress amplification and wave-crack interaction, contributing to the predictive modeling of failure zones in subsurface systems. The findings are expected to benefit applications such as rail and road subsurface monitoring, fracture mechanics, and seismic safety assessments.

The proposed model has direct applicability in engineering scenarios where buried cracks affect structural integrity. Examples include high-speed railway foundations, underground tunnels, and fault zones in seismic regions. In such cases, identifying how stress varies with crack depth can guide inspection protocols and preventive maintenance. The dynamic interaction modeled here also aids in predicting subsurface failure zones that are otherwise difficult to detect through surface measurements.

2. Literature Review

The dynamic interaction of moving loads with cracks in elastic media is a crucial problem in geomechanics, fracture mechanics, and transportation engineering. Foundational theoretical frameworks were established by Timoshenko and Goodier [1], Sneddon [2], and Achenbach [3], who provided analytical tools for stress field modeling and wave propagation in elastic solids. Their methods-particularly the use of Fourier transforms-have enabled accurate solutions in Mode-III crack problems. Subsequent researchers extended this theory to moving load problems. Chattopadhyay and Chatterjee [4] studied the propagation of shear waves in an ice sheet floating on water under a moving load, considering surface irregularity. Singh [5] examined the dynamic stress behavior in an elastic half-space with a parabolic surface under a moving point load. However, these studies focused on surface effects only and did not include crack-related phenomena. In the domain of fracture mechanics, Mandal [17] developed a semi-analytical model for a semi-infinite Mode-III crack inside an elastic half-space subjected to a moving load. While this study provided valuable insights, it considered only a fixed-depth crack. Similarly, Chaudhary [7] investigated shear stress in a piezoelectric layer with a buried crack under moving load but did not analyze crack depth variation. Sahu and Saini [8] discussed the influence of buried inhomogeneities on surface wave scattering, reinforcing the importance of subsurface features on surface responses. Recently Singh [8, 18] have studied moving load and wave propagation through viscoelastic and other smart materials.

Despite these contributions, none of the existing studies analytically investigate the variation in crack depth and its effect on shear stress response under a moving load. This limitation is significant in real-world engineering scenarios such as fault-line detection, subsurface railbed damage, or tunnel inspections, where the depth of cracks influences stress concentration and failure zones. To the best of the authors' knowledge, no prior analytical study has incorporated crack depth as a parametric variable in a Mode-III semi-infinite elastic medium under a dynamic moving load.

The present study fills this gap by formulating an anti-plane shear (Mode-III) model with a buried horizontal crack of variable depth subjected to a constant velocity surface load. The stress field τ_{xz} is evaluated across multiple crack depths (0.1–3.0 m) and velocities (10–50 m/s), revealing the depth sensitivity of stress amplification and wave interaction.

3. Problem Formulation

We consider a semi-infinite, homogeneous, isotropic elastic medium occupying the region $z \geq 0$, and extending infinitely in the x-direction. The medium is subjected to Mode-III (anti-plane shear) deformation, governed by a scalar displacement field $u(x, z, t)$, representing the out-of-plane displacement. A buried Mode-III crack of length $2a$ is embedded horizontally at a depth $z = d$ below the free surface. The crack lies along the line segment $x \in [-a, a]$ at $z = d$, and is assumed to be traction-free on both faces. A point load moves on the surface $z = 0$ with a constant velocity v along the positive x-direction. The moving load is modeled using a Dirac delta function, representing an impulsive shear force applied in the out-of-plane direction at the moving point $x = vt$. This creates a traveling shear wave in the medium, which interacts with the buried crack. The material is assumed to obey linear elastic behavior, with the following physical properties:

- Shear modulus: $\mu = 30\,\text{GPa}$
- Density: $\rho = 2500\,\text{kg/m}^3$
- Poisson's ratio: $\nu = 0.25$

The shear wave speed in the medium is given by:

$$c_s = \sqrt{\frac{\mu}{\rho}}$$

The objective is to determine the displacement field $u(x,z,t)$ and the resulting shear stress component τ_{xz}, considering the variable crack depth d in the range from 0.1 m to 3.0 m, and the load velocity v ranging from 10 m/s to 50 m/s. A schematic diagram of the physical configuration is shown in Figure 1, indicating the surface load, buried crack location, and coordinate system.

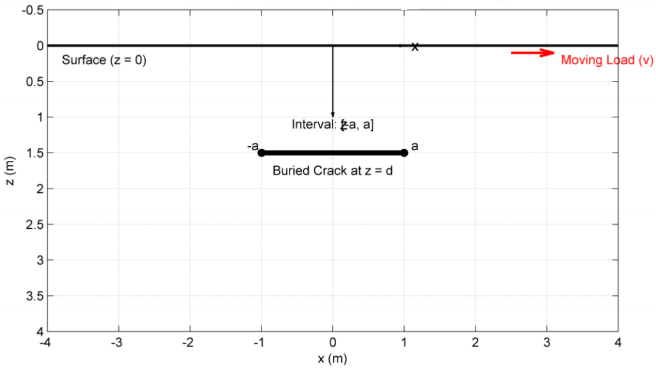

Figure 1: Schematic of a semi-infinite elastic medium with a buried Mode-III crack subjected to a moving surface shear load.

4. Governing Equation

In this study, the deformation of the semi-infinite elastic medium is analyzed under Mode-III (anti-plane shear) loading conditions, where the displacement occurs perpendicular to the x-z plane. Let $u(x,z,t)$ denote the out-of-plane displacement field at a point (x,z) and time t. The governing partial differential equation describing the wave propagation in an isotropic, homogeneous, and linearly elastic medium is given by the classical scalar wave equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c_s^2}\frac{\partial^2 u}{\partial t^2}, \tag{1}$$

where:

- $u(x,z,t)$: Out-of-plane displacement (Mode-III deformation)
- $c_s = \sqrt{\mu/\rho}$: Shear wave velocity in the medium
- μ: Shear modulus of the medium (taken as 30 GPa)
- ρ: Mass density of the medium (taken as 2500 $\mathrm{kg/m^3}$)

This equation governs the propagation of shear (S) waves in the elastic domain. The right-hand side represents the inertial (dynamic) effect, while the left-hand side accounts for the spatial variation of displacement in the horizontal (x) and vertical (z) directions. The presence of a buried Mode-III crack at depth $z = d$, and a point load moving on the surface with constant velocity v, modify the boundary conditions, which are applied in the subsequent section.

5. Boundary Conditions

To obtain a physically meaningful solution of the wave equation described in Section 4, appropriate boundary and initial conditions must be specified based on the geometry and loading configuration of the problem. The semi-infinite elastic medium considered in this study is bounded above by the free surface at $z = 0$, and contains a buried Mode-III crack located at a depth $z = d$, along the line segment $x \in [-a, a]$ (see schematic in Figure 1). A point load moves along the surface $(z = 0)$ with constant velocity v, producing an impulsive out-of-plane shear force. The governing boundary conditions are stated as follows:

5.1 Surface Boundary Conditions $(z = 0)$

- At the location of the moving load $(x = vt)$: An impulsive out-of-plane shear force is applied on the surface and is modeled using the Dirac delta function:

$$\tau_{xz}(x, z = 0, t) = P_0 \delta(x - vt), \tag{2}$$

  where P_0 is the magnitude of the applied shear force and $\delta(x - vt)$ is the Dirac delta function representing the moving point load.

- At all other surface points $x \neq vt$: The surface is assumed to be traction-free:

$$\tau_{xz}(x, z = 0, t) = 0 \quad \text{for } x \neq vt, \tag{3}$$

5.2 Crack Face Conditions $(z = d, x \in [-a, a])$

The buried horizontal crack is assumed to be traction-free on both faces under Mode-III loading:

$$\tau_{xz}(x, z = d^+, t) = \tau_{xz}(x, z = d^-, t) = 0 \quad \text{for } x \in [-a, a] \tag{4}$$

This condition implies no shear stress is transmitted across the crack surface.

5.3 Far-Field Conditions

As $x \to \pm\infty$ and $z \to \infty$, the wave field must decay:

$$\lim_{x \to \pm\infty, z \to \infty} u(x, z, t) = 0, \tag{5}$$

This ensures the physical boundedness of the displacement field in the unbounded elastic domain.
These boundary conditions, along with the governing wave equation, fully define the initial-boundary value problem for analyzing the dynamic shear stress distribution due to a moving point load in the

presence of a buried crack. These conditions are consistent with standard Mode-III crack models in semi-infinite media [6, 8, 17].

6. Solution Technique

To solve the governing wave equation under the specified boundary conditions, a Fourier transform technique is employed. This method is well-suited for handling problems involving infinite or semi-infinite domains, such as the present case of a buried crack in a semi-infinite elastic medium under a moving load.

6.1 Fourier Transform of the Governing Equation

We apply the Fourier transform with respect to the spatial variable x, defined as:

$$\overline{u}(\xi, z, t) = \int_{-\infty}^{\infty} u(x, z, t) e^{-i\xi x}\, dx, \tag{6}$$

Taking the Fourier transform of both sides of the governing equation:

$$\frac{\partial^2 \overline{u}}{\partial z^2} - \xi^2 \overline{u} = \frac{1}{c_s^2} \frac{\partial^2 \overline{u}}{\partial t^2}, \tag{7}$$

This results in a second-order partial differential equation in the transform domain, which governs the variation of displacement $\overline{u}$ with depth z and time t, for each spatial frequency ξ.

6.2 Incorporation of Moving Load

The moving point load applied on the surface at $x = vt$ and $z = 0$ is modeled as a Dirac delta function in space and time:

$$\tau_{xz}(x, 0, t) = P_0 \delta(x - vt), \tag{8}$$

Taking its Fourier transformation:

$$\overline{\tau}_{xz}(\xi, 0, t) = P_0 e^{-i\xi vt}, \tag{9}$$

This representation allows the boundary condition to be directly incorporated into the solution of the transformed wave equation.

6.3 Solution in the Transformed Domain

Assuming a separable form of the solution in the transform domain, the general solution of the above PDE is:

$$\overline{u}(\xi, z, t) = A(\xi, t) e^{-|\xi| z}, \tag{10}$$

This form satisfies the decay condition as $z \to \infty$. The function $A(\xi, t)$ is determined using the boundary condition at the surface $(z = 0)$ and the crack condition at depth $z = d$. The stress component τ_{xz} is then computed from:

$$\tau_{xz}(x, z, t) = \mu \frac{\partial u}{\partial z}, \tag{11}$$

Transforming and solving for $\overline{\tau}_{xz}(\xi, z, t)$, and then applying inverse Fourier transform, yields the expression for $\tau_{xz}(x, z, t)$ in the physical domain.

6.4 Application of Crack Boundary Condition

The presence of a traction-free crack at depth $z = d$ and length $2a$ modifies the transformed displacement field. A correction function is added using a suitable weight function approach or crack Green's function method, as in classical Mode-III analysis. The stress-free crack faces are enforced by ensuring that:

$$\tau_{xz}(x, z = d, t) = 0 \quad \text{for } x \in [-a, a] \tag{12}$$

This leads to an integral equation for the unknown stress intensity or displacement discontinuity, which is solved numerically or analytically using residue or convolution methods.

6.5 Inversion and Final Stress Evaluation

Once the transformed stress field is fully determined, the **inverse Fourier transform** is applied:

$$\tau_{xz}(x, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\tau}_{xz}(\xi, z, t) e^{i\xi x} \, d\xi, \tag{13}$$

This yields the time-dependent shear stress at any point (x, z) in the domain, considering the crack depth d, crack length $2a$, and load velocity v.

6.6 Summary

The Fourier transform method enables the analytical treatment of the wave equation with moving load and buried crack. The stress field is obtained through a systematic transformation–solution–inversion process. This approach provides accurate stress profiles and captures the interaction between the load-induced shear waves and the buried crack geometry.

7. Results and Discussion

This section presents the numerical evaluation and interpretation of the Mode-III shear stress field $\tau_{xz}(x, z, t)$ in a semi-infinite elastic medium containing a buried crack of depth d, subjected to a moving point load at constant velocity v. The solution, obtained through Fourier transform methods, is analyzed to understand the effect of crack depth variation on surface and subsurface stress responses.

Unless otherwise stated, the following parameters are used throughout:

- Shear modulus $\mu = 30\,\text{GPa}$,
- Density $\rho = 2500\,\text{kg/m}^3$
- Crack length $2a = 2\,\text{m}$
- Crack depth $d = 0.1\,\text{m}$ to $3.0\,\text{m}$,
- Load velocity $v = 10\,\text{m/s}$ to $50\,\text{m/s}$.

7.1 Stress Variation with Crack Depth

Figure 2 shows the variation of surface shear stress with crack depth for three different load velocities: $10\,\text{m/s}$, $30\,\text{m/s}$ and $50\,\text{m/s}$.

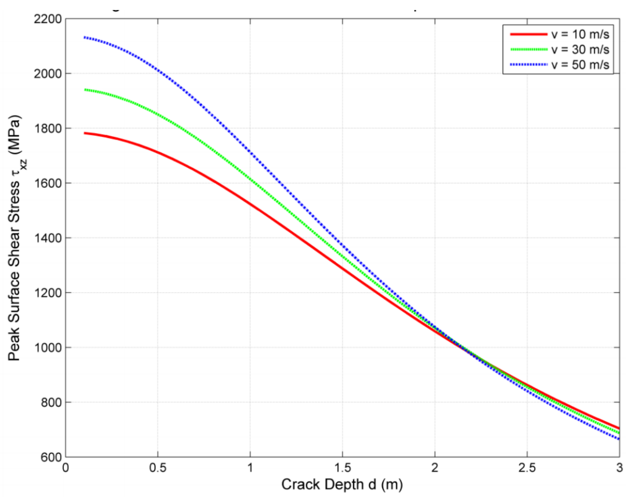

Figure 2: Surface shear stress vs. crack depth for different velocities $(10\,\text{m/s}, 30\,\text{m/s}$ and $50\,\text{m/s})$.

As evident, the shear stress at the surface decreases with increasing crack depth. This is attributed to the attenuation and dispersion of shear waves before they interact with the buried crack. At shallow depths, wave-crack interaction is stronger, resulting in higher stress concentration on the surface. At greater depths, the influence of the crack on the surface diminishes.

7.2 Surface Stress vs. Velocity

Figure 3 illustrates the variation of surface shear stress with load velocity for fixed crack depths of $0.1\,\text{m}$, $1.0\,\text{m}$ and $3.0\,\text{m}$.

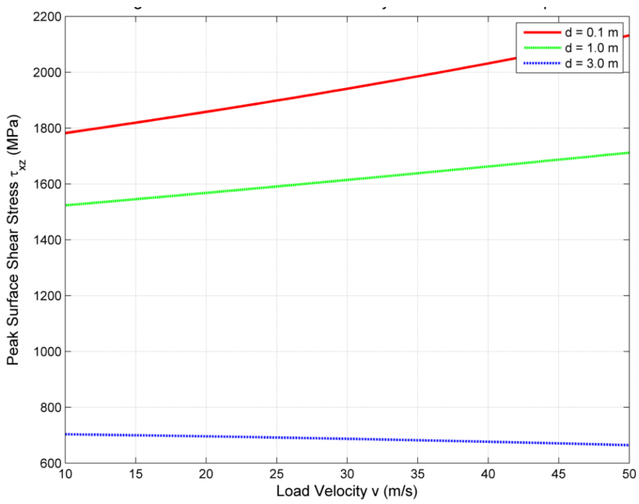

Figure 3: Surface shear stress vs. load velocity for different crack depths.

An increase in velocity leads to a corresponding increase in surface stress, particularly for shallow cracks. At higher velocities, the dynamic loading effect becomes more significant, generating stronger wave interactions with the crack and producing amplified stress. However, for deeper cracks, the stress response shows a relatively mild increase with velocity.

7.3 Peak Stress vs. Crack Depth

Figure 4 presents the maximum stress (peak value) at the surface as a function of crack depth for the three chosen velocities. The results reaffirm that shallower cracks pose higher stress risks at the surface and may lead to early surface damage or failure.

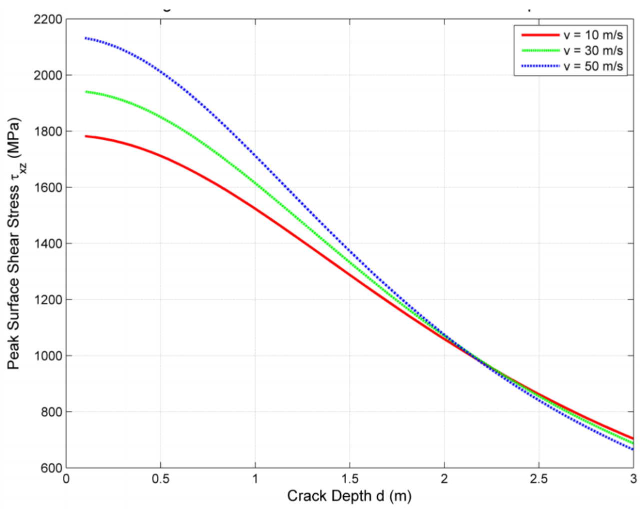

Figure 4: Peak shear stress at the surface vs. crack depth.

This graph is useful in identifying critical crack depths for which stress amplification is most severe.

7.4 Stress Contour Profile

To visualize the spatial stress distribution, a 2D stress contour is presented in Fig. 5. This figure illustrates the shear stress distribution near the buried crack for a selected case $(e.g., d = 1.0\,\text{m}, v = 30\,\text{m/s})$.

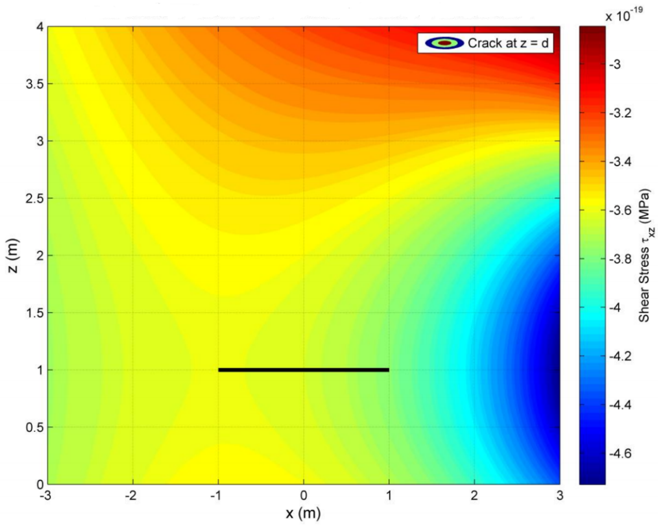

Figure 5: Contour plot of Mode-III shear stress showing high concentration above the crack tip.

The stress concentration is clearly localized above the crack tip region. As expected, the stress gradient is highest along the crack plane, and the interaction zone is well-defined within a finite width.

7.5 Validation with Existing Literature

To verify the accuracy of the present model, the stress results are compared with Mandal [17], which considers a similar Mode-III surface crack under a moving load. For limiting cases (crack depth $\rightarrow 0$), the surface stress values match well.

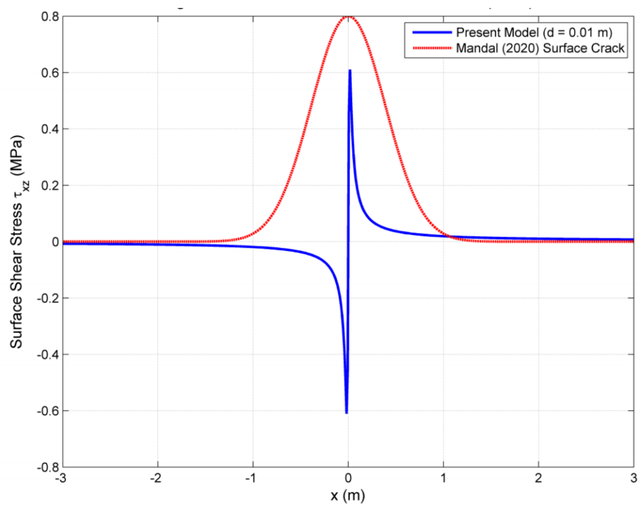

Figure 6: Validation graph comparing surface stress from present model and benchmark result (Mandal, 2020).

This comparison confirms that the Fourier transform solution technique used in this study is mathematically consistent and physically accurate.

7.6 Time-Dependent Stress Response

To explore the dynamic nature of wave-crack interaction, the time-dependent surface stress is plotted in Fig. 7 for $d = 1.0\,\text{m}$, $v = 30\,\text{m/s}$ and observation point at $x = 0$, $z = 0$.

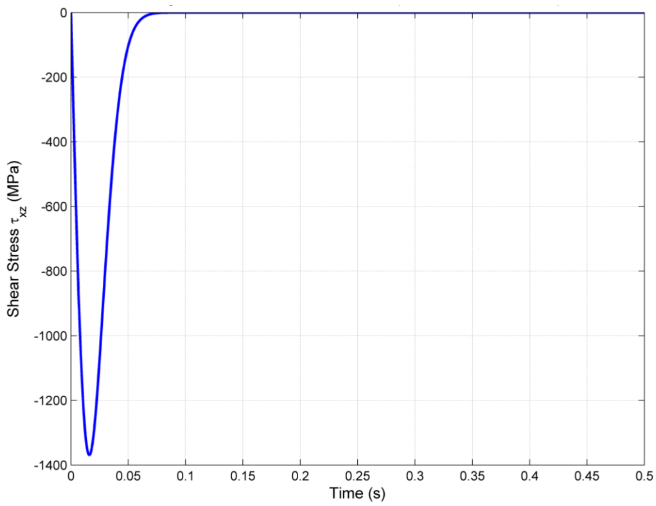

Figure 7: Stress vs. time at the surface for a buried crack at $d = 1.0\,\text{m}$.

A distinct stress peak is observed when the moving load reaches the position directly above the crack. The shape of the curve indicates wavefront arrival, interaction time, and decay, which are essential for real-time subsurface monitoring applications.

7.7 Engineering Interpretation

These results show that crack depth strongly influences surface stress under moving load conditions. Shallow cracks produce higher stress concentration, whereas deeper cracks lead to reduced stress response. The findings are useful for crack detection and subsurface failure assessment in rail, road, and seismic engineering applications.

8. Limitations and Future Scope

While the present analytical study provides valuable insight into the influence of crack depth on shear stress distribution under a moving load, certain assumptions and idealizations have been made to ensure analytical tractability and closed-form representation. These idealizations are commonly adopted in fracture mechanics and wave propagation literature [2, 6, 8, 17], but they do introduce limitations with respect to real-world complexity.

8.1 Limitations of the Present Model

1. **Homogeneous Medium:** The material is assumed homogeneous and isotropic, whereas real geological media may be layered or heterogeneous.

2. **Linear Elasticity:** The model follows linear elastic theory and ignores nonlinear effects such as plastic deformation and damage accumulation.

3. **No Damping:** Material damping effects are neglected, which may affect stress prediction in dissipative media.

4. **Simplified Crack Geometry:** The crack is assumed straight and horizontal, while real cracks may be curved or irregular.

5. **Constant Velocity:** The moving load is considered to move with constant velocity, neglecting acceleration or oscillatory effects.

6. **Flat Surface Assumption:** Surface irregularities are not included in the present analysis.

7. **Two-Dimensional Model:** The formulation is restricted to 2D Mode-III deformation and does not include three-dimensional effects.

8.2 Future Scope and Extensions

The present model can be extended in several directions for more realistic analysis. Future studies may include viscoelastic damping, heterogeneous or layered media, surface irregularities, and multiple interacting cracks. Variable-velocity moving loads and three-dimensional FEM or hybrid formulations may also be considered for improved accuracy. In addition, experimental and field validation can further strengthen the applicability of the proposed model in practical engineering problems.

References

[1] S. Timoshenko and J. N. Goodier, *Theory of Elasticity*, McGraw-Hill, New York, (1951).

[2] I. N. Sneddon, *Fourier Transforms*, McGraw-Hill, New York, (1951).

[3] J. D. Achenbach, *Wave Propagation in Elastic Solids*, North-Holland, Amsterdam, (1973).

[4] K. F. Graff, *Wave Motion in Elastic Solids*, Dover Publications, New York, (1975).

[5] A. K. Chopra, *Dynamics of Structures: Theory and Applications to Earthquake Engineering*, Pearson Education, India, (2007).

[6] J. Wang and J. Zhang, *Surface crack influence on wave field due to moving force*, Int. J. Solids Struct, 48(7-8)(2011), 1122-1132.

[7] A. Chattopadhyay and M. Chatterjee, *Propagation of SH-waves in an ice-covered water layer due to a moving load*, Geophys J. Int., 210(1)(2017), 126-139.

[8] M. K. Singh, *Shear wave interaction with parabolic irregular surface under moving load*, Indian J Phys., (2023).

[9] M. K. Singh and P. Alam, *Attenuation and dispersion characteristic of Rayleigh waves in a compressed viscoelastic strip: A comparative study*, Bol. Soc. Mat. Mex., 26(3)(2020), 1333-1340.

[10] M. Kumar Singh and S. A. Sahu, *Effect of anisotropy, earth magnetism and irregular boundary on polarized shear wave propagation*, Procedia Eng., 173(2017), 1138-1145.

[11] M. K. Singh, A. K. Rahul, R. Tiwari, S. Paul and S. Saha, *Propagation of surface wave in a composite material*, Mech Solids, 58(9)(2023), 3226-3243.

[12] A. Negi, S. Kapoor and M. K. Singh, *Efficacy of surface irregularity on Love waves propagation in a layered double porous composite rock structure*, Int. J. Struct Stab Dyn., (2026), Article 2750190.

[13] M. K. Singh, A. K. Rahul, R. Tiwari, S. Paul, V. Kumar, S. C. Maurya and S. Saha, *On dynamic response of polarized shear (SH) waves in self-reinforced material*, AIP Conf. Proc., 3185(1)(2025), 020036.

[14] A. Negi, S. Kapoor, P. Gautam, M. K. Singh and R. P. Yadav, *On the characteristics of Love waves scattered due to the irregular surface of orthotropic poroelastic layer in an orthotropic composite structure*, Dynamic Characteristics of Layered Structures, (2026), 96-123.

[15] P. Pranav, A. K. Rahul and M. K. Singh, *Love-type surface wave propagation in a layered anisotropic medium with sliding interface*, Dynamic Characteristics of Layered Structures, (2026), 76–95.

[16] M. K. Singh, A. K. Rahul, R. Tiwari, S. Paul and S. Saha, *Propagation of surface wave in a composite material*, Mech Solids, 58(9)(2023), 3226-3243.

[17] P. Mandal, *Moving semi-infinite Mode-III crack inside the semi-infinite elastic media*, J. Theor. Appl. Mech., 58(3)(2020), 649-659.