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# A Delayed Host-Parasite Model with Partial Host Cover and Parasite Harvesting

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## Abstract

This paper improves and investigates a delayed host parasite communication model that encompasses partial host cover and parasite harvesting. The partial cover represents a constant fraction of the host population that is temporarily shielded from parasitic attack due to behavioural, spatial, or physiological refuge. The delay accounts for the time lag in the parasites growing process or infection transmission. A system of nonlinear delay differential equations formulated to describe the population dynamics. The equilibria and their stability are examined, and analytical conditions for the occurrence of the Hopf bifurcation are derived. The results show that intensifying time delay can dislocate the positive equilibrium, ahead to periodic oscillations, whereas increasing host cover or harvesting strength has a steadying influence. The numerical simulations back up the theory and show just how complicated these systems can get. We see stable coexistence, ongoing oscillations, and even points where everything collapses. This study really shines a light on how delayed reactions, safe zones, and different harvesting strategies shape the long-term behaviour of host parasite relationships, passive implications for sustainable management and biological control.

**Keywords:** Host-Parasite Dynamics; DDE; Host Refuge; Hopf Bifurcation; Parasite Harvesting.

## 1. Introduction

Host-parasite relationships really shape how ecosystems work. They affect everything, who survives, how many species stick around, and even how we control pests. At first, scientists leaned on Lotka and Volterra's [1,2] classic predator prey models to explain these dynamics. Then C. S. Holling [3] things up by introducing his functional response theory, which added some nonlinear spice and helped ecologists make sense of nature's unpredictability. Time delays are just part of the story. Think about it: animals need time to grow, develop or even just be born. G.E. Hutchinson was the first to show that when you let this natural delay into the math, populations can start swinging into cycles booming, crashing, then

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booming again [4]. After him, researchers like J.D. Murray [5] and Y. Kuang [6] took it further and laid out the basics for using delay differential equations in biological systems. There is another layer refuge. When some hosts can hide or get partial protection, it changes the whole game. Experiments [7,8] have shown that giving prey a safer corner actually calms things down, makes populations steadier over time. On top of that, scientists have spent years figuring out how harvesting basically, picking off a portion of the population in a controlled way, lets us manage species without wiping them out [9,10]. So, here's where this paper comes in. We are diving into a new host parasite model that includes delays. The catch? It also throws in partial protection for hosts and looks at what happens when we harvest parasites.

## 2. Mathematical Model

Let $H(t)$ denote the host population density and $P(t)$ denote the parasite population density at any time $t$. This model incorporates

- Logistic growth
- Partial refuge: a constant fraction $0 < m < 1$ of host shielded
- Parasite growth depends on delayed host availability.
- Parasites are harvested at constant rate $h$.

The system of delayed differential equations is

$$\frac{dH}{dt} = rH\left(1 - \frac{H}{k}\right) - \beta(1-m)HP \tag{1}$$

$$\frac{dP}{dt} = \alpha\beta(1-m)H(t-\tau)P(t-\tau) - dP - hP \tag{2}$$

where

- $r$ is intrinsic growth rate
- $k$ is carrying capacity
- $\alpha$ is conversion efficiency
- $\beta$ is infection rate
- $d$ is parasite death rate
- $h$ is harvesting rate
- $\tau$ is time delay

## 3. Co-existing Equilibrium

The trivial $(0, 0)$ and the host only $(k, 0)$ equilibria exist. From parasite equation

$$\alpha\beta(1-m)H^* = d + h \tag{3}$$

Thus

$$H^* = \frac{d+h}{\alpha\beta(1-m)} \tag{4}$$

Substituting into the host equation

$$P^* = \frac{r}{\beta(1-m)}\left(1 - \frac{H^*}{k}\right) \tag{5}$$

Then the existence condition is

$$\frac{d+h}{\alpha\beta(1-m)} < k \tag{6}$$

## 4. Local Stability Analysis

Linearizing around $(H^*, P^*)$ to obtain the characteristic equation

$$\lambda^2 + a_1\lambda + a_2 + a_3 e^{-\lambda\tau} = 0 \tag{7}$$

For $\tau = 0$ (no delay), stability holds if trace $< 0$ and determinant $> 0$ (Routh-Hurwitz). For $\tau > 0$, stability switches occur as $\tau$ increases.

## 5. Hopf Bifurcation

Hopf bifurcation arises when pure imaginary roots $\lambda = \pm i\omega$ cross the imaginary axis at critical $\tau_c$, transversality, where

$$\tau_c = \frac{1}{\omega}\cos^{-1}\left(\frac{a_2 - \omega^2}{a_3}\right) \tag{8}$$

For $\tau > \tau_c$ the system undergoes Hopf bifurcation leading to periodic oscillations. Increasing refuge or harvesting typically enlarges stability regions by reducing effective infection or parasite growth. From the above, increasing the delay destabilizes equilibrium.

## 6. Global Stability

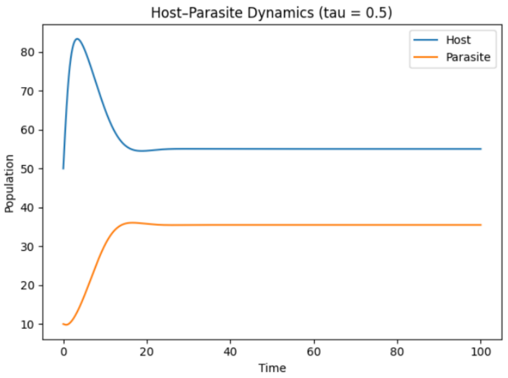
Global stability can be shown using the Lyapunov functional of the form

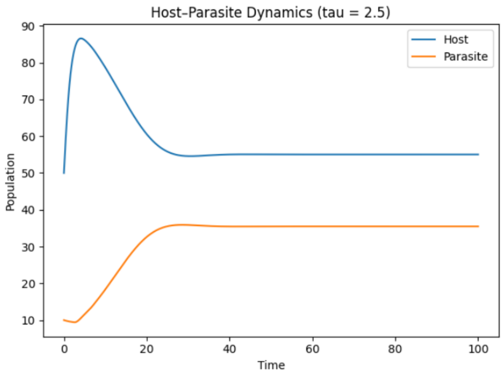
$$V = \left(H - H^* - H^*\ln\left(\frac{H}{H^*}\right)\right) + c\left(P - P^* - P^*\ln\left(\frac{P}{P^*}\right)\right) \tag{9}$$

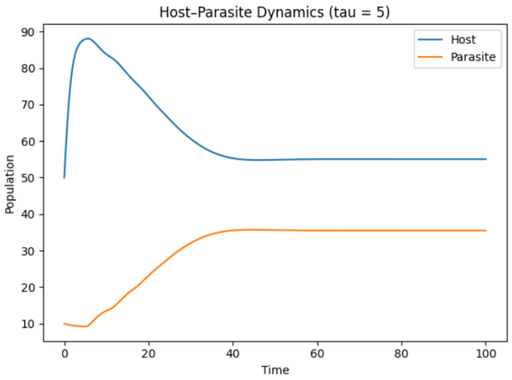
We can show $\frac{dV}{dt} \leq 0$ if $r < \alpha\beta(1-m)k$; then equilibrium is globally asymptotically stable when $\tau < \tau_c$.

## 7. Numerical Simulation

Numerical simulations confirm analytics. For the baseline parameters: $r = 1.2$, $k = 100$, $\alpha = 0.49$, $\beta = 0.019$, $d = 0.31$, $h = 0.1$ and $m = 0.2$.







## 8. Conclusion

This delayed host parasite model really highlights an opposing force: on one side, time delays stir up oscillations and can push the system toward instability via Hopf bifurcations. But then, when you add partial host cover and parasite harvesting, they exert a stabilizing effect by reducing the parasitism pressure. When we run the numbers, the system shows all sorts of behaviour: sometimes you get stable coexistence, sometimes endless cycles, and in extreme cases, total collapse. These findings underscore the importance of behavioural/spatial refuges and strategic harvesting in modulating long-term host parasite outcomes. This matters a lot for things like sustainable biological control, pest management and conservation ecology.

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