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Degcity Indices of Some Networks

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Abstract

Topological indices are important tools for analyzing structural properties of graphs and interconnection networks. In this paper, we compute seven eccentricity-based degcity indices for honeycomb, oxide, and hypertree networks. By employing edge partition techniques based on vertex degree and eccentricity, explicit formulae are derived for each network class. The results contribute to the theoretical study of degree-eccentricity interactions and provide useful descriptors for the analysis and design of complex network structures.

Keywords: Degcity indices; Honeycomb Network; Oxide Network; Hypertree Network.

2020 Mathematics Subject Classification: 05C05, 05C07, 05C09, 05C10, 05C12, 05C90.

1. Introduction

Graph theory has emerged as a valuable branch of mathematics and contributed to the analysis of networks in recent years. Topological indices are numerical parameters that capture some aspects of the structure and behavior of graphs, which can be used to model various networks and systems. Graphene network, honeycomb network, oxide network and hypertree network are some examples of networks that have applications in chemistry, physics, computer science and engineering. It explores the relationship between the structure and properties of networks and their applications in various fields. Topological indices are used to design and optimize networks for specific purposes, such as interconnection, communication, computation, and encryption. It can also be used to model and simulate the behavior and performance of networks under different conditions, such as temperature, pressure, voltage and current. Further, topological indices helps to discover new properties and phenomena of networks, such as symmetry, domination, coloring and enumeration. Moreover, structural properties of some networks like honeycomb, oxide and hypertree have been studied extensively in [3–7], where several degree and eccentricity related indices were derived. These works

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highlight the growing importance of degree-eccentricity interactions in network analysis. Motivated by these developments, we investigate degcity indices for honeycomb, oxide, and hypertree networks and derive explicit closed-form expressions using edge partition techniques. In this paper, we compute the following degcity indices[9] of these networks.

$$DC_1(G) = \sum_{uv \in E(G)} [e_u + e_v]\,[d_u + d_v]$$
$$DC_2(G) = \sum_{uv \in E(G)} \frac{e_u + e_v}{d_u + d_v}$$
$$DC_3(G) = \sum_{uv \in E(G)} \frac{d_u + d_v}{e_u + e_v}$$
$$DC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{e_u + e_v}{d_u + d_v}}$$
$$DC_5(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v}{e_u + e_v}}$$
$$DC_6(G) = \sum_{uv \in E(G)} \frac{e_u + e_v}{d_u d_v}$$
$$DC_7(G) = \sum_{uv \in E(G)} \frac{d_u + d_v}{e_u e_v}$$

where d_u, d_v represent the degree of the vertices u, v in the vertex set $V(G)$ respectively, uv is an edge in the edge set $E(G)$ and e_u, e_v denote the eccentricity of the vertices u, v respectively of a graph G. For any undefined graph terminologies and notations, refer [10].

2. Main Results

2.1 Honeycomb Network

Honeycomb networks are a class of interconnection networks derived from hexagonal tessellations of the plane, known for their regular structure and efficient communication properties. Due to these features, they have been widely applied in parallel computing, image processing, cellular communication, and graph-theoretic modeling [1,8]. In recent years, honeycomb-based graph structures have attracted considerable attention for studying structural properties such as symmetry, connectivity, domination, coloring, and enumeration.

Definition 2.1 ([7]). *The honeycomb network HC_1 is just a hexagon. The honeycomb network HC_2 is constructed by adding six hexagons to the boundary edges of HC_1. Similarly, the honeycomb network HC_n is obtained by adding a layer of hexagons around the boundary edges of HC_{n-1}. The variable n in HC_n denotes the number of hexagons from the centre to the boundary. The number of vertices and edges in HC_n are respectively $6n^2$ and $9n^2 - 3n$. Figure 1 represents honeycomb network HC_3.*

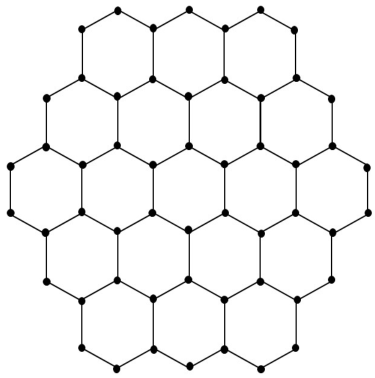

Figure 1: Honeycomb Network HC_3

Theorem 2.2. *Let HC_n be the n-dimensional honeycomb network. Then*

$$\begin{aligned}
DC_1(HC_n) &= 360n^3 - 294n^2 + 90n - 12,\\
DC_2(HC_n) &= \frac{100n^3 - 23n^2 + 11n + 2}{10},\\
DC_3(HC_n) &= \frac{240n^2 - 204n + 24}{32n^2 - 20n + 3} + 36\left[\sum_{m=1}^{n-1}\frac{1}{4n+4m-2} + \sum_{m=1}^{n-1}\frac{m}{4n+4m-1}\right.\\
&\left. +2\sum_{m=1}^{n-2}\frac{m}{4n+4m+1}\right],\\
DC_4(HC_n) &= 3\sqrt{8n-2} + 12(n-1)\sqrt{\frac{8n-3}{5}}\\
&+6\left[\sum_{m=1}^{n-1}\sqrt{\frac{4n+4m-2}{6}} + \sum_{m=1}^{n-1}m\sqrt{\frac{4n+4m-1}{6}} + 2\sum_{m=1}^{n-2}m\sqrt{\frac{4n+4m+1}{6}}\right],\\
DC_5(HC_n) &= 12\sqrt{\frac{1}{8n-2}} + 12(n-1)\sqrt{\frac{5}{8n-3}}\\
&+6\left[\sum_{m=1}^{n-1}\sqrt{\frac{6}{4n+4m-2}} + \sum_{m=1}^{n-1}m\sqrt{\frac{6}{4n+4m-1}} + 2\sum_{m=1}^{n-2}m\sqrt{\frac{6}{4n+4m+1}}\right],\\
DC_6(HC_n) &= \frac{96n^2 - 112n + 23}{3},\\
DC_7(HC_n) &= \frac{24}{(4n-1)^2} + \frac{60(n-1)}{(4n-2)(4n-1)} + 36\left[\sum_{m=1}^{n-1}\frac{1}{(2n+2m-1)^2}\right.\\
&\left. + \sum_{m=1}^{n-1}\frac{m}{(2n+2m-1)(2n+2m)} + 2\sum_{m=1}^{n-2}\frac{m}{(2n+2m)(2n+2m+1)}\right].
\end{aligned}$$

Proof. The edge sets in HC_n are given by

$$E_{2,2}(HC_n) = \{(d_u, d_v) | d_u = 2,\ d_v = 2, uv \in E(HC_n)\},$$

$$E_{2,3}(HC_n) = \{(d_u, d_v) | d_u = 2,\ d_v = 3, uv \in E(HC_n)\},$$

$$E_{3,3}(HC_n) = \{(d_u, d_v) | d_u = 3,\ d_v = 3, uv \in E(HC_n)\}.$$

(d_u, d_v)	(e_u, e_v)	**Frequency**	**Range of m and n**
$(2,2)$	$(4n-1,\, 4n-1)$	6	$n \geq 1$
$(2,3)$	$(4n-2,\, 4n-1)$	$12(n-1)$	$n > 1$
$(3,3)$	$(2n+2m-1,\, 2n+2m-1)$	6	$1 \leq m \leq n-1,\ n > 1$
$(3,3)$	$(2n+2m-1,\, 2n+2m)$	$6m$	$1 \leq m \leq n-1,\ n > 1$
$(3,3)$	$(2n+2m,\, 2n+2m+1)$	$12m$	$1 \leq m \leq n-2,\ n > 2$

Table 1: Edge partition of the honeycomb network HC_n with respect to degree and eccentricity.

Using the edge partition with respect to the degree and eccentricity as given in the Table 2.1, the result follows easily. □

2.2 Oxide Network

An oxide network is a class of structures composed exclusively of metal-oxygen linkages, as observed in compounds such as copper oxides and iron oxides. These networks can be naturally represented using graph-theoretic models, where atoms correspond to vertices and bonds to edges. Such graph representations provide a useful framework for analyzing structural and chemical properties of oxide compounds, including symmetry, connectivity, structural complexity, and reactivity.

Definition 2.3 ([7]). *A silicate network is a silicate sheet of a tetrahedron. The corner vertices and the centre vertex of a tetrahedron represents oxygen and silicon respectively. An oxide network is a new network obtained by deleting all the silicon vertices from the silicate network. The n-dimensional oxide network is denoted by OX_n. The number of vertices and edges in OX_n are respectively $9n^2 + 3n$ and $18n^2$. Figure 2 represents 2-dimensional oxide network OX_2.*

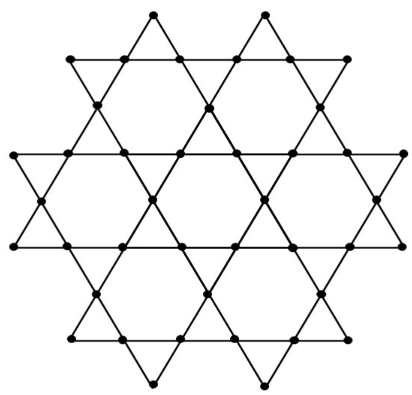

Figure 2: **Oxide Network** OX_2

Theorem 2.4. *Let OX_n be the n-dimensional oxide network. Then*

$$DC_1(OX_n) = 960n^3 - 192n^2 + 24n,$$

$$
\begin{aligned}
DC_2(OX_n) &= \frac{30n^3 - 24n^2 + 35n - 4}{2}, \\
DC_3(OX_n) &= \frac{72n}{8n-1} + 48\left[\sum_{m=1}^{n} \frac{2m-1}{4n+4m-2} + 2\sum_{m=1}^{n-1} \frac{m}{4n+4m-1} + 2\sum_{m=1}^{n-1} \frac{m}{4n+4m+1}\right], \\
DC_4(OX_n) &= 12n\sqrt{\frac{8n-1}{6}} + 3\sum_{m=1}^{n} (2m-1)\sqrt{2n+2m-1} \\
&\quad + 3\sqrt{2}\sum_{m=1}^{n-1} m\left[\sqrt{4n+4m-1} + \sqrt{4n+4m+1}\right], \\
DC_5(OX_n) &= 12n\sqrt{\frac{6}{8n-1}} + 12\left[\sum_{m=1}^{n} (2m-1)\sqrt{\frac{1}{2n+2m-1}}\right. \\
&\quad \left. + \sum_{m=1}^{n-1} m\left(\sqrt{\frac{8}{4n+4m-1}} + \sqrt{\frac{8}{4n+4m+1}}\right)\right], \\
DC_6(OX_n) &= \frac{30n^3 - 24n^2 + 51n - 6}{4}, \\
DC_7(OX_n) &= \frac{18n}{n(4n-1)} + 48\left[\sum_{m=1}^{n} \frac{2m-1}{(2n+2m-1)^2} + 4\sum_{m=1}^{n-1} \frac{m}{(2n+2m)^2-1}\right].
\end{aligned}
$$

Proof. The edge sets in OX_n are given by

$$
\begin{aligned}
E_{2,4}(OX_n) &= \{(d_u, d_v) | d_u = 2,\ d_v = 4, uv \in E(OX_n)\}, \\
E_{4,4}(OX_n) &= \{(d_u, d_v) | d_u = 4,\ d_v = 4, uv \in E(OX_n)\}.
\end{aligned}
$$

$(\mathbf{d_u}, \mathbf{d_v})$	$(\mathbf{e_u}, \mathbf{e_v})$	**Frequency**	**Range of m and n**
$(4,4)$	$(2n+2m-1,\ 2n+2m-1)$	$6(2m-1)$	$1 \le m \le n,\ n \ge 1$
$(4,4)$	$(2n+2m-1,\ 2n+2m)$	$12m$	$1 \le m \le n-1,\ n > 1$
$(2,4)$	$(4n-1,\ 4n)$	$12n$	$n > 1$
$(4,4)$	$(2n+2m,\ 2n+2m+1)$	$12m$	$1 \le m \le n-1,\ n > 1$

Table 2: Edge partition of the oxide network OX_n with respect to degree and eccentricity.

Using the edge partition with respect to the degree and eccentricity as given in the Table 2.2, the result follows. □

2.3 Hypertree Network

Hypertree network graphs constitute an important and versatile class of interconnection networks with applications in parallel computing, data communication, and network architecture design [1]. These structures are built upon a binary tree framework, which serves as the fundamental backbone of the hypertree, and are enhanced through additional hierarchical and cross-level connections to improve connectivity and communication efficiency [2].

Definition 2.5 ([7]). *Let BT_s denote the complete binary tree in which each level $t, 0 \le t \le s$ contains 2^t vertices. The root vertex of BT_s is labeled as 1 and the siblings of the vertex i are labeled as $2i$ and $2i+1$. The* *s-level hypertree network denoted by HT_s is obtained from BT_s by adding edges in each level t, $1 \leq t \leq s$ between the vertices whose label difference is 2^{t-1}. The number of vertices and edges in HT_s are respectively $2^{s+1}-1$ and $3(2^s-1)$. Figure 3 represents 3-level hypertree network HT_3.*

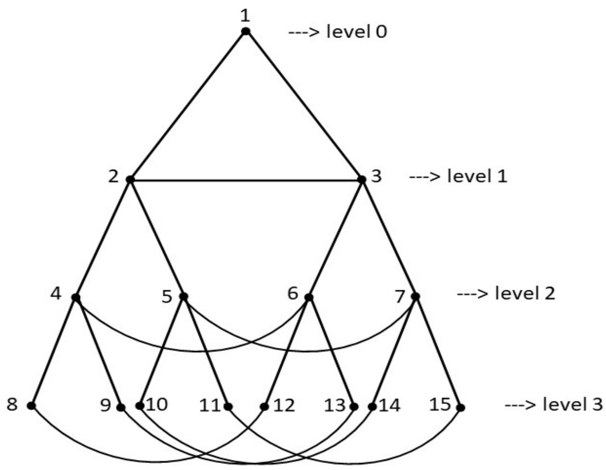

Figure 3: **Hypertree Network** HT_3

Theorem 2.6. *Let HT_s be the s-level hypertree network. Then*

$$DC_1(HT_s) = (80s-102)2^s - 56s + 128,$$

$$DC_2(HT_s) = \frac{(23s-24)2^s - 7s + 24}{12},$$

$$DC_3(HT_s) = \frac{6}{s} + 7\frac{2^s}{4s-3} + \frac{2^s}{2s-1} + 9\sum_{t=1}^{s-2}\frac{2^{t+1}}{2s+2t-1},$$

$$DC_4(HT_s) = 2\sqrt{\frac{s}{3}} + 2^s\sqrt{\frac{4s-3}{6}} + 2^{s-2}\sqrt{4s-2} + \sum_{t=1}^{s-2}2^t\sqrt{\frac{2s+2t-1}{2}} + \sum_{t=1}^{s-1}2^{t-2}\sqrt{s+t-1},$$

$$DC_5(HT_s) = 2\sqrt{\frac{3}{s}} + 2^s\sqrt{\frac{6}{4s-3}} + 2^{s-1}\sqrt{\frac{2}{2s-1}} + \sum_{t=1}^{s-2}2^{t+2}\sqrt{\frac{2}{2s+2t-1}} + \sum_{t=1}^{s-1}2^t\sqrt{\frac{1}{s+t-1}},$$

$$DC_6(HT_s) = \frac{(11s-10)2^s - s + 8}{8},$$

$$DC_7(HT_s) = \frac{12}{s^2} + 6\frac{2^s}{(2s-2)(2s-1)} + 4\frac{2^{s-1}}{(2s-1)^2} + 8\sum_{t=1}^{s-2}\frac{2^{t+1}}{(s+t-1)(s+t)} + 8\sum_{t=1}^{s-1}\frac{2^{t-1}}{(s+t-1)^2}.$$

Proof. The edge sets in HT_s are given by

$$E_{2,2}(HT_s) = \{(d_u, d_v)|d_u = 2,\ d_v = 2, uv \in E(HT_s)\},$$
$$E_{2,4}(HT_s) = \{(d_u, d_v)|d_u = 2,\ d_v = 4, uv \in E(HT_s)\},$$
$$E_{4,2}(HT_s) = \{(d_u, d_v)|d_u = 4,\ d_v = 2, uv \in E(HT_s)\},$$
$$E_{4,4}(HT_s) = \{(d_u, d_v)|d_u = 4,\ d_v = 4, uv \in E(HT_s)\}.$$

$(\mathbf{d_u}, \mathbf{d_v})$	$(\mathbf{e_u}, \mathbf{e_v})$	**Frequency**	**Range of** t
$(2,4)$	(s,s)	$2 \cdot 2^t$	$t = 0$
$(4,4)$	$(s+t-1,\ s+t)$	2^{t+1}	$1 \leq t \leq s-2,\ s > 2$
$(4,2)$	$(s+t-1,\ s+t)$	2^{t+1}	$t = s-1$
$(4,4)$	$(s+t-1,\ s+t-1)$	2^{t-1}	$1 \leq t \leq s-1,\ s > 1$
$(2,2)$	$(s+t-1,\ s+t-1)$	2^{t-1}	$t = s$

Table 3: Edge partition of the hypertree network HT_s with respect to degree and eccentricity.

Using the edge partition with respect to the degree and eccentricity as given in the Table 2.3, the result follows. □

3. Conclusion

In this work, we derived explicit expressions for seven degcity indices (DC_1-DC_7) of honeycomb, oxide, and hypertree networks. Using edge partition methods based on vertex degree and eccentricity, closed-form results were obtained for each class of networks. These findings enrich the study of eccentricity-based topological indices and provide useful structural descriptors for analyzing interconnection networks. The results may serve as a foundation for further investigations on other network families and potential applications in mathematical chemistry and network analysis.

References

[1] W. J. Dally and B. P. Towles, *Principles and practices of interconnection networks*, Elsevier, (2004).

[2] A. Grama, G. Karypis, Vipin Kumar and Anshul Gupta, *Introduction to Parallel Computing*, Second Edition, Addison-Wesley, (2003).

[3] L. Gu, S. Yousaf, A. A. Bhatti, P. Xu and A. Aslam, *Computing some degree-based topological indices of honeycomb networks*, Complexity, 2022(1)(2022), 2771059.

[4] S. Hayat and M. Imran, *Computation of topological indices of certain networks*, Applied Mathematics and Computation, 240(2014), 213-228.

[5] M. Imran, M. K. Siddiqui, A. A. E. Abunamous, D. Adi, S. H. Rafique and A. Q. Baig, *Eccentricity based topological indices of an oxide network*, Mathematics, 6(7)(2018), 126.

[6] M. Imran, A. A. E. Abunamous, D. Adi, S. H. Rafique, A. Q. Baig and M. R. Farahani, *Eccentricity based topological indices of honeycomb networks*, Journal of Discrete Mathematical Sciences and Cryptography, 22(7)(2019), 1199-1213.

[7] G. Kirithiga Nandini, S. Klavžar, T. M. Rajalaxmi and R. Sundara Rajan, *A note on eccentricity based topological indices of honeycomb, oxide and 2-power interconnection networks*, Journal of Discrete Mathematical Sciences and Cryptography, (2021), 1-23.

[8] I. Stojmenovic, *Honeycomb networks: Topological properties and communication algorithms*, IEEE Transactions on parallel and distributed systems, 8(10)(2002), 1036-1042.

[9] K. B. Sudhakara, P. S. Guruprasad and M. A. Sriraj, *Prediction potential of degcity indices for physico-chemical properties of polycyclic aromatic hydrocarbons: A QSPR study*, Biointerface Research in Applied Chemistry, 13(6)(2023), 599.

[10] D. B. West, *Introduction to Graph Theory*, 2nd ed., Prentice Hall, Upper Saddle River, (2001).