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The m-Ranking of Nodes in Weighted Networks

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Abstract

Identifying pivotal nodes within complex systems has emerged as a cornerstone of network science, offering both profound theoretical insights and significant practical utility. However, only a limited number of centrality measures are applicable to weighted networks. Addressing this limitation, the m-Ranking method has been introduced as a robust analytical framework that evaluates node hierarchy by integrating connection density with the specific intensity of link weights. This approach is notably versatile and maintains high precision in weighted network environments.

Keywords: Weighted networks; node ranking; centrality; m-ranking; air transportation network.
2020 Mathematics Subject Classification: 05C82, 91D30.

1. Introduction

Complex systems are frequently mapped as relational networks, providing a framework to quantify the interactions between diverse components. By modelling these as mathematical graphs, researchers can analyse the structural properties of everything from financial markets to physical phenomena. The primary advantage of this approach is the ability to monitor temporal dynamics. As a system evolves, its topology changes accordingly. Centrality is fluid; "hubs" (highly connected nodes) are not static entities but are subject to the shifting pressures and interactions of the network, allowing for a rigorous study of how systems transform over time. Nodes with a large number of connections typically hold greater importance and influence within a network. In contact-driven disease transmission models, such highly connected nodes (hubs) are especially critical, since an infection emerging from them can spread quickly and extensively throughout the system. As a result, identifying and immunizing these nodes is an effective strategy for mitigating epidemic outbreaks [2]. An analogous pattern appears in the diffusion of rumours, information, or social practices within a population. Messages propagate more efficiently when initiated by individuals who maintain strong and numerous connections. Consequently, a thorough understanding of information spread requires

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close examination of the key actors occupying central positions in the network. Across these contexts, it is clear that certain nodes play a decisive role in influencing network behaviour and overall outcomes.

The significance of nodes within a network has been extensively investigated in several studies [1,6]. The importance of a node is not static; nodes that initially appear insignificant may later emerge as highly influential, while those occupying central positions may gradually lose their prominence. Certain nodes play a crucial role in shaping the dynamics of a network. In particular, hubs are often regarded as popular or highly influential. These nodes naturally attract greater attention from other members and tend to accumulate even more connections over time [4,5]. Nodes with fewer connections are often inclined to link with hubs, recognizing that such associations can enhance their own connectivity and, consequently, elevate their status within the network.

Understanding how the importance of nodes evolves has been a central problem in the study of complex networks. Traditionally, measures such as degree, k-shell, Mixed Degree Decomposition (MDD), closeness, neighbourhood coreness and betweenness have been widely employed to identify influential nodes. In recent years, several additional centrality measures have been developed, with PageRank [3] and LeaderRank [7] serving as prominent examples. However, designing an effective method to identify the node importance is still an open issue. In practical applications, most real-world networks used in modelling studies are weighted in nature. However, there is currently no universally accepted centrality measure that effectively ranks nodes in weighted networks. To address this limitation, we propose a new **m-ranking** method for weighted complex networks. The following section presents the mathematical formulation used to determine node ranks, along with an algorithm outlining the computational procedure. The method is then illustrated using an example graph, and the resulting node rankings are summarized in a table.

2. The m-Ranking Method for Weighted Complex Networks

In this paper we propose a new method m-Ranking [8–10] to rank the nodes of a weighted network. The ranking is based on the following formula, which helps us to find the total power of a node i. It is denoted by $T(i)$.

$$T(i) = \sum W_i^0 + \frac{1}{\beta}\sum W_i^1 + \frac{1}{\beta^2}\sum W_i^2 + \dots \tag{1}$$

It is good practice to choose β an integer close to the average degree of the nodes of the graph. The series contains at most D terms, where D is the diameter of the graph. Here, $\sum W_i^0$ is the sum of the weights of incident edges of the node i and $\sum W_i^j$ is the sum of the weights of edges j away from node i. Since we are considering weights of all links, usually total power of nodes will be different. Therefore, there are very little chance for two nodes have equal rank. Next, we calculate the ranks of an example weighted network. This network contains six nodes and nine edges. All edges are weighted. The network is shown in the following Figure 1.

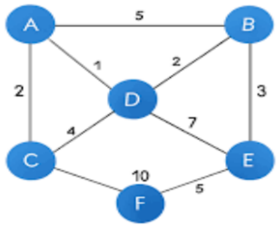

Figure 1: Example weighted network with six nodes and nine edges.

We calculate the total power of each node and the corresponding ranks of all nodes. The total power $T(i)$ and the ranks are tabulated below.

Node Name	$T(i)$	Rank
A	20.25	6
B	22.5	5
C	26.75	1
D	24.5	4
E	26.5	2
F	25	3

Table 1: Total power and rank of nodes in the example network.

Since the edge weights differ, the ranks of nodes depend strongly on their weights. In the above example, an edge with weight 10 significantly increases the ranks of the vertices incident to it, making them more prominent in the overall ranking.

3. An Air Transportation Network - A Real Example

The real air transportation network formed in India by the private air transportation company 'Spice Jet' used to further illustrate the method m-ranking. This network contains 33 airports (airports denoted by nodes in the network) and 140 routes (routes are denoted by links) connecting these nodes. Each edge has a weight w_{ij} which represents the number of flights between two airports i and j in a particular day. The data is available in the web page of the company [11]. The network is given in the following Figure 2. The weights are not given in the figure because they may not be visible.

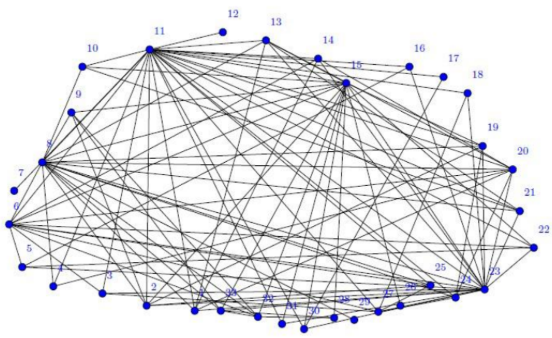

Figure 2: Spice Jet air transportation network in India (33 airports, 140 routes).

Using the m-ranking formula (1) we can calculate the total power of each node of the network by fixing

$\beta = 2$. Using this we can calculate total power $T(i)$ and rank the 33 airports. The ranks of first 10 airports are tabulated below.

Rank	Total Power $T(i)$	Name of the Airport (node number)
1	379	Delhi (11)
2	367	Mumbai (23)
3	362	Chennai (8)
4	333	Bengaluru (6)
5	331	Hyderabad (15)
6	312	Kolkata (20)
7	310	Goa (13)
8	298	Kochi (19)
9	295	Ahmedabad (2)
10	283	Agartala (1)

Table 2: Ranking of top 10 airports using the m-ranking method.

The airport Delhi with $T(i) = 379$ ranked first and Mumbai comes in the second position. We can see that the nodes which have different network related characteristics are ranked differently.

4. Conclusion

In this paper we proposed a new method named m-ranking to find node ranking in a weighted network. This method shows its difference from many other methods in the form of its ability to assign different ranks to non-similar nodes and rank nodes in weighted networks. We applied the method to find ranks of the nodes in an example network and to a real air transportation network.

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