

Transportation Problem of Multi-objective Multi-item Solid Using Fuzzy Environment

Aashish Bankar^{1,*}, Makhansingh Chouhan¹

¹*Department of Mathematics, Institute of Excellence in Higher Education, Bhopal, Madhya Pradesh, India*

Abstract

The transportation issue pertains to the reservation of fully loaded vehicles, which may include light commercial vehicles, medium-duty trucks, and heavy-duty trucks, among others. The transportation costs are incurred irrespective of whether the vehicle's capacity is fully utilized. Moreover, instead of transporting a single item, it may be essential to transfer various types of items from multiple sources to different destinations utilizing a variety of conveyances. The optimal transportation approach can be affected by several factors, such as the volume and weight of each product unit, as well as the insufficient availability of a sufficient number of specific vehicle types. This paper introduces a formulation of a multi-objective, multi-item solid transportation problem that considers all these factors. The problem is characterized by transportation cost and time parameters that are treated as fuzzy variables. By utilizing the credibility theory of fuzzy variables, a chance-constraint programming model is created and subsequently transformed into its deterministic equivalent. Lastly, a numerical example is included to illustrate the problem.

Keywords: Solid transportation problem; Credibility theory; Chance-constrained programming.

1. Introduction

As a generalization of classical transportation problem (TP), the solid transportation problem (STP) has been extended by considering some extra constraints along with source constraints and destination constraints. The STP was first presented by [14] considering the conveyance capacity constraints. Recently the STP has been studied by many researchers by describing it with many models and methods under different uncertain environments.

The majority of the papers referenced earlier examined STPs by taking into account the total available capacities (space) of conveyances, as well as the transportation cost associated with each unit of product transported. However, in transportation systems where full vehicles must be considered for the movement of products, various issues arise in the formulation of the problem. For instance, the transportation cost of a vehicle does not depend on whether the vehicle's capacity is fully utilized; there

*Corresponding author (doctorateaashishbankar@gmail.com)(Research Scholar)

are also considerations regarding the volume and weight capacities of the vehicles, as well as limitations on the number of certain types of vehicles, among other factors. Furthermore, prior research primarily focused on the travel time of vehicles, yet it is crucial to also consider the loading and unloading times of products, which are influenced by both the types of vehicles and the characteristics of the products. In this paper, we introduce a multi-objective multi-item solid transportation model that addresses all these concerns. The problem presented is formulated with transportation time and cost parameters treated as fuzzy variables.

2. Preliminaries

A fuzzy variable [34] is defined as a function from the possibility space (Θ, p, Pos) to the set of real numbers \Re to describe fuzzy phenomena, where possibility measure (Pos) of a fuzzy event $\{\tilde{\zeta} \in B\}$, $B \subset \Re$ is defined as

$$Pos\{\tilde{\zeta} \in B\} = \sup_{x \in B} \mu_{\tilde{\zeta}}(x),$$

where $\mu_{\tilde{\zeta}}(x)$ is the possibility distribution of $\tilde{\zeta}$.

For normalized fuzzy variable ($\sup_{x \in \Re} \mu_{\tilde{\zeta}}(x) = 1$), necessity measure (Nec) is defined as

$$Nec\{\tilde{\zeta} \in B\} = 1 - Pos\{\tilde{\zeta} \in B^c\} = 1 - \sup_{x \in B^c} \mu_{\tilde{\zeta}}(x)$$

and the credibility measure [27] of $\{\tilde{\zeta} \in B\}$ is defined as

$$Cr\{\tilde{\zeta} \in B\} = \frac{1}{2}(Pos\{\tilde{\zeta} \in B\} + Nec\{\tilde{\zeta} \in B\}).$$

Optimistic and pessimistic value: Let $\tilde{\zeta}$ be a fuzzy variable and $\alpha \in [0, 1]$. Then, α -optimistic value and α -pessimistic values of $\tilde{\zeta}$ are defined as follows.

$$\tilde{\zeta}_{sup}(\alpha) = \sup\{r : Cr\{\tilde{\zeta} \geq r\} \geq \alpha\},$$

$$\tilde{\zeta}_{inf}(\alpha) = \inf\{r : Cr\{\tilde{\zeta} \leq r\} \geq \alpha\}.$$

Example 2.1. Let $\tilde{\zeta} = (r_1, r_2, r_3, r_4)$ be a trapezoidal fuzzy variable. Then its α -optimistic and α -pessimistic values are as given below.

$$\tilde{\zeta}_{sup}(\alpha) = \begin{cases} 2\alpha r_3 + (1 - 2\alpha)r_4, & \text{if } \alpha \leq 0.5; \\ (2\alpha - 1)r_1 + 2(1 - \alpha)r_2, & \text{if } \alpha > 0.5. \end{cases}$$

$$\tilde{\zeta}_{inf}(\alpha) = \begin{cases} (1 - 2\alpha)r_1 + 2\alpha r_2, & \text{if } \alpha \leq 0.5; \\ 2(1 - \alpha)r_3 + (2\alpha - 1)r_4, & \text{if } \alpha > 0.5. \end{cases}$$

3. Problem Explanation and Model Development

In many practical transportation scenarios, full vehicles?such as light commercial vehicles, medium-duty trucks, heavy-duty trucks for road transport, and coaches for rail transport?must be reserved, and the number of vehicles is determined based on the quantity of products to be transported along a specific route. In this case, the full transportation cost of a vehicle must be paid regardless of whether the vehicle's capacity is fully utilized by the products.

In this study, we have taken into account the travel time, as well as the loading and unloading times for each type of vehicle. We also consider the weight capacities of the vehicles. Furthermore, the number of vehicles of a specific type of conveyance may be restricted for certain routes. In such cases, it is essential to factor in a constraint regarding the number of available vehicles. This limitation on the number of vehicles can significantly impact the optimal transportation strategy. For instance, if there are not enough vehicles of a particular type of conveyance available, it may necessitate the use of a different type of conveyance that incurs higher costs.

3.1 Model formulation

Different parameters and decision variables as used to formulate the mathematical model are given below:

Parameters

p	Type of Items/products; $p = 1, 2, \dots, l$
i	Source of transportation /origin of transportation; $i = 1, 2, \dots, m$
j	Destination of transportation/demand point; $j = 1, 2, \dots, n$
k	Types of vehicles used; $k = 1, 2, \dots, K$
t_{ijk}	Time required to travel from source i to destination j through used vehicle of type k
α_{pk}	Time of loading and unloading one unit of item p for the vehicle of type k
c_{ijk}	Per trip transportation cost of a vehicle of type k for traveling from origin i to destination j
V_k	Volume capacity of a vehicle of type k
W_k	Weight capacity of a vehicle of type k
v^p	Volume of one unit of product p
w^p	Weight of one unit of product p
a_i^p	Amount of a product p available at origin i
b_j^p	Demand of the product p at destination j
Q_k	Number of available vehicles of type k

f_i The objective value

Decision variables

x_{ijk}^p Amount of item p transported from source i to destination j using vehicles of type k

z_{ijk} Number of required vehicles of type k for transporting goods from source i to destination j

The proposed bi-objective MISTP model with vehicle cost, volume and weight capacity is formulated as follows:

$$\text{Min } f_1 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K c_{ijk} z_{ijk}, \quad (1)$$

$$\text{Min } f_2 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K (t_{ijk} z_{ijk} + \sum_{p=1}^l \alpha_{pk} x_{ijk}^p), \quad (2)$$

$$\text{subject to } \sum_{j=1}^n \sum_{k=1}^K x_{ijk}^p \leq a_i^p, \quad i = 1, 2, \dots, m; p = 1, 2, \dots, l, \quad (3)$$

$$\sum_{i=1}^m \sum_{k=1}^K x_{ijk}^p \geq b_j^p, \quad j = 1, 2, \dots, n; p = 1, 2, \dots, l, \quad (4)$$

$$\sum_{p=1}^l v^p x_{ijk}^p \leq z_{ijk} \cdot V_k, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n; k = 1, 2, \dots, K, \quad (5)$$

$$\sum_{p=1}^l w^p x_{ijk}^p \leq z_{ijk} \cdot W_k, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n; k = 1, 2, \dots, K, \quad (6)$$

$$\sum_{i=1}^m \sum_{j=1}^n z_{ijk} \leq Q_k, \quad k = 1, 2, \dots, K, \quad (7)$$

$$x_{ijk}^p \geq 0, \quad \forall i, j, k, p. \quad (8)$$

We consider, the first objective function is, f_1 represents the total transportation cost, and the second objective function, f_2 represents the total time (one complete trip durations), where t_{ijk} represents the the travel time for each vehicle of type k from source i to destination j , and $\sum_{p=1}^l \alpha_{pk} x_{ijk}^p$ represents the loading and unloading time of all types of items transported from source i to destination j for the vehicle of type k .

The constraint (3) ensures that total transported amount of each type of item from some source must be equal to or less than the availability (a_i^p) of the item in that source. The constraint (4) indicates that total transported amount of each type of item from the sources should satisfy the demand (b_j^p) of destination.

The constraint (5) ensures that total transported amount of products must be equal to or less than the total volume capacities of all types of allocated vehicles from a source i to a destination j . The

constraint (6) ensures that weights of total transported products must be equal to or less than the total weight capacities of all types of allocated vehicles from a source i to a destination j . The constraint (7) is imposed on the availability of vehicles of type k for the source i to destination j .

Consider that transportation cost c_{ijk} , travel time t_{ijk} , loading and unloading time α_{pk} in the above model are represented by fuzzy variable respectively as follows:

$$\tilde{c}_{ijk} = (c_{ijk}^1, c_{ijk}^2, c_{ijk}^3, c_{ijk}^4),$$

$$\tilde{t}_{ijk} = (t_{ijk}^1, t_{ijk}^2, t_{ijk}^3, t_{ijk}^4),$$

$$\tilde{\alpha}_{pk} = (\alpha_{pk}^1, \alpha_{pk}^2, \alpha_{pk}^3, \alpha_{pk}^4)$$

for all i, j, k and p . Then the problem (1)-(8) becomes

$$\text{Min } \tilde{f}_1 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K \tilde{c}_{ijk} z_{ijk}, \quad (9)$$

$$\text{Min } \tilde{f}_2 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K (\tilde{t}_{ijk} z_{ijk} + \sum_{p=1}^l \tilde{\alpha}_{pk} x_{ijk}^p), \quad (10)$$

$$\text{subject to (3) - (8)}. \quad (11)$$

Since \tilde{c}_{ijk} are trapezoidal fuzzy numbers and $z_{ijk} \geq 0$ for all i, j, k , so

$$\tilde{f}_1 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K \tilde{c}_{ijk} z_{ijk}$$

is also trapezoidal fuzzy number for any feasible solution and given by

$$\tilde{f}_1 = (r_1, r_2, r_3, r_4),$$

where

$$r_1 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K c_{ijk}^1 z_{ijk}, \quad r_2 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K c_{ijk}^2 z_{ijk}, \quad (12)$$

$$r_3 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K c_{ijk}^3 z_{ijk}, \quad r_4 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K c_{ijk}^4 z_{ijk}. \quad (13)$$

Similarly $\tilde{t}_{ijk}, \tilde{\alpha}_{pk}$ are trapezoidal fuzzy numbers and $x_{ijk}^p \geq 0$ for all i, j, k, p . So \tilde{f}_2 can be represented by

$$\tilde{f}_2 = (s_1, s_2, s_3, s_4),$$

where

$$s_1 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K (t_{ijk}^1 z_{ijk} + \sum_{p=1}^l \alpha_{pk}^1 x_{ijk}^p), \quad s_2 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K (t_{ijk}^2 z_{ijk} + \sum_{p=1}^l \alpha_{pk}^2 x_{ijk}^p), \quad (14)$$

$$s_3 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K (t_{ijk}^3 z_{ijk} + \sum_{p=1}^l \alpha_{pk}^3 x_{ijk}^p), \quad s_4 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K (t_{ijk}^4 z_{ijk} + \sum_{p=1}^l \alpha_{pk}^4 x_{ijk}^p). \quad (15)$$

4. Solution Methodology: Chance-constrained Programming

Applying this method using credibility measure for the above problem with fuzzy transportation costs and time parameters, the following chance-constrained programming (CCP) model is formulated:

$$\text{Min (Min}_{\bar{f}_1} \bar{f}_1), \quad (16)$$

$$\text{Min (Min}_{\bar{f}_2} \bar{f}_2), \quad (17)$$

$$\text{s.t. Cr} \left\{ \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K \tilde{c}_{ijk} z_{ijk} \leq \bar{f}_1 \right\} \geq \eta, \quad (18)$$

$$\text{Cr} \left\{ \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K (\tilde{t}_{ijk} y_{ijk} + \sum_{p=1}^l \tilde{\alpha}_{pk} x_{ijk}^p) \leq \bar{f}_2 \right\} \geq \gamma, \quad (19)$$

$$\text{subject to (3) – (8)}. \quad (20)$$

Since our problem is minimization problem, for the objective functions (9) and (10) we want to minimize η -pessimistic and γ -pessimistic values of \tilde{f}_1 and \tilde{f}_2 respectively, where η and γ ($0 < \eta, \gamma \leq 1$) are preassigned values. More specifically, for the objective function (9) we want to minimize

$$\inf\{\bar{f}_1 : \text{Cr}\{\tilde{f}_1 \leq \bar{f}_1\} \geq \eta\}$$

which is represented by (16) and (18) together. Similar explanation follows for (17) and (19). so that (16) and (18) together represent that we are going to minimize smallest possible \bar{f}_1 with the condition that the credibility degree that the respective objective function \tilde{f}_1 less than or equal to it should be at least the preassigned numbers η .

4.1 Deterministic form of the CCP Model

In the above CCP model, $\text{Min}_{\bar{f}_1} \bar{f}_1$, s.t.

$$\text{Cr}\left\{ \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K \tilde{c}_{ijk} z_{ijk} \leq \bar{f}_1 \right\} \geq \eta$$

can be equivalently computed as

$$f'_1 = \inf\{r : \text{Cr}\{\tilde{f}_1 \leq r\} \geq \eta\},$$

which is nothing but η -pessimistic value to \tilde{f}_1 and so is equal to f'_1 , where

$$f'_1 = \begin{cases} (1 - 2\eta)r_1 + 2\eta r_2, & \text{if } \eta \leq 0.5; \\ 2(1 - \eta)r_3 + (2\eta - 1)r_4, & \text{if } \eta > 0.5. \end{cases} \quad (21)$$

Here r_1, r_2, r_3, r_4 are given in equations (12) and (13). Similarly $\text{Min}_{\tilde{f}_2} \tilde{f}_2$, s.t.

$$\text{Cr}\left\{\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K (\tilde{t}_{ijk} y_{ijk} + \sum_{p=1}^l \tilde{\alpha}_{pk} x_{ijk}^p) \leq \tilde{f}_2\right\} \geq \gamma$$

is equivalent to

$$f'_2 = \inf\{s : \text{Cr}\{\tilde{f}_2 \leq s\} \geq \gamma\},$$

which is nothing but γ -pessimistic value to \tilde{f}_2 (i.e. $\tilde{f}_{2_{\text{inf}}}(\gamma)$) and so is equal to f'_2 , where

$$f'_2 = \begin{cases} (1 - 2\gamma)s_1 + 2\gamma s_2, & \text{if } \gamma \leq 0.5; \\ 2(1 - \gamma)s_3 + (2\gamma - 1)s_4, & \text{if } \gamma > 0.5. \end{cases} \quad (22)$$

Here s_1, s_2, s_3, s_4 are given in equations (14) and (15).

Finally crisp form of the above CCP Model can be written as

$$\begin{aligned} & \text{Min } f'_1, \\ & \text{Min } f'_2, \\ & \text{subject to (3) - (8)}. \end{aligned}$$

To solve the deterministic multi-objective problem, we apply two multi-objective optimization methods, namely, the fuzzy programming technique [3,46] and global criterion method which are discussed briefly in the next section.

5. Method Used to Solve Multi-objective Optimization Problem

Consider a multi-objective optimization problem with R objective functions:

$$\text{Min } F(x) = (f_1(x), f_2(x), \dots, f_R(x))^T \quad \text{s.t. } x \in D,$$

where D is the set of feasible solutions.

Fuzzy Technique

The steps to solve the multi-objective models using fuzzy programming technique are as follows:

Step 1: Address the multi-objective challenge by treating it as a single objective issue, focusing solely

on one objective f_t ($t = 1, 2, \dots, R$) at a time (disregarding all other objectives) to derive the optimal solution x^{t*} for R distinct single objective problems.

Step 2: Calculate the values of each objective function at all these R optimal solutions x^{t*} ($t = 1, 2, \dots, R$) and find the upper and lower bound for each objective given by

$$U_t = \text{Max}\{f_t(x^{1*}), f_t(x^{2*}), \dots, f_t(x^{R*})\}$$

and $L_t = f_t(x^{t*})$, $t = 1, 2, \dots, R$ respectively.

Step 3: Then an initial model is given by

Find x

$$\text{subject to } f_t(x) \leq L_t, \quad t = 1, 2, \dots, R$$

and $x \in D$.

However, generally due to conflicting nature of the objective functions, feasible solution of the above model does not always exist.

Step 4: Construct the linear membership function $\mu_t(f_t)$ corresponding to t -th objective as

$$\mu_t(f_t) = \begin{cases} 1, & \text{if } f_t \leq L_t; \\ \frac{U_t - f_t(x)}{U_t - L_t}, & \text{if } L_t < f_t < U_t; \\ 0, & \text{if } f_t \geq U_t, \end{cases} \quad \forall t.$$

Step 5: Formulate fuzzy linear programming problem using max-min operator as

Max λ

$$\text{subject to } \lambda \leq \mu_t(f_t) = (U_t - f_t)/(U_t - L_t), \quad \forall t$$

and $x \in D$,

$$\lambda \geq 0 \text{ and } \lambda = \min_t\{\mu_t(f_t)\}.$$

Step-6: Now the reduced problem is solved by a linear optimization technique and the optimum compromise solutions are obtained.

5.1 Global Criteria Method

The steps of this method to solve the multi-objective models are as follows:

Step-1: Solve the multi-objective problem as a single objective problem using, each time, only one objective f_t ($t = 1, 2, \dots, R$) ignoring all other objectives.

Step-2: From the results of step-1, determine the ideal objective vector, say $(f_1^{\min}, f_2^{\min}, \dots, f_R^{\min})$ and corresponding values of $(f_1^{\max}, f_2^{\max}, \dots, f_R^{\max})$.

Step-3: Formulate the following auxiliary problem

$$\text{Min } G(x)$$

$$\text{s.t. } x \in D,$$

$$G(x) = \text{Min} \left\{ \sum_{t=1}^R \left(\frac{f_t(x) - f_t^{\min}}{f_t^{\min}} \right)^q \right\}^{\frac{1}{q}},$$

$$\text{or, } G(x) = \text{Min} \left\{ \sum_{t=1}^R \left(\frac{f_t(x) - f_t^{\min}}{f_t^{\max} - f_t^{\min}} \right)^q \right\}^{\frac{1}{q}},$$

where $1 \leq q \leq \infty$.

An usual value of q is 2. This method is then called global criterion method in L_2 norm.

References

- [1] E. E. Ammar and E. A. Youness, *Study on multiobjective transportation problem with fuzzy numbers*, Applied Mathematics and Computation, 166(2005), 241-253.
- [2] C. R. Bector and S. Chandra, *Fuzzy mathematical programming and fuzzy matrix games*, Springer, Berlin, Heidelberg, (2005).
- [3] A. K. Bit, M. P. Biswal and S. S. Alam, *Fuzzy programming approach to multi-objective solid transportation problem*, Fuzzy Sets and Systems, 57(1993), 183-194.
- [4] J. J. Buckley and E. Eslami, *An Introduction to Fuzzy Logic and Fuzzy Sets*, Physica-Verlag Heidelberg, New York, (2002).
- [5] S. Chanas and D. Kuchta, *A concept of the optimal solution of the transportation problem with fuzzy cost coefficients*, Fuzzy Sets and Systems, 82(1996), 299-305.
- [6] M. Chen and S. A. Ludwig, *Color image segmentation using fuzzy C-regression model*, Advances in Fuzzy Systems, 2017(2017), 1-15.
- [7] H. Dalman, N. Guzel and M. Sivri, *A fuzzy set-based approach to multi-objective multi-item solid transportation problem under uncertainty*, International Journal of Fuzzy Systems, 18(4)(2016), 716-729.
- [8] A. Das, U. Bera and M. Maiti, *Defuzzification of trapezoidal type-2 fuzzy variables and its application to solid transportation problem*, Journal of Intelligent & Fuzzy Systems, 30(2016), 2431-2445.

- [9] D. Dubois and H. Prade, *Possibility Theory: An Approach to Computerized Processing of Uncertainty*, Plenum, New York, (1998).
- [10] M. Fei, P. Yi, Z. Kedong and Z. Jianyong, *On-line hybrid fault diagnosis method for high voltage circuit breaker*, Journal of Intelligent & Fuzzy Systems, 33(5)(2017), 2763-2774.
- [11] M. R. Fegad, V. A. Jadhav and A. A. Muley, *Finding an optimal solution of transportation problem using interval and triangular membership functions*, European Journal of Scientific Research, 60(3)(2011), 415-421.
- [12] P. K. Giri, M. K. Maiti and M. Maiti, *Fully fuzzy fixed charge multi-item solid transportation problem*, Applied Soft Computing, 27(2015), 77-91.
- [13] M. Hassaballah and A. Ghareeb, *A framework for objective image quality measures based on intuitionistic fuzzy sets*, Applied Soft Computing, 57(2017), 48-59.
- [14] K. B. Haley, *The solid transportation problem*, Operations Research, 10(1962), 448-463.
- [15] G. Jeon, M. Anisetti, E. Damiani and O. Monga, *Real-time image processing systems using fuzzy and rough sets techniques*, Soft Computing, 22(5)(2018), 1381-1384.
- [16] F. Jiménez and J. L. Verdegay, *Solving fuzzy solid transportation problems by an evolutionary algorithm based parametric approach*, European Journal of Operational Research, 117(1999), 485-510.
- [17] P. Kundu, S. Kar and M. Maiti, *Multi-objective solid transportation problems with budget constraint in uncertain environment*, International Journal of Systems Science, 45(8)(2014), 1668-1682.
- [18] P. Kundu, S. Kar and M. Maiti, *Multi-item solid transportation problem with type-2 fuzzy parameters*, Applied Soft Computing, 31(2015), 61-80.
- [19] P. Kundu, S. Majumder, S. Kar and M. Maiti, *A method to solve linear programming problem with interval type-2 fuzzy parameters*, Fuzzy Optimization and Decision Making, (2018).
- [20] S. M. Lee and L. J. Moor, *Optimizing transportation problems with multiple objectives*, AIEE Transactions, 5(1973), 333-338.
- [21] C. Li, Z. Ding, D. Qian and Y. Lv, *Data-driven design of the extended fuzzy neural network having linguistic outputs*, Journal of Intelligent & Fuzzy Systems, 34(1)(2018), 349-360.
- [22] Y. Li, K. Ida and M. Gen, *Improved genetic algorithm for solving multi-objective solid transportation problem with fuzzy numbers*, Computer and Industrial Engineering, 33(3-4)(1997), 589-592.
- [23] L. Li and K. K. Lai, *A fuzzy approach to the multi-objective transportation problem*, Computers and Operation Research, 27(2000), 43-57.

- [24] B. Liu, *Minimax chance constrained programming model for fuzzy decision systems*, Information Sciences, 112(1-4)(1998), 25-38.
- [25] B. Liu, *A survey of credibility theory*, Fuzzy Optimization and Decision Making, 5(4)(2006), 387-408.
- [26] B. Liu and K. Iwamura, *Chance constrained programming with fuzzy parameters*, Fuzzy Sets and Systems, 94(2)(1998), 227-237.
- [27] B. Liu and Y. K. Liu, *Expected value of fuzzy variable and fuzzy expected value models*, IEEE Transactions On Fuzzy Systems, 10(2002), 445-450.
- [28] P. Liu, *Multiple attribute group decision making method based on interval-valued intuitionistic fuzzy power Heronian aggregation operators*, Computers & Industrial Engineering, 108(2017), 199-212.
- [29] P. Liu and S. M. Chen, *Multiattribute group decision making based on intuitionistic 2-tuple linguistic information*, Information Sciences, 430-431(2018), 599-619.
- [30] P. Liu and S. M. Chen, *Group decision making based on Heronian aggregation operators of intuitionistic fuzzy numbers*, IEEE Transactions on Cybernetics, 47(9)(2017), 2514-2530.
- [31] P. Liu, J. Liu and J. M. Merigó, *Partitioned Heronian means based on linguistic intuitionistic fuzzy numbers for dealing with multi-attribute group decision making*, Applied Soft Computing 62(2018), 395-422.
- [32] P. Liu, S. M. Chen and J. Liu, *Some intuitionistic fuzzy interaction partitioned Bonferroni mean operators and their application to multi-attribute group decision making*, Information Sciences, 411(2017), 98-121.
- [33] J. M. Mendel, *Uncertain rule-based fuzzy logic systems: Introduction and new directions*, Prentice-Hall, NJ, (2001).
- [34] S. Nahmias, *Fuzzy variable*, Fuzzy Sets and Systems, 1(1978), 97-101.
- [35] P. Wang, *Fuzzy contactability and fuzzy variables*, Fuzzy Sets and Systems, 8(1982), 81-92.
- [36] A. Nagarjan and K. Jeyaraman, *Solution of chance constrained programming problem for multi-objective interval solid transportation problem under stochastic environment using fuzzy approach*, International Journal of Computer Applications, 10(9)(2010), 19-29.
- [37] A. Ojha, B. Das, S. Mondal and M. Maity, *An entropy based solid transportation problem for general fuzzy costs and time with fuzzy equality*, Mathematical and Computer Modeling, 50(1-2)(2009), 166-178.
- [38] B. Sinha, A. Das and U. K. Bera, *Profit maximization solid transportation problem with trapezoidal interval type-2 fuzzy numbers*, International Journal of Applied and Computational Mathematics, 2(1)(2016), 41-56.

- [39] L. Yang and Y. Feng, *A bicriteria solid transportation problem with fixed charge under stochastic environment*, Applied Mathematical Modelling, 31(2007), 2668-2683.
- [40] L. Yang and L. Liu, *Fuzzy fixed charge solid transportation problem and algorithm*, Applied Soft Computing, 7(2007), 879-889.
- [41] F. Waiel and Abd El-wahed, *A multi-objective transportation problem under fuzziness*, Fuzzy Sets and Systems, 117(2001), 27-33.
- [42] W. Wang, *Finite-time synchronization for a class of fuzzy cellular neural networks with time-varying coefficients and proportional delays*, Fuzzy Sets and Systems, 338(2018), 40-49.
- [43] Y. Wu and J. Dong, *Fault detection for T-S fuzzy systems with partly unmeasurable premise variables*, Fuzzy Sets and Systems, 338(2018), 136-156.
- [44] L. A. Zadeh, *Fuzzy sets*, Information and Control, 8(3)(1965), 338-353.
- [45] L. A. Zadeh, *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems, 1(1978), 3-28.
- [46] H. -J. Zimmermann, *Fuzzy programming and linear programming with several objective functions*, Fuzzy Sets and Systems, 1(1978), 45-55.