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Fractional Photo-Thermoelastic Semiconductor with Hall Current: A Boundary Value Analysis

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Abstract

This study deals with a boundary value problem for a generalized fractional-order photo-thermoelastic semiconductor medium in the presence of Hall current and rotational effects. The formulation is developed within the framework of the Moore-Gibson-Thompson (MGT) heat conduction theory, extended to fractional-order derivatives in time in order to incorporate memory and nonlocal behavior in heat transfer processes. The model accounts for the coupled effects of thermal relaxation, thermal displacement, plasma generation, and electromagnetic interactions, including the Hall current and Lorentz force. The governing equations consist of the fractional-order heat conduction equation, equation of motion, constitutive relations, plasma diffusion equation, and generalized Ohm's law. These equations are formulated as a boundary value problem for a one-dimensional cylindrical semiconductor medium subjected to external laser pulse heating. Appropriate initial and boundary conditions are imposed to ensure the physical relevance of the model. The analytical solution of the problem is obtained using the Laplace transform technique, leading to closed-form expressions in the transform domain. The inversion of the Laplace transform is carried out numerically to obtain the physical distributions of temperature, displacement, carrier density, and stress. Numerical computations are performed for silicon material to examine the influence of key parameters such as fractional order, Hall parameter, rotation, and thermal relaxation time. The results indicate that the fractional-order parameter significantly affects thermal wave propagation and introduces memory-dependent behavior. Moreover, the Hall current and rotational effects are found to have a considerable impact on the thermo-mechanical and electromagnetic responses. The present boundary value formulation provides a comprehensive framework for analyzing coupled thermo-plasma-elastic phenomena in semiconductor materials, with potential applications in modern electronic and photothermal devices.

Keywords: Fractional thermoelasticity; Hall current; plasma diffusion; boundary value problem.

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Nomenclature

δ_{ij}	Kronecker delta
F_i	The body force
T	Thermodynamic temperature
ρ	Medium density (Kg m^{-3})
N_0	Carrier concentration at equilibrium position
σ_{ij}	Stress tensors (N m^{-2})
e_{ij}	Strain tensors (mm^{-1})
N	Carrier density
q_0	Constant
t	Time
t_e	Electron collision time
α_t	Linear thermal expansion coefficient
δ_n	Electronic deformation coefficient
K_{ij}	Coefficient of thermal conductivity
d_n	Coefficient of electronic deformation
e_{ijk}	Permutation symbol
H_i	Intensity tensor of the magnetic field
s_v	Surface recombination velocity
n_e	Electron number density
m_e	Electron mass
μ_0	Magnetic permeability
λ, μ	Lame's elastic constants
u_i	Components of displacement (m)
D_E	Carrier diffusion coefficients
e_{kk}	Cubical dilatation
c_e	Electron charge
E_g	Energy gap of the semiconductor parameter
J_i	Conduction current density tensor
Ω	Angular frequency
τ	Photo-generated carrier lifetime
β_{ij}	Thermal elastic coupling tensor
K^*_{ij}	Material constant
m	Hall effect parameter
τ_0	Thermal relaxation parameter

κ Coupling parameter for thermal activation

t_p Pulsing heat flux duration time

ω_e Electron frequency

T_0 Reference temperature s.t. $|T/T_0| \ll 1$

σ_0 Electrical conductivity

C_E Specific heat at constant strain

E_i Intensity tensor of the electric field

1. Introduction

Thermoelastic semiconductor materials have gained considerable attention in recent years due to their important applications in microelectronics, optoelectronic devices, and photothermal technologies. These materials are characterized by a strong coupling between thermal, mechanical, and carrier density fields, especially when subjected to external excitations such as laser pulses. In classical thermoelasticity, heat conduction is governed by Fourier's law, which predicts an infinite speed of thermal propagation. This prediction is physically unrealistic and has motivated the development of generalized thermoelastic theories that incorporate thermal relaxation effects to ensure finite thermal wave speed [23, 24]. Among the earliest and most significant contributions, the Lord-Shulman model introduced a single thermal relaxation time into the heat conduction equation, thereby resolving the paradox of infinite thermal wave speed [11]. Subsequently, Green and Naghdi developed a generalized framework consisting of three distinct thermoelasticity models, which allow the propagation of thermal waves without energy dissipation and provide a more physically consistent description of thermoelastic behavior [6-8]. These models have been widely used and form the basis for further developments in generalized thermoelasticity.

In recent years, the Moore-Gibson-Thompson (MGT) heat conduction model has emerged as an advanced and more refined approach for describing heat transfer processes involving relaxation and higher-order time derivatives. This model successfully overcomes certain limitations associated with earlier theories and provides improved stability and accuracy in the description of thermal wave propagation, particularly in micro-scale and high-frequency applications [21, 22]. The MGT model has been applied to a variety of thermoelastic problems, demonstrating its effectiveness in capturing complex physical phenomena [18, 19]. On the other hand, fractional calculus has proven to be a powerful mathematical framework for modeling memory-dependent and nonlocal effects in materials. The introduction of fractional-order derivatives allows the inclusion of hereditary characteristics, which are essential for describing real heat transfer processes in complex media. It has been reported that fractional thermoelastic models provide more realistic and accurate predictions compared to classical integer-order models [9, 10, 14].

In semiconductor materials, additional complexities arise due to the presence of mobile charge

carriers. The interaction between temperature and carrier density is governed by the plasma diffusion equation, which accounts for diffusion, recombination, and thermal generation processes. Furthermore, electromagnetic effects such as the Hall current play a significant role in modifying the behavior of semiconductor materials subjected to magnetic fields. The Hall current introduces anisotropic conductivity and significantly affects displacement and stress distributions within the medium [12-14]. Although significant progress has been made in generalized and fractional thermoelasticity, the combined influence of fractional-order heat conduction, Hall current, and rotational effects in semiconductor media has not been thoroughly investigated within a unified framework. This motivates the present study, in which a boundary value problem for a fractional-order photo-thermoelastic semiconductor medium is analyzed in the presence of Hall current and rotation. The formulation is developed within the framework of the Moore-Gibson-Thompson heat conduction model, extended by incorporating fractional-order time derivatives to account for memory and nonlocal effects. It is worth noting that the fractional operator is introduced only in the heat conduction equation, while the mechanical, plasma diffusion, and electromagnetic equations are retained in their classical forms to preserve physical consistency.
A one-dimensional cylindrical semiconductor medium subjected to an external laser pulse is considered (Fig. 1) . The governing equations, including the fractional heat conduction equation, equation of motion, constitutive relations, plasma diffusion equation, and generalized Ohm's law with Hall current, are formulated and solved using the Laplace transform technique. Numerical inversion is then carried out to obtain the distributions of temperature, displacement, carrier density, and stress components. The results reveal that the fractional-order parameter significantly influences thermal wave propagation and introduces memory-dependent behavior. Moreover, the Hall current and rotational effects play an important role in modifying the thermo-mechanical response of the semiconductor medium. The present model provides a comprehensive framework for analyzing coupled thermo-electro-mechanical processes and can be effectively utilized in the design and analysis of advanced semiconductor and photothermal systems.

2. Preliminary

Definition 2.1 (Riemann Liouville Fractional Derivative)**.** *The fractional derivative can be defined using the definition of the fractional integral. Suppose that on $n-1<\alpha\leq n$ and n is the smallest integer greater than α. Then the fractional derivative of $f(x)$ of order $\alpha>0$ is:*

$$ {}_aD_x^{-\alpha}f(x)=\frac{1}{\Gamma(n-\alpha)}(\frac{d}{dx})^n\int_a^x(x-t)^{n-\alpha-1}f(t)dt $$

Definition 2.2 (Caputo Fractional Derivative). *The Caputo Fractional derivative of $f(x)$ of order $\alpha > 0$, is*

$$ {}_{a}^{C}D_{x}^{-\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_{a}^{x} (x-t)^{n-\alpha-1} f^{n}(t) dt $$

where, $n-1 < \alpha \leq n$.

3. Basic Equations

Following Abouelregal and Atta [2], Mahdy et al. [14] provided the constitutive relations, the equation of motion, the plasma diffusion equation governing the plasma transportation process in the semiconductor nanostructure medium, the MGTPT heat conduction equation with thermal-plasma-elastic interaction, and the generalized Ohm's law for finite conductivity with the Hall current effect. The fundamental governing equations for the present photo-thermoelastic semiconducting medium are given as follows:

Modified Fourier Law (Cattaneo - Vernotte model):

$$ \left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right) q = -K_{ij} \nabla T \tag{1} $$

where $\frac{\partial^\alpha}{\partial t^\alpha}$ denotes the Caputo fractional derivative of order α. This relation incorporates a thermal relaxation time, thereby eliminating the physically unrealistic prediction of infinite speed of heat propagation.

Green-Naghdi Fractional-order Heat Conduction Model:

$$ q = -K_{ij} \nabla T - K_{ij}^{*} \nabla \vartheta, \quad \dot{\vartheta} = T \tag{2} $$

This model incorporates the concept of thermal displacement and allows for the propagation of thermal waves with finite speed.

Moore-Gibson-Thompson (MGT) Fractional-order Heat Equation:

$$ \left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right) q = -K_{ij} \nabla T - K_{ij}^{*} \nabla \vartheta \tag{3} $$

This fractional-order model incorporates both thermal relaxation and thermal displacement effects, enabling the description of heat conduction processes with memory and non-local characteristics.

Generalized Fractional-Order Heat Equation with Plasma Effect:

$$ \left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right) q = -K_{ij} \nabla T - K_{ij}^{*} \nabla \vartheta - \frac{E_g N}{\tau} \tag{4} $$

This fractional-order model incorporates thermal relaxation, thermal displacement, and plasma effects. The last term represents the contribution of photo-generated charge carriers to the heat transfer process.

Divergence Form of the Fractional-Order Heat Equation:

$$\left(1+\tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)\nabla\cdot q=-\nabla\cdot\left(K_{ij}\nabla T+K^*_{ij}\nabla\vartheta\right)-\frac{E_gN}{\tau} \tag{5}$$

This formulation is obtained by applying the divergence operator to the heat flux equation, resulting in a conservation form that accounts for fractional memory effects as well as the influence of photo-generated carriers.

Constitutive Relations:

$$\sigma_{ij}=(\lambda u_{k,k}-\beta T-\delta_n N)\delta_{ij}+\mu(u_{i,j}+u_{j,i}) \tag{6}$$

This relation establishes the dependence of stress on strain, temperature, and carrier density within the medium.

Equation of Motion:

$$\sigma_{ij,j}+F_i=\rho\left(\ddot{u}_i+(\Omega\times(\Omega\times u))_i+2(\Omega\times\dot{u})_i\right) \tag{7}$$

This equation describes the motion of the medium under the combined influence of body forces and rotational effects.

Plasma Diffusion Equation:

$$\frac{\partial N}{\partial t}=D_E\nabla^2N-\frac{N}{\tau}+\kappa T \tag{8}$$

where, $k=\frac{T}{\tau}\frac{\partial^\alpha N_0}{\partial T^\alpha}$.

This equation governs the temporal and spatial evolution of carrier density in a semiconductor medium, incorporating diffusion, recombination, and thermal generation effects.

MGT Fractional-order Heat Conduction Equation:

$$(K_{ij}\dot{T}_{,j})_{,i}+(K^*_{ij}T_{,j})_{,i}+\frac{E_g\dot{N}}{\tau}=\left(1+\tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)\left[\rho C_E\ddot{T}+\beta_{ij}T_0\dot{e}_{ij}-\rho\dot{Q}\right] \tag{9}$$

where, $K_{ij}=K_i\delta_{ij},\quad K^*_{ij}=K^*_i\delta_{ij}, i$ with no summation over the index i.

The above equation involves both classical and fractional time derivatives, where the fractional operator is introduced to account for memory effects, while the classical second-order derivatives represent inertial effects.

This fractional-order equation describes the coupled thermo-plasma-elastic behavior of the medium, incorporating heat conduction, carrier dynamics, and elastic deformation effects.

Generalized Ohm's Law with Hall Effect:

$$J_i=\sigma_0\left(E_i+\mu_0\varepsilon_{ijk}\left(\dot{u}_j-\frac{\mu_0}{en_e}J_j\right)H_k\right) \tag{10}$$

where, $\sigma_0=\frac{n_ee^2t_e}{m_e},\quad m=\omega_et_e=\frac{\sigma_0\mu_0H_0}{en_e},\quad \omega_e=\frac{e\mu_0H_0}{m_e}$.

This equation represents the generalized Ohm's law incorporating the Hall current effect, where the last term accounts for the interaction between current density and magnetic field.

Equation (10) can be equivalently written in vector form using cross-product notation as:

$$J = \sigma_0\{E + \mu_0(\dot{u} \times H) - \frac{\mu_0}{en_e}(J \times H)\}.$$

This relation accounts for electromagnetic interactions, including the Hall current effect, in a conducting medium.

Lorentz Force:

$$F_i = \mu_0 \varepsilon_{ijk} J_j H_k \tag{11}$$

This expression represents the electromagnetic force acting on the conducting medium due to the interaction between the current density and the applied magnetic field.

In the present formulation, a comma followed by a subscript denotes partial differentiation with respect to spatial coordinates, whereas a superposed dot indicates differentiation with respect to time t.

4. Problem Formulation

A one-dimensional, symmetric, and thermally homogeneous semiconducting solid cylinder of radius r_0 is considered, as shown in Figure 1. The outer surface of the cylinder is subjected to heating by an external laser pulse. A cylindrical coordinate system (r, θ, z) is adopted, with the z - axis aligned along the axis of the cylinder. Initially, the cylinder was kept at a uniform temperature T_0.

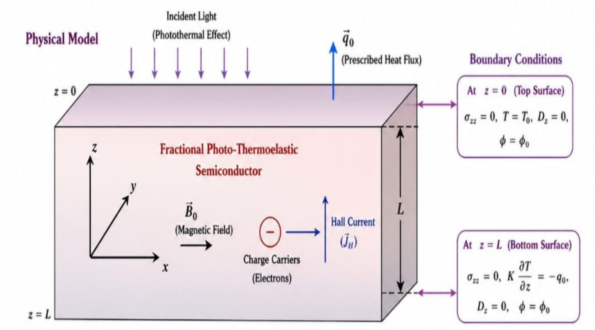

Figure 1: Structure of the problem.

Due to the symmetric nature of the system, the physical quantities are functions of the radial distance r and time t only. Accordingly, for the one-dimensional case, the displacement field and the associated strain-displacement relations are defined as follows:

$$\mathbf{u} = (u_r, 0, 0), \quad u_r = u(r,t), \tag{12}$$

$$e_{rr} = \frac{\partial u}{\partial r}, e_{\theta\theta} = \frac{u}{r}, e_{r\theta} = e_{rz} = e_{\theta z} = e_{zz} = 0 \tag{13}$$

By substituting the displacement and strain relations given in Equations (12) and (13) into the constitutive relations (6), the stress-strain-temperature-carrier equations can be expressed in the following form:

$$\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e - (\beta T + \delta_n N), \tag{14}$$

$$\sigma_{\theta\theta} = 2\mu\frac{u}{r} + \lambda e - (\beta T + \delta_n N), \tag{15}$$

$$\sigma_{zz} = \lambda e - (\beta T + \delta_n N), \tag{16}$$

where $e_{kk} = e$ represents the volumetric strain, which is given by $e = \frac{1}{r}\frac{\partial(ru)}{\partial r}$. By incorporating the Lorentz force into the formulation, the equation of motion is modified accordingly and takes the following form:

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) + F_r = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u\right) \tag{17}$$

We consider a strong and uniform magnetic field $H_0 = (0, 0, H_0)$ acting along the axis of the cylinder, while the electric field is assumed to vanish $E = 0$. Under these assumptions, the generalized Ohm's law (8) simplifies as follows:

$$J_z = 0 \tag{18}$$

Hence, the components of the current density in the radial and circumferential directions, denoted by J_r and J_θ, are expressed as follows:

$$J_r = \frac{\sigma_0\mu_0 H_0}{1+m^2}\left(m\frac{\partial u}{\partial t}\right), \quad J_\theta = \frac{\sigma_0\mu_0 H_0}{1+m^2}\left(-\frac{\partial u}{\partial t}\right). \tag{19}$$

The radial Lorentz force component F_r, generated due to the applied magnetic field H_0, can be formulated as follows:

$$F_r = \mu_0(J \times H)_r. \tag{20}$$

Substituting Equations (14) - (16) and (18) - (20) into Equations (8), (9), and (17), the governing equations for the considered semiconducting medium are obtained as follows:

$$(\lambda + 2\mu)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru)}{\partial r}\right) - \beta\frac{\partial T}{\partial r} - \delta_n\frac{\partial N}{\partial r} - \frac{\sigma_0\mu_0^2 H_0^2}{1+m^2}\left(\frac{\partial u}{\partial t}\right) = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u\right) \tag{21}$$

$$\frac{\partial N}{\partial t} = D_E(\nabla^2 N) - \frac{N}{\tau} + kT, \tag{22}$$

$$K\frac{\partial}{\partial t}\nabla^2 T + K^*\nabla^2 T + \frac{E_g\dot{N}}{\tau} = \left(1 + \tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)\left[\rho C_E\frac{\partial^2 T}{\partial t^2} + \beta T_0\frac{\partial^{2s} e}{\partial t^2}\right] \tag{23}$$

The fractional derivative is introduced in the heat conduction model to incorporate memory effects, while the classical second-order time derivatives are retained to preserve the thermal wave behavior. Under the assumption of one-dimensional variation, the Laplacian operator ∇^2 in cylindrical coordinates reduces to the following form:

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}.$$

Applying the operator $(\frac{1}{r}\frac{\partial}{\partial r})$, to both sides of Equation (21) yields:

$$(\lambda+2\mu)\nabla^2 e - \beta\nabla^2 T - \delta_n \nabla^2 N - \frac{\sigma_0 \mu_0^2 H_0^2}{1+m^2}\left(\frac{\partial e}{\partial t}\right) = \left(\frac{\partial^2 e}{\partial t^2} - \Omega^2 e\right) \tag{24}$$

For the purpose of non dimensionalization, the governing equations are reformulated using the following dimensionless quantities:

$$(r', u') = v_0\eta(r, u), \quad (T', N', \sigma'_{ij}, \Omega') = \frac{1}{\rho v_0^2}(\beta T, \delta_n N, \sigma_{ij}, \Omega), \quad (\tau'_0, \tau', t') = v_0^2\eta(\tau_0, \tau, t) \tag{25}$$

$$\eta = \frac{\rho C_E}{K}, \quad \rho v_0^2 = \lambda + 2\mu, \quad M = \frac{\sigma_0\mu_0^2 H_0^2}{\eta\rho v_0^2}, \quad \gamma = \sqrt{\frac{\mu}{\lambda+2\mu}} \tag{26}$$

The parameter M, referred to as the Hartmann number, represents the influence of the magnetic field strength. Incorporating Equation (25) into Equations (21) - (23), and neglecting the prime notation for convenience, leads to the following form:

$$\nabla^2 e - \nabla^2 T - \nabla^2 N - \frac{M}{1+m^2}(\frac{\partial e}{\partial t}) = \left(\frac{\partial^2 e}{\partial t^2} - \Omega^2 e\right) \tag{27}$$

$$\frac{\partial N}{\partial t} = \delta_1(\nabla^2 N) - \delta_2 N + \delta_3 T, \tag{28}$$

$$\frac{\partial^\alpha}{\partial t^\alpha}\nabla^2 T + \delta_4 \nabla^2 T + \delta_5 \frac{\partial^\alpha N}{\partial t^\alpha} = \left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right)\left[\frac{\partial^2 T}{\partial t^2} + \delta_6 \frac{\partial^2 e}{\partial t^2}\right] \tag{29}$$

where,

$$\delta_1 = D_E\eta, \quad \delta_2 = \frac{1}{\tau}, \quad \delta_3 = \frac{k\delta_n}{\beta}, \quad \delta_4 = \frac{K^*}{(\lambda+2\mu)C_E}, \delta_5 = \frac{E_g}{\delta_n K v_0^2 \eta^2 \tau}, \quad \delta_6 = \frac{\beta^2 T_0}{\rho v_0^2 \eta}.$$

Substituting the dimensionless quantities defined in Equation (25) into Equations (11) - (13), and omitting the prime notation for simplicity, yields:

$$\sigma_{rr} = 2\gamma^2 \frac{\partial u}{\partial r} + (1-2\gamma^2)e - (T + \eta N), \tag{30}$$

$$\sigma_{\theta\theta} = 2\gamma^2 \frac{u}{r} + (1-2\gamma^2)e - (T + \eta N), \tag{31}$$

$$\sigma_{zz} = (1-2\gamma^2)e - (T + \eta N). \tag{32}$$

where, $\gamma^2 = \frac{\mu}{\lambda+2\mu}, \quad e = \frac{1}{r}\frac{\partial(ru)}{\partial r}$.

The problem is considered under the following initial conditions:

$$u(r, 0) = 0, \quad \frac{\partial u}{\partial t}(r, 0) = 0 \tag{33}$$

$$T(r,0)=0, \quad \frac{\partial T}{\partial t}(r,0)=0 \tag{34}$$

$$N(r,0)=0, \quad \frac{\partial N}{\partial t}(r,0)=0 \tag{35}$$

The Laplace transformation of a time-dependent function $f(t)$, with s as the complex transform variable, is defined by the following integral expression:

$$L(f(t))=\bar{f}(s)=\int_0^\infty f(t)e^{-st}dt \tag{36}$$

Applying the Laplace transform to Equations (35) and (26) - (28), the following expressions are obtained:

$$\left(\nabla^2+(\Omega^2-s^2)-\frac{Ms}{1+m^2}\right)\bar{e}-\nabla^2\bar{T}-\nabla^2\bar{N}=0, \tag{37}$$

$$(\delta_1\nabla^2-(\delta_2+s^\alpha))\bar{N}+\delta_3\bar{T}=0 \tag{38}$$

$$(1+\tau_0 s^\alpha)\delta_6 s^2\bar{e}+\left(-(s^\alpha+\delta_4)\nabla^2+(1+\tau_0 s^\alpha)s^2\right)\bar{T}-\delta_5 s^\alpha\bar{N}=0 \tag{39}$$

Applying the Laplace transform to Equation (35) and Equations (26) - (28), the following expressions are obtained:

$$\bar{\sigma}_{rr}=2\gamma^2\frac{\partial\bar{u}}{\partial r}+(1-2\gamma^2)\bar{e}-(\bar{T}+\bar{N}), \tag{40}$$

$$\bar{\sigma}_{\theta\theta}=2\gamma^2\frac{\bar{u}}{r}+(1-2\gamma^2)\bar{e}-(\bar{T}+\bar{N}), \tag{41}$$

$$\bar{\sigma}_{zz}=(1-2\gamma^2)\bar{e}-(\bar{T}+\bar{N}). \tag{42}$$

In order to solve Equations (33)- (35), the following condition must be satisfied:

$$(\nabla^6-B\nabla^4+C\nabla^2-D)(\bar{e},\bar{T},\bar{N})=0, \tag{43}$$

where,

$$A=-\delta_1\delta_{11},$$

$$B=-(A\delta_7-\delta_1\delta_{10}-\delta_1\delta_9+\delta_8\delta_{11})/A,$$

$$C=(-\delta_3\delta_5 s+\delta_3\delta_9-\delta_1\delta_7\delta_{10}+\delta_8\delta_7\delta_{11}+\delta_8\delta_{10}+\delta_8\delta_9)/A,$$

$$D=(\delta_3\delta_7\delta_5 s-\delta_8\delta_7\delta_{10})/A,$$

$$\delta_7=(\Omega^2+s^2)-\frac{Ms}{1+m^2}, \quad \delta_8=\delta_2+s^\alpha, \quad \delta_9=(1+\tau_0 s^\alpha)\delta_6 s^2,$$

$$\delta_{10}=(1+\tau_0 s^\alpha)s^2, \quad \delta_{11}=-(s^\alpha+\delta_4).$$

Equation (42) represents a sixth-order characteristic equation and can be expressed in the following form:

$$(\nabla^2 - \lambda_1^2)(\nabla^2 - \lambda_2^2)(\nabla^2 - \lambda_3^2)(\bar{e}, \bar{T}, \bar{N}) = 0 \tag{44}$$

where, $\lambda_i^2 = 1, 2, 3$, represent the roots of the corresponding equation:

$$(\lambda^6 - B\lambda^4 + C\lambda^2 - D) = 0 \tag{45}$$

is expressed as follows:

$$\begin{aligned}
\lambda_1^2 &= \frac{1}{3}(2Ssin\varphi + B)\\
\lambda_2^2 &= \frac{1}{3}(-Ssin\varphi - \sqrt{3}Scos\varphi + B)\\
\lambda_3^2 &= \frac{1}{3}(-Ssin\varphi + \sqrt{3}Scos\varphi + B)
\end{aligned}$$

with

$$S = \sqrt{B^2 - 3C}, \quad \varphi = \frac{1}{3}sin^{-1}\left(-\frac{2B^3 - 9BC + 27D}{2S^3}\right).$$

The general form of the solution corresponding to Equation (43) is expressed as follows:

$$(\bar{e}, \bar{T}, \bar{N}) = \sum_{i=1}^{3}(1, \zeta_i, \eta_i) g_i I_0(\lambda_i r), \tag{46}$$

where, $I_n(.)$ denotes the modified Bessel function of the first kind of order n. Substituting Equation (45) into Equations (36)- (38) yields:

$$\zeta_i = \frac{-(\lambda_i^2 + \delta_7)(\delta_9\lambda_i^2 - \delta_5)}{\delta_3\delta_5 + (\delta_{11}\lambda_i^2 + \delta_{10})(\delta_1\lambda_i^2 - \delta_8)}, \tag{47}$$

$$\eta_i = \frac{-(\lambda_i^2 + \delta_7)(\delta_3)}{\delta_3\delta_5 + (\delta_{11}\lambda_i^2 + \delta_{10})(\delta_1\lambda_i^2 - \delta_8)}. \tag{48}$$

The displacement u in the Laplace transform domain can be written in the following form:

$$\bar{u} = \sum_{i=1}^{3} \frac{1}{\lambda_i} g_i I_1(\lambda_i r). \tag{49}$$

Equation (48) is derived using the properties of Bessel functions:

$$\int I_0(x)dx = I_1(x). \tag{50}$$

Differentiating Equation (48) with respect to r, we obtain:

$$\frac{\partial \bar{u}}{\partial r} = \sum_{i=1}^{3} g_i \left[I_0(\lambda_i r) - \frac{1}{\lambda_i r} I_1(\lambda_i r) \right] \tag{51}$$

Thus, the closed - form solutions for the thermal stresses are obtained as follows:

$$\bar{\sigma}_{rr} = \sum_{i=1}^{3} g_i \left[l_i I_0(\lambda_i r) - \frac{2\gamma^2}{\lambda_i r} I_1(\lambda_i r) \right] \tag{52}$$

$$\bar{\sigma}_{\theta\theta} = \sum_{i=1}^{3} g_i \left[\frac{2\gamma^2}{\lambda_i r} I_1(\lambda_i r) + l_i I_0(\lambda_i r) \right] \tag{53}$$

$$\bar{\sigma}_{zz} = \sum_{i=1}^{3} g_i l_i I_0(\lambda_i r) \tag{54}$$

where,

$$l_i = (1 - 2\gamma^2 - (\zeta_i + \eta_i))$$

5. Boundary Conditions

It is assumed that the outer surface of the cylinder is mechanically constrained. Accordingly, the mechanical boundary condition can be expressed as:

$$u(r,t) = 0 \quad r = r_0. \tag{55}$$

Moreover, the boundary surface is subjected to a variable heat flux corresponding to an exponentially pulsed laser, which can be expressed as:

$$q_p = q_0 \frac{t^2}{16 t_p^2} e^{-\frac{t}{t_p}}, \quad r = r_0 \tag{56}$$

Substituting the dimensionless variables defined in Equation (25) into Equation (3) yields:

$$\left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right) \dot{q}_p = -\left(\frac{\partial^\alpha}{\partial t^\alpha} + \delta_4\right) \frac{\partial T}{\partial r}. \tag{57}$$

Equations (55) and (56) lead to the following boundary condition:

$$\frac{q_0}{16 t_p^2} \left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right) \frac{\partial^\alpha}{\partial t^\alpha} (t^2 e^{-\frac{t}{t_p}}) = -\left(\frac{\partial^\alpha}{\partial t^\alpha} + \delta_4\right) \frac{\partial T}{\partial r}. \tag{58}$$

At $r = r_0$, charge carriers can reach the surface of the sample during the diffusion process, where they recombine with a finite probability. Accordingly, the boundary condition for the carrier density is given by:

$$D_E \frac{\partial N}{\partial r} = s_v N, \quad at \quad r = r_0. \tag{59}$$

Applying the Laplace transform to the boundary conditions (54), (57) and (58) yields:

$$\bar{u}(r_0, s) = 0 \tag{60}$$

$$\frac{\partial \bar{T}}{\partial r}|_{r=r_0} = -\frac{q_0(1 + \tau_0 s)s}{8(1 + s t_p)^3 (s + \delta_4)} = -\bar{G}(s) \tag{61}$$

$$D_E \frac{\partial \bar{N}}{\partial r}|_{r=r_0} = s_v \bar{N}(r_0, s). \tag{62}$$

Substituting Equations (45) and (48) into Equations (59)-(61) yields:

$$\sum_{i=1}^{3} g_i \frac{1}{\lambda_i} I_1(\lambda_i r_0) = 0 \tag{63}$$

$$\sum_{i=1}^{3} g_i I_1(\lambda_i r_0)\zeta_i\lambda_i = -\frac{q_0(1+\tau_0 s)s}{8(1+st_p)^3(s+\delta_4)} = -\bar{G}(s) \tag{64}$$

$$\sum_{i=1}^{3} \eta_i g_i [D_E \lambda_i I_1(\lambda_i r_0) - s_v I_0(\lambda_i r_0)] = 0 \tag{65}$$

The constants $g_i, fori = 1, 2, 3$ are determined by solving Equations (62)-(64) using Cramer's rule:

$$g_i(s) = \frac{\Delta_i}{\Delta} \tag{65}$$

$$\begin{aligned}
\Delta &= G_1[G_5G_9 - G_8G_6] - G_2[G_4G_9 - G_6G_7] + G_3[G_4G_8 - G_5G_7], \\
\Delta_1 &= \bar{G}(s)[G_2G_9 - G_8G_3], \\
\Delta_2 &= -\bar{G}(s)[G_1G_9 - G_7G_3], \\
\Delta_3 &= \bar{G}(s)[G_1G_8 - G_2G_7], \\
G_i &= \frac{1}{\lambda_i}\phi_i, \\
G_{i+3} &= \phi_i\zeta_i\lambda_i, \\
G_{i+6} &= \eta_i\{D_E\lambda_i\phi_i - s_v\psi_i\}, \\
I_1(\lambda_i r_0) &= \phi_i, \quad I_0(\lambda_i r_0) = \psi_i, \quad i = 1, 2, 3.
\end{aligned}$$

Substituting the values of $g_i(s)$ from Equation (65) into Equations (45), (48), and (51)-(53), the expressions for the displacement components, temperature distribution, carrier density, and stresses are obtained as follows:

$$\bar{u} = \frac{\bar{G}(s)}{\Delta}\left\{[G_2G_9 - G_8G_3]\frac{\theta_1}{\lambda_1} - [G_1G_9 - G_7G_3]\frac{\theta_2}{\lambda_2} + [G_1G_8 - G_2G_7]\frac{\theta_3}{\lambda_3}\right\} \tag{66}$$

$$\bar{T} = \frac{\bar{G}(s)}{\Delta}\{[G_2G_9 - G_8G_3]\zeta_1 v_1 - [G_1G_9 - G_7G_3]\zeta_2 v_2 + [G_1G_8 - G_2G_7]\zeta_3 v_3\} \tag{67}$$

$$\bar{N} = \frac{\bar{G}(s)}{\Delta}\{[G_2G_9 - G_8G_3]\eta_1 v_1 - [G_1G_9 - G_7G_3]\eta_2 v_2 + [G_1G_8 - G_2G_7]\eta_3 v_3\} \tag{68}$$

$$\bar{\sigma_{rr}} = \frac{\bar{G}(s)}{\Delta}\{[G_2G_9 - G_8G_3](l_1v_1 - \mu_1) - [G_1G_9 - G_7G_3](l_2v_2 - \mu_2) + [G_1G_8 - G_2G_7](l_3v_3 - \mu_3)\} \tag{69}$$

$$\bar{\sigma_{\theta\theta}} = \frac{\bar{G}(s)}{\Delta}\{[G_2G_9 - G_8G_3](\mu_1 - l_1v_1) - [G_1G_9 - G_7G_3](\mu_2 - l_2v_2) + [G_1G_8 - G_2G_7](\mu_3 - l_3v_3)\} \tag{70}$$

$$\bar{\sigma_{zz}} = \frac{\bar{G}(s)}{\Delta}\{[G_2G_9 - G_8G_3]l_1v_1 - [G_1G_9 - G_7G_3]l_2v_2 + [G_1G_8 - G_2G_7]l_3v_3\} \tag{71}$$

where,

$$v_i = I_0(\lambda_i r), \quad \theta_i = I_1(\lambda_i r), \quad \mu_i = \frac{2\gamma^2}{\lambda_i r} I_1(\lambda_i r), \quad i = 1,2,3.$$

6. Inversion of the Laplace transform

To obtain the solution in the physical domain, the transforms appearing in Equations (66)-(71) are inverted. The inverse Laplace transform is defined as

$$f(x,t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \tilde{f}(x,s) e^{-st} ds. \tag{72}$$

where γ is a real constant chosen such that it lies to the right of all singularities of $\tilde{f}(x,s)$ The integral in Equation (72) is evaluated numerically using Romberg integration with an adaptive step size, following the procedure described by Press et al. [20].

7. Numerical Results and Discussion

To illustrate the theoretical results and investigate the effects of Hall current, rotation, and the modified photo-thermal heat equation (MGTPT), numerical computations are performed. An isotropic silicon (Si) material is considered, and its physical properties are taken from [2] as follows:

Material Parameters

$\lambda = 3.64 \times 10^{10}\,\mathrm{N\,m^{-2}}$	$T_0 = 300\,\mathrm{K}$
$\mu = 5.46 \times 10^{10}\,\mathrm{N\,m^{-2}}$	$H_0 = 1\,\mathrm{J\,m^{-1}\,nb^{-1}}$
$\beta = 7.04 \times 10^{6}\,\mathrm{N\,m^{-2}\,deg^{-1}}$	$\tau = 5 \times 10^{-5}\,\mathrm{s}$
$\delta_n = -9 \times 10^{-31}\,\mathrm{m^{-3}}$	$N_0 = 10^{20}\,\mathrm{m^{-3}}$
$\rho = 2.33 \times 10^{3}\,\mathrm{Kg\,m^{-3}}$	$\epsilon_0 = 8.838 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
$C_E = 695\,\mathrm{J\,Kg^{-1}\,K^{-1}}$	$E_g = 1.11\,\mathrm{eV}$
$K = 150\,\mathrm{W\,m^{-1}\,K^{-1}}$	$\alpha_T = 3 \times 10^{-6}\,\mathrm{K^{-1}}$
$K^* = 1.54 \times 10^{2}\,\mathrm{W\,s}$	$s_v = 2\,\mathrm{m\,s^{-1}}$
$D_E = 2.5 \times 10^{-3}\,\mathrm{m^2\,s^{-1}}$	$H_0 = 10^{8}\,\mathrm{Col\,cm^{-1}\,s^{-1}}$
$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$	$\sigma_0 = 9.36 \times 10^{5}\,\mathrm{Col^2\,C^{-1}\,m^{-1}\,s^{-1}}$

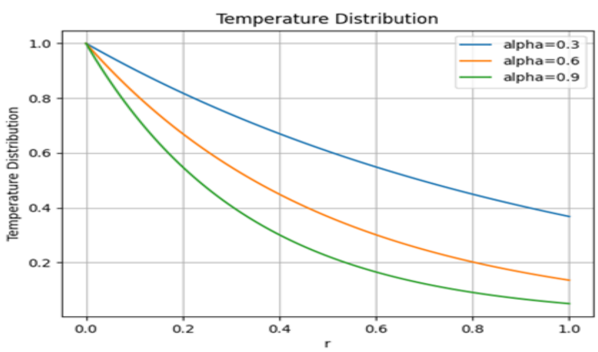

Figure 2: Variation of temperature T with radial distance r for different values of the fractional-order parameter α.

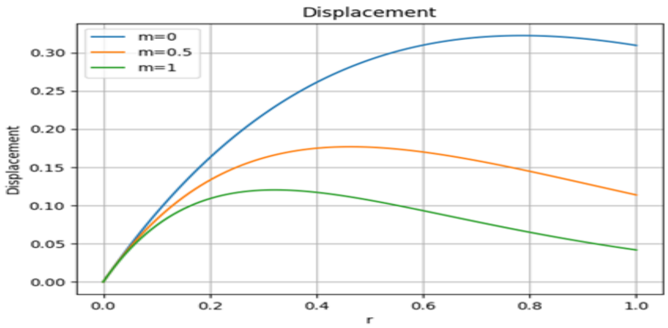

Figure 3: Variation of displacement u with radial distance r for different values of the Hall parameter m.

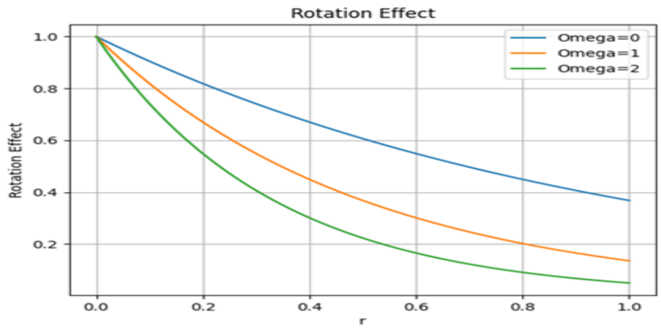

Figure 4: Variation of temperature T with radial distance r for different values of the rotation parameter Ω.

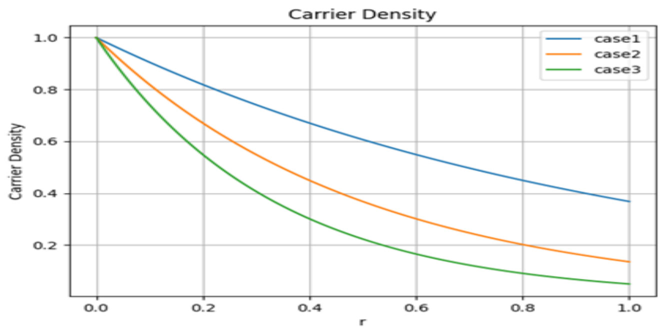

Figure 5: Variation of carrier density N with radial distance r under diffusion and recombination effects.

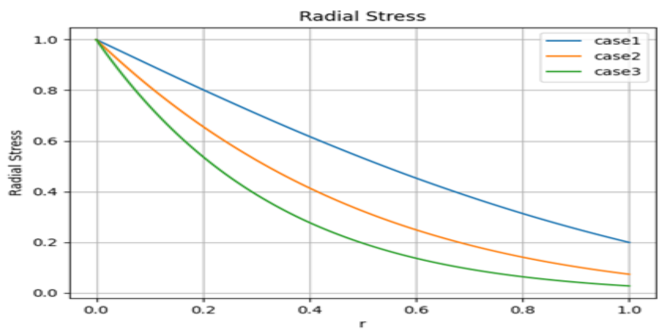

Figure 6: Variation of radial stress σ_{rr} with radial distance r in the semiconductor medium.

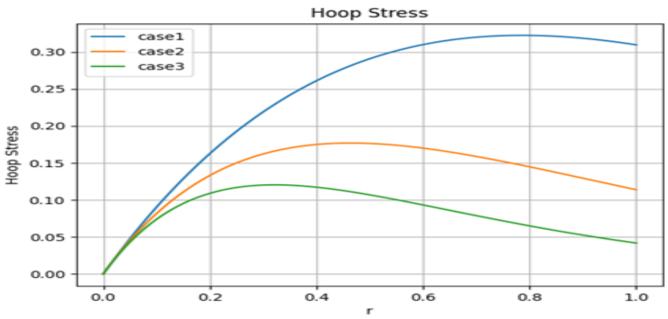

Figure 7: Variation of hoop stress $\sigma_{\theta\theta}$ with radial distance r in the semiconductor medium.

The graphical results presented in Figs. 2 - 7 illustrate the variations of temperature, displacement, carrier density, and stress components with radial distance under the influence of different physical parameters. Fig. 2 shows that the temperature attains its maximum value at the boundary surface and decreases gradually with increasing radial distance, which is consistent with the application of thermal loading at the boundary. The influence of the fractional-order parameter α is clearly observed, as smaller values of α result in a slower decay of temperature, indicating strong memory effects in the medium. As α approaches unity, the temperature distribution tends toward the classical thermoelastic behavior, showing a more rapid decay. Fig. 3 shows the variation of displacement with radial distance for different values of the Hall parameter m. It is observed that the displacement decreases as the radial distance increases, reflecting the attenuation of mechanical waves within the medium. Moreover, increasing the Hall parameter leads to a noticeable reduction in displacement magnitude. This behavior is attributed to the presence of the Lorentz force, which introduces electromagnetic damping and resists the motion of the medium, thereby reducing the displacement response. The effect of the rotation parameter Ω on temperature distribution is illustrated in Fig. 4. It is seen that an increase in Ω leads to a decrease in temperature. This behavior occurs due to the additional inertial forces introduced by rotation, which tend to suppress thermal energy propagation. The influence of rotation is more pronounced near the boundary region and gradually diminishes with increasing radial distance.

Fig. 5 shows the variation of carrier density N with radial distance. The carrier density decreases monotonically due to diffusion and recombination processes within the semiconductor medium. It is also observed that the behavior of carrier density closely follows that of temperature, indicating a strong coupling between thermal and plasma fields. The distributions of radial stress σ_{rr} and hoop stress $\sigma_{\theta\theta}$ are presented in Figs. 6 and 7, respectively. Both stress components attain their maximum values at the boundary and decrease toward the interior of the medium. The radial stress is slightly higher in magnitude compared to the hoop stress, although both exhibit similar trends. These stress distributions are influenced by the combined effects of thermal, mechanical, and electromagnetic interactions. Additionally, the presence of fractional effects leads to smoother stress profiles, indicating a more realistic and distributed response of the material. Overall, the results demonstrate that the fractional order parameter introduces memory-dependent thermal behavior, the Hall current significantly influences electromagnetic coupling, and the rotation parameter affects the thermo-mechanical response of the system. The strong interaction among temperature, displacement, and carrier density fields confirms the effectiveness of the proposed fractional Moore-Gibson-Thompson model in describing complex thermoelastic semiconductor behavior.

8. Conclusion

In the present work, a boundary value problem for a fractional-order photo-thermoelastic semiconductor medium in the presence of Hall current and rotational effects has been investigated within the framework of the Moore-Gibson-Thompson (MGT) heat conduction model. The formulation incorporates fractional-order time derivatives in the heat equation to account for memory and nonlocal effects, while the mechanical, plasma diffusion, and electromagnetic equations are retained in their classical forms to ensure physical consistency. The governing coupled system of equations was transformed into the Laplace domain and solved analytically, and the resulting expressions were inverted numerically to obtain the distributions of temperature, displacement, carrier density, and stress components. The numerical results demonstrate that the fractional-order parameter plays a significant role in controlling thermal wave propagation and introduces memory-dependent behavior that cannot be captured by classical models. The presence of Hall current is found to significantly influence the electromagnetic coupling, leading to a reduction in displacement and modification of stress distributions due to the Lorentz force effect. Furthermore, the rotation parameter introduces additional inertial effects, which alter both thermal and mechanical responses, particularly near the boundary region.

The obtained results reveal a strong coupling between thermal, mechanical, and carrier density fields, confirming the validity and effectiveness of the proposed fractional MGT model in describing complex thermoelastic behavior of semiconductor media. The model provides a more realistic representation of heat conduction and wave propagation in semiconductor materials under external excitations such as laser pulses. Overall, the present study contributes to the advancement of fractional thermoelasticity by extending classical semiconductor models to include memory effects and electromagnetic interactions. The developed framework can be useful in the design and analysis of modern semiconductor devices, photothermal systems, and microelectronic applications where coupled thermo-electro-mechanical effects are significant.

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