

Domination Parameters of Bistar Graph and its Transformation Graph

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Abstract

This paper investigates the domination number of transformation graph of bistar graph with parameters like independence domination and connected domination. We determined the domination number for all eight transformation graphs of the bistar graph. Also, we established the relationship of domination number among transformation graphs.

Keywords: Domination Number; Bistar Graph; Transformation of Bistar Graph.

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1. Introduction

Let $G = (V, E)$. For any vertex $u \in V$, the open neighborhood of u is the set $N(u) = \{v \mid (u, v) \in E\}$ and the closed neighborhood of u is the set $N[u] = N(u) \cup \{u\}$. A set $D \subseteq V$ is said to be dominating set if every vertex $v \in V \setminus D$ is adjacent to at least one element in D [2]. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set in G . An independent dominating set in G is a set that is both dominating and independent in G . The independent domination number of G , denoted by $\gamma_i(G)$, is the minimum cardinality of an independent dominating set [4]. A set $D \subseteq V$ is a connected dominating set if D is a dominating set and the subgraph D induced by V is connected. The connected domination number of G is denoted by $\gamma_c(G)$ is the minimum cardinality of a connected dominating set [4]. The bistar graph $B_{m,n}$ is a graph derived by joining the center vertices by an edge of a two-star graph [1]. The concept of the transformation graph G^{xyz} was formally introduced by Chinese mathematicians B. Wu and J. Meng (sometimes referred to as Wu and Meng) [11]. Transformation graph G^{xyz} where, xyz is either $+$ or $-$. For any graph we can obtained eight transformation graph that is G^{+++} and its compliment G^{---} , G^{++-} and its compliment G^{-+-} , G^{+-+} and its compliment G^{-+-} , G^{+--} and its compliment G^{--+} . Let $G = (V(G), E(G))$ be a graph and x, y, z be three variables taking values $+$ or $-$. The transformation graph G^{xyz} is the graph having $V(G) \cup E(G)$ as the vertex set and for $\alpha, \beta \in V(G) \cup E(G)$, α and β are adjacent in G^{xyz} if and only if the following holds: (i) α ,

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$\beta \in V(G)$, α and β are adjacent in G^{xyz} if $x = +$; α and β are not adjacent in G^{xyz} if $x = -$. (ii) $\alpha, \beta \in E(G)$. α and β are adjacent in G^{xyz} if $y = +$; α and β are not adjacent in G if $y = -$. (iii) $\alpha \in V(G), \beta \in E(G)$, α and β are incident in G^{xyz} if $z = +$; α and β are not incident in G^{xyz} if $z = -$ [5]. we apply this definition on bistar graph and also on its parameter. In this paper we established the results of domination number of bistar graph and its transformation graph.

2. Main Results

Theorem 2.1. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma(G) = 2$.

Proof. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges.

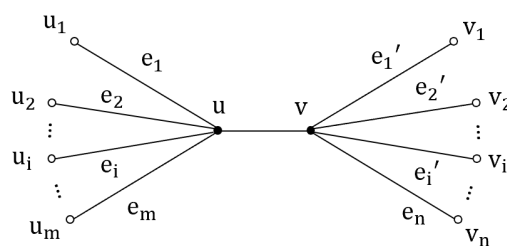


Figure 1: $B_{m,n}$

Let $V(G) = \{u_1, u_2, u_3, \dots, u_m, u, v_1, v_2, v_3, \dots, v_n, v\}$. Consider $V_1 = \{u_1, u_2, u_3, \dots, u_m, u\}$, $V_2 = \{v_1, v_2, v_3, \dots, v_n, v\}$, $E = \{e_1, e_2, e_3, \dots, e_m, \dots, e, e'_1, e'_2, e'_3, \dots, e_n\}$. Here u and v are central vertices of $B_{m,n}$. Clearly, $N[u] = \{u_1, u_2, u_3, \dots, u_m, u, v\}$ and $N[v] = \{v_1, v_2, v_3, \dots, v_n, v, u\}$. i.e., u and v dominate all the vertices of $B_{m,n}$. Therefore, dominating set of $B_{m,n}$ is $\{u, v\}$. Thus, $\gamma(G) = \gamma(B_{m,n}) = 2$. $\square$

Theorem 2.2. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma(G^{+++}) = \gamma(G^{+-+}) = \gamma(G^{-++}) = 2$.

Proof. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges.

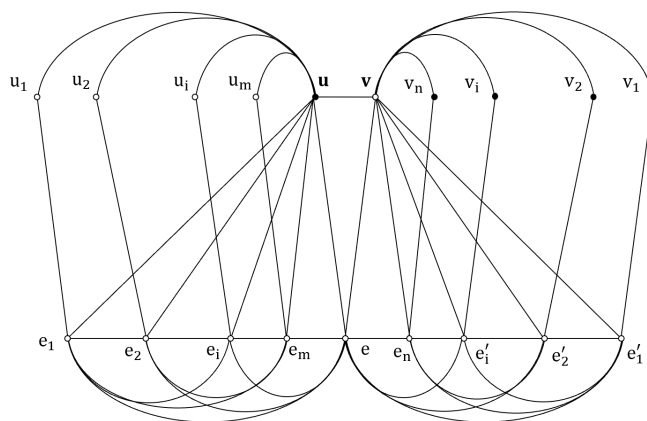


Figure 2: G^{+++}

Let $V(G) = \{u_1, u_2, u_3, \dots, u_m, u, v_1, v_2, v_3, \dots, v_n, v\}$. Consider $V_1 = \{u_1, u_2, u_3, \dots, u_m, u\}$, $V_2 = \{v_1, v_2, v_3, \dots, v_n, v\}$, $E = \{e_1, e_2, e_3, \dots, e_m, \dots, e, e'_1, e'_2, e'_3, \dots, e'_n\}$. Here u and v are central vertices of $B_{m,n}$. Clearly, $N[u] = \{u_1, u_2, u_3, \dots, u_m, u, v\}$ and $N[v] = \{v_1, v_2, v_3, \dots, v_n, v, u\}$. i.e., u and v dominate all the vertices of $B_{m,n}$. Therefore, dominating set of $B_{m,n}$ is $\{u, v\}$. Thus, $\gamma(G) = \gamma(B_{m,n}) = 2$. $\square$

Theorem 2.3. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma(G^{---}) = \gamma(G^{+-}) = 2$.

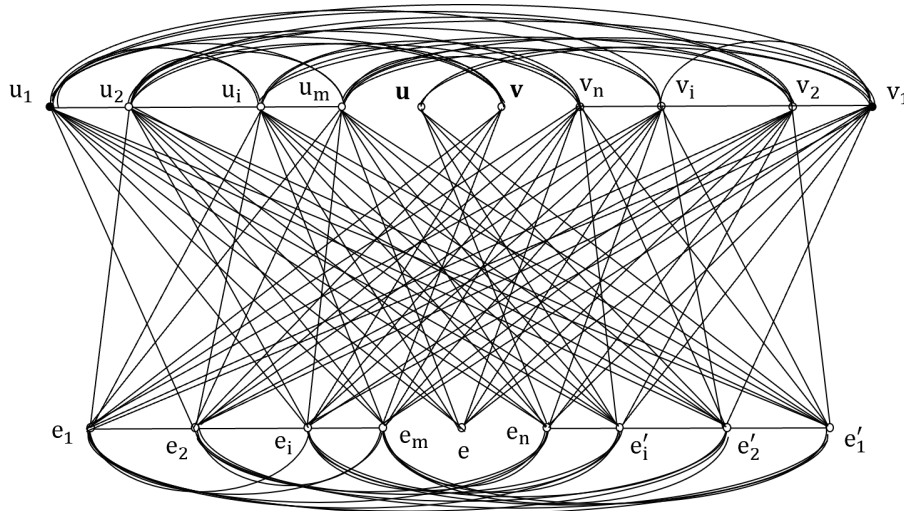


Figure 3: G^{---}

Proof. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges and G^{---}, G^{+-} are transformation graph of $B_{m,n}$. Let

$$V(G^{---}) = \{u_1, u_2, u_3, \dots, u_m, u, v_1, v_2, v_3, \dots, v_n, v, e_1, e_2, e_3, \dots, e_m, \dots, e, e'_1, e'_2, e'_3, \dots, e'_n\}$$

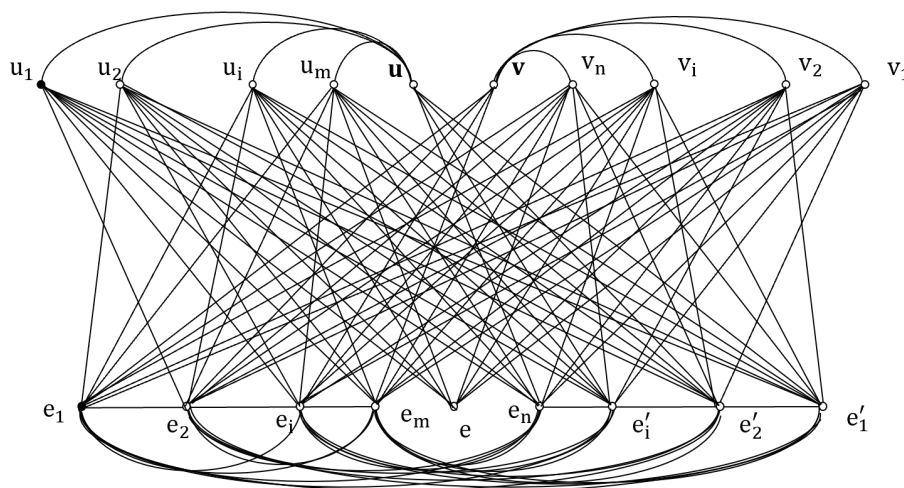
Take $u_1 \in V(G^{---})$ be such that

$$N[u_1] = \{u_1, u_2, u_3, \dots, u_m, v_1, v_2, v_3, \dots, v_n, v, e_2, e_3, \dots, e_m, \dots, e, e'_1, e'_2, e'_3, \dots, e'_n\}$$

Here, $N[u_1]$ does not contain vertex u and e_1 . So, we need only the vertex whose neighbourhood contains u and e_1 . There are many possibilities for that, but we consider $v_1 \in V(G^{---})$ be such that $N[v_1] = \{u, e_1, e_2, \dots\}$. Therefore, $N[u_1] \cup N[v_1] = V(G^{---})$. Thus, $\gamma(G^{---}) = \gamma(G^{+-}) = 2$. $\square$

Theorem 2.4. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma(G^{+-}) = \gamma(G^{++}) = 2$.

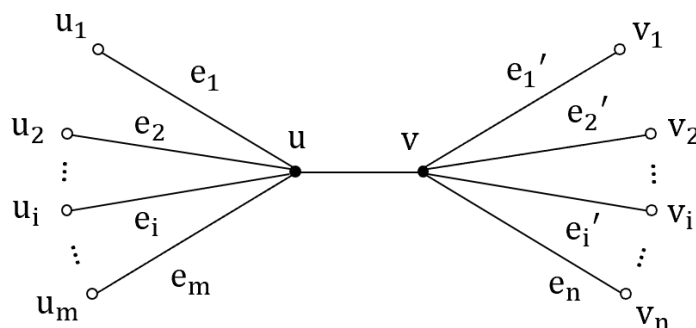
Proof. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges and G^{+-}, G^{++} are transformation graph of $B_{m,n}$.

Figure 4: G^{+--}

Let $V(G^{+--}) = \{u_1, u_2, u_3, \dots, u_m, u, v_1, v_2, v_3, \dots, v_n, v, e_1, e_2, e_3, \dots, e_m, \dots, e, e'_1, e'_2, e'_3, \dots, e_n\}$. Take $u_1, e_1 \in V(G^{xyz})$ be such that $N[u_1] = \{u, u_1, e_2, e_3, \dots, e_m, \dots, e, e'_1, e'_2, e'_3, \dots, e_n\}$ and $N[e_1] = \{e_1, u_2, u_3, \dots, u_m, v_1, v_2, v_3, \dots, v_n, v\}$. Therefore, $N[u_1] \cup N[e_1] = V$. Thus, $\gamma(G^{+--}) = \gamma(G^{++-}) = 2$. $\square$

Theorem 2.5. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma_i(G) = n + 1$.

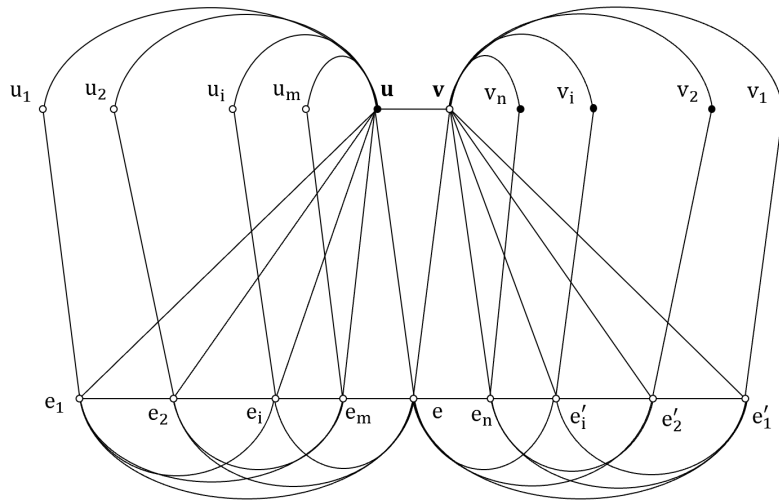
Proof. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges.

Figure 5: $B_{m,n}$

Consider $V_1 = \{u, u_1, u_2, u_3, \dots, u_m\}$, $V_2 = \{v, v_1, v_2, v_3, \dots, v_n\}$. Now, as per the definition of independence domination number we could not take vertex u and v in dominating set because, they are adjacent (dependent) to each other. Let $u \in V_1$ be such that, $N[u] = \{u, v, u_1, u_2, \dots, u_m\}$. Also, it is necessary to include all vertices of right side (i.e., vertices of V_2) in dominating set. Therefore, dominating set is $\{u, v_1, v_2, v_3, \dots, v_n\}$. Thus, $\gamma_i(G) = n + 1$. $\square$

Theorem 2.6. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma_i(G^{+++}) = \gamma_i(G^{+-+}) = n + 1$.

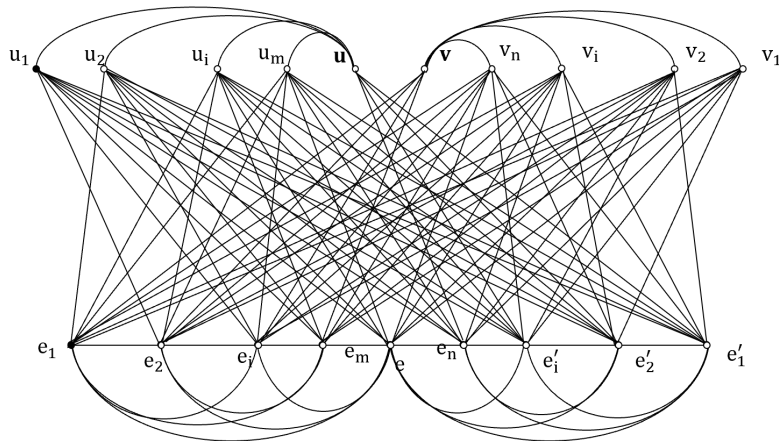
Proof. Let G^{+++} and G^{+-+} are transformation graph of $B_{m,n}$.

Figure 6: G^{+++}

Let $V(G^{+++}) = V(G^{++-}) = \{u, u_1, u_2, \dots, u_m, v, v_1, v_2, \dots, v_n, e, e_1, e_2, \dots, e_m, e'_1, e'_2, \dots, e'_n\}$. Let $N[u] = \{u, v, u_1, u_2, \dots, u_m, e_1, e_2, \dots, e_m, e\}$. Now, consider $v_1, v_2, v_3, \dots, v_n \in V(G^{++-})$ such that $N[v_1] = \{v_1, e'_1, v\}$, $N[v_2] = \{v_2, e'_2, v\}$, $N[v_3] = \{v_3, e'_3, v\}$, ..., $N[v_n] = \{v_n, e_n, v\}$. Note that, here $u, v_1, v_2, v_3, \dots, v_n$ all are independent. Therefore, $N[u] \cup N[v_1] \cup N[v_2] \cup N[v_3] \cup \dots \cup N[v_n] = V(G^{+++}) = (G^{++-})$. Thus, $\gamma_i(G^{+++}) = \gamma_i(G^{++-}) = n + 1$. $\square$

Theorem 2.7. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma_i(G^{++-}) = \gamma_i(G^{+--}) = 2$.

Proof. Let G^{++-} and G^{+--} are transformation graph of $B_{m,n}$.

Figure 7: G^{++-}

Let $V(G^{++-}) = V(G^{+--}) = \{u, u_1, u_2, u_3, \dots, u_m, v, v_1, v_2, v_3, \dots, v_n, e, e_1, e_2, e_3, \dots, e_m, e'_1, e'_2, e'_3, \dots, e'_n\}$. Consider $V_1 = \{u, u_1, u_2, u_3, \dots, u_m, v, v_1, v_2, v_3, \dots, v_n\} \subset V(G^{++-})$, $V_2 = \{e, e_1, e_2, e_3, \dots, e_m, e'_1, e'_2, e'_3, \dots, e'_n\} \subset V(G^{++-})$, Take, $u_1 \in V_1$ be such that, $N[u_1] = (V_2 \setminus \{e_1\}) \cup \{u, u_1\}$ and $e_1 \in V_2$ be such that $N[e_1] = (V_1 \setminus \{u, u_1\}) \cup \{e_1\}$. Therefore $N[u_1] \cup N[e_1] = ((V_2 \setminus \{e_1\}) \cup \{u, u_1\}) \cup ((V_1 \setminus \{u, u_1\}) \cup \{e_1\}) = V_1 \cup V_2 = V(G^{++-}) = V(G^{+--})$. Thus, $\gamma_i(G^{++-}) = \gamma_i(G^{+--}) = 2$. $\square$

Theorem 2.8. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma_i(G^{-++}) = \gamma_i(G^{--+}) = 2$.

Proof. Let G^{-++} and G^{--+} are transformation graph of $B_{m,n}$.

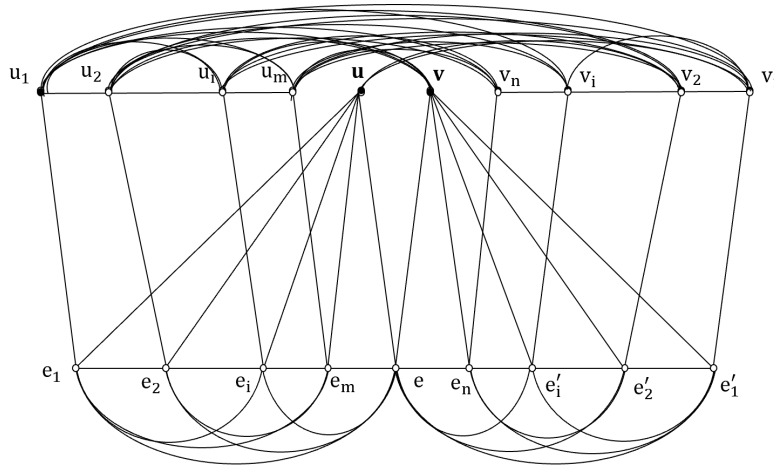


Figure 8: G^{-++}

Let $V(G^{-++}) = V(G^{--+}) = \{u, u_1, u_2, u_3, \dots, u_m, v, v_1, v_2, v_3, \dots, v_n, e, e_1, e_2, e_3, \dots, e_m, e'_1, e'_2, e'_3, \dots, e_n\}$. Consider $V_1 = \{u, u_1, u_2, u_3, \dots, u_m\} \subset V(G^{-++})$, $V_2 = \{v, v_1, v_2, v_3, \dots, v_n\} \subset V(G^{-++})$, $V_3 = \{e, e_1, e_2, e_3, \dots, e_m\} \subset V(G^{-++})$, $V_4 = \{e, e'_1, e'_2, e'_3, \dots, e_n\} \subset V(G^{-++})$. Take $u \in V_1$ and $v \in V_2$ be such that $N[u] = V_3 \cup (V_2 \setminus \{v\}) \cup \{u\}$, $N[v] = (V_1 \setminus \{u\}) \cup V_4 \cup \{v\}$. Therefore, $N[u] \cup N[v] = (V_3 \cup (V_2 \setminus \{v\}) \cup \{u\}) \cup ((V_1 \setminus \{u\}) \cup V_4 \cup \{v\}) = V_1 \cup V_2 \cup V_3 \cup V_4 = V(G^{-++}) = V(G^{--+})$. Thus, $\gamma_i(G^{-++}) = \gamma_i(G^{--+}) = 2$. $\square$

Theorem 2.9. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma_i(G^{---}) = \gamma_i(G^{-+-}) = 3$.

Proof. Let G^{---} and G^{-+-} are transformation graph of $B_{m,n}$.

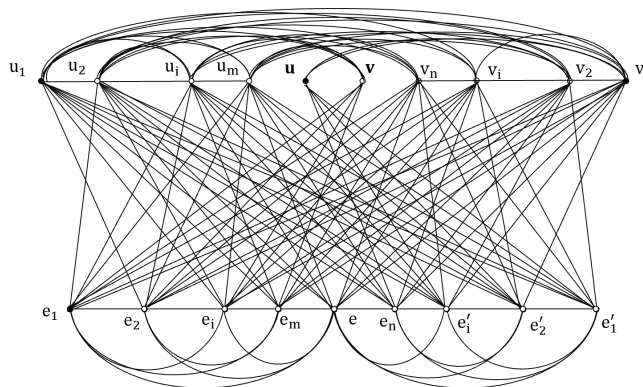


Figure 9: G^{-+-}

Let $V(G^{---}) = V(G^{-+-}) = \{u, u_1, u_2, u_3, \dots, u_m, v, v_1, v_2, v_3, \dots, v_n, e, e_1, e_2, e_3, \dots, e_m, e'_1, e'_2, e'_3, \dots, e_n\}$. Consider $V_1 = \{u, u_1, u_2, u_3, \dots, u_m, v, v_1, v_2, v_3, \dots, v_n\} \subset V(G^{-+-})$, $V_2 = \{e, e_1, e_2, e_3, \dots, e_m, e'_1, e'_2, e'_3, \dots, e_n\} \subset V(G^{-+-})$. Take $u_1, u \in V_1$ be such that $N[u_1] = (V_1 \setminus \{u\}) \cup (V_2 \setminus \{e_1\})$, $N[u] = \{u, \dots\}$

and $e_1 \in V_2$ such that $N[e_1] = \{e_1, \dots\}$. Therefore, $N[u_1] \cup N[u] \cup N[e_1] = ((V_1 \setminus \{u\}) \cup (V_2 \setminus \{e_1\})) \cup \{e_1, \dots\} \cup \{u, \dots\} = V_1 \cup V_2$. So, dominating set is $\{u_1, u, e_1\}$. Thus, $\gamma_i(G^{---}) = \gamma_i(G^{+-}) = 3$. $\square$

Theorem 2.10. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma_c(G) = 2$.

Proof. Let $G = B_{m,n}$ be a bistar graph with $m + n + 2$ vertices and $m + n + 1$ edges.

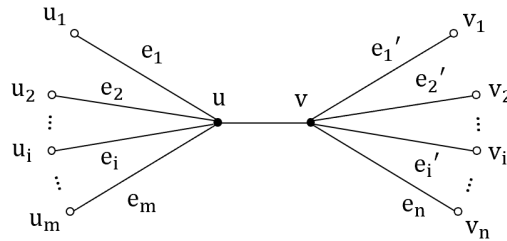


Figure 10: $B_{m,n}$

Let, $V(G) = \{u_1, u_2, u_3, \dots, u_m, u, v_1, v_2, v_3, \dots, v_n, v\}$. Consider $V_1 = \{u_1, u_2, u_3, \dots, u_m, u\}$, $V_2 = \{v_1, v_2, v_3, \dots, v_n, v\}$, $E = \{e_1, e_2, e_3, \dots, e_m, \dots, e, e'_1, e'_2, e'_3, \dots, e'_n\}$. Here u and v are central vertices of $B_{m,n}$. Clearly, $N[u] = \{u_1, u_2, u_3, \dots, u_m, u, v\}$ and $N[v] = \{v_1, v_2, v_3, \dots, v_n, v, u\}$. i.e., u and v dominate all the vertices of $B_{m,n}$. Therefore, dominating set is $\{u, v\}$. Thus, $\gamma_c(G) = 2$. $\square$

Theorem 2.11. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma_c(G^{+++}) = \gamma_c(G^{+-+}) = 2$.

Proof. Let $G = B_{m,n}$ be a bistar graph with $m + n + 2$ vertices and $m + n + 1$ edges. and G^{+++}, G^{+-+} are transformation graph of $B_{m,n}$.

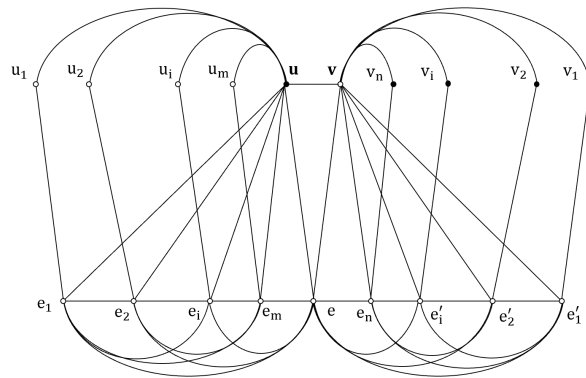


Figure 11: G^{+++}

Let $V(G^{+++}) = \{u_1, u_2, u_3, \dots, u_m, u, v_1, v_2, v_3, \dots, v_n, v, e_1, e_2, e_3, \dots, e_m, \dots, e, e'_1, e'_2, e'_3, \dots, e'_n\}$. Consider $v \in V(G^{+++})$, such that $N[u] = \{u_1, u_2, u_3, \dots, u_m, u, v, e_1, e_2, e_3, \dots, e_m, \dots, e\}$ and $N[v] = \{v_1, v_2, v_3, \dots, v_n, u, v, e'_1, e'_2, e'_3, \dots, e'_n, \dots, e\}$. Therefore, $N[u] \cup N[v] = V$. i.e., u and v dominate all the vertices of G^{xyz} . Where, xyz is $+++$, $+-+$. Thus, $\gamma_c(G^{+++}) = \gamma_c(G^{+-+}) = 2$. $\square$

Theorem 2.12. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma_c(G^{---}) = \gamma_c(G^{+-}) = 2$.

Proof. Let $G = B_{m,n}$ be a bistar graph with $m + n + 2$ vertices and $m + n + 1$ edges. and G^{---}, G^{-+-} are transformation graph of $B_{m,n}$.

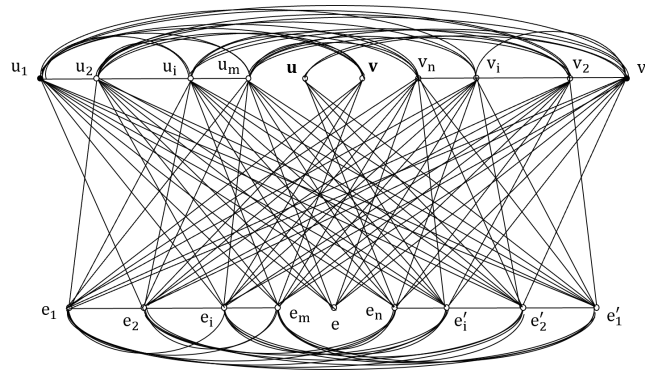


Figure 12: G^{---}

Let $V(G^{---}) = \{u_1, u_2, u_3, \dots, u_m, u, v_1, v_2, v_3, \dots, v_n, v, e_1, e_2, e_3, \dots, e_m, \dots, e, e'_1, e'_2, e'_3, \dots, e_n\}$. Consider $u_1 \in V(G^{---})$ be such that $N[u_1] = \{u_1, u_2, u_3, \dots, u_m, v_1, v_2, v_3, \dots, v_n, v, e_2, e_3, \dots, e_m, \dots, e, e'_1, e'_2, e'_3, \dots, e_n\}$. Here $N[u_1]$ does not contain vertex u and e_1 in G^{---} and G^{-+-} . So, we need only vertex whose neighborhood contains u and e_1 . There are many possibilities for that but, we consider $v_1 \in V(G^{---})$ be such that $N[v_1] = \{u, e_1, \dots\}$. Therefore, $N[u_1] \cup N[v_1] = V$. Thus, $\gamma_c(G^{---}) = \gamma_c(G^{-+-}) = 2$. $\square$

Theorem 2.13. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma_c(G^{-++}) = \gamma_c(G^{--+}) = 3$.

Proof. Let $G = B_{m,n}$ be a bistar graph with $m + n + 2$ vertices and $m + n + 1$ edges. and G^{-++}, G^{--+} are transformation graph of $B_{m,n}$.

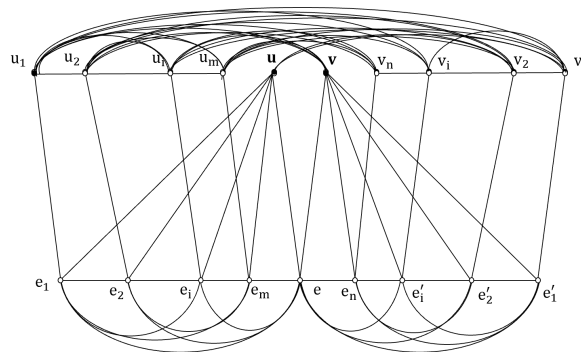


Figure 13: G^{-++}

Let $V(G^{-++}) = V(G^{--+}) = \{u, u_1, u_2, u_3, \dots, u_m, v, v_1, v_2, v_3, \dots, v_n, e, e_1, e_2, e_3, \dots, e_m, e'_1, e'_2, e'_3, \dots, e_n\}$. Also, $V_1 = \{u, u_1, u_2, u_3, \dots, u_m\} \subset V(G^{-++})$, $V_2 = \{v, v_1, v_2, v_3, \dots, v_n\} \subset V(G^{-++})$, $V_3 = \{e, e_1, e_2, e_3, \dots, e_m\} \subset V(G^{-++})$, $V_4 = \{e, e'_1, e'_2, e'_3, \dots, e_n\} \subset V(G^{-++})$. Consider $u \in V_1$ and $v \in V_2$ be such that $N[u] = V_3 \cup (V_2 \setminus \{v\}) \cup \{u\}$ and $N[v] = (V_1 \setminus \{u\}) \cup V_4 \cup \{v\}$. Therefore, $N[u] \cup N[v] = (V_3 \cup (V_2 \setminus \{v\}) \cup \{u\}) \cup ((V_1 \setminus \{u\}) \cup V_4 \cup \{v\}) = V_1 \cup V_2 \cup V_3 \cup V_4 = V(G^{-++}) = V(G^{--+})$. Note

that note that for the connectedness we must have to add vertex e in dominating set. Thus, Dominating set is $D = \{u, v, e\}$. Hence, $\gamma_c(G^{-++}) = \gamma_c(G^{--+}) = 3$. $\square$

Theorem 2.14. Let G be any bistar graph $B_{m,n}$ with $m + n + 2$ vertices and $m + n + 1$ edges then $\gamma_c(G^{++-}) = \gamma_c(G^{+--}) = 3$.

Proof. Let $G = B_{m,n}$ be a bistar graph with $m + n + 2$ vertices and $m + n + 1$ edges. and G^{++-}, G^{+--} are transformation graph of $B_{m,n}$.

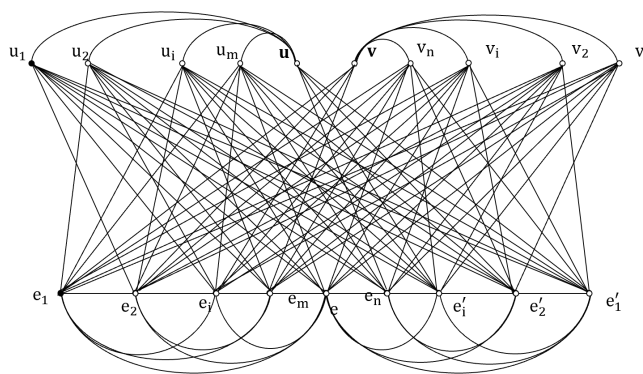


Figure 14: G^{++-}

Let $V(G^{++-}) = V(G^{+--}) = \{u, u_1, u_2, u_3, \dots, u_m, v, v_1, v_2, v_3, \dots, v_n, e, e_1, e_2, e_3, \dots, e_m, e'_1, e'_2, e'_3, \dots, e'_n\}$. Also, $V_1 = \{u, u_1, u_2, u_3, \dots, u_m\} \subset V(G^{++-})$, $V_2 = \{v, v_1, v_2, v_3, \dots, v_n\} \subset V(G^{++-})$, $V_3 = \{e, e_1, e_2, e_3, \dots, e_m\} \subset V(G^{++-})$, $V_4 = \{e, e'_1, e'_2, e'_3, \dots, e'_n\} \subset V(G^{++-})$. Consider $u_1 \in V_1$, $v_1 \in V_2$ and $e \in V_3$ be such that $N[u_1] = V_4 \cup (V_3 \setminus \{e_1\}) \cup \{u\}$, $N[v_1] = (V_4 \setminus \{e'_1\}) \cup V_3 \cup \{v\}$, $N[e] = (V_1 \cup V_2) \setminus \{u, v\}$. Therefore, $N[u_1] \cup N[v_1] \cup N[e] = (V_4 \cup (V_3 \setminus \{e_1\}) \cup \{u\}) \cup ((V_4 \setminus \{e'_1\}) \cup V_3 \cup \{v\}) \cup ((V_1 \cup V_2) \setminus \{u, v\}) = V_1 \cup V_2 \cup V_3 \cup V_4 = V(G^{++-}) = V(G^{+--})$. Thus, Dominating set $D = \{u_1, v_1, e\}$. Hence, $\gamma_c(G^{++-}) = \gamma_c(G^{+--}) = 3$. $\square$

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