

Neighborhood Augmented Sombor and Reciprocal Neighborhood Augmented Sombor Indices of Certain Networks

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Abstract

In this study, we introduce the neighborhood augmented Sombor and reciprocal neighborhood augmented Sombor indices of a graph. Furthermore, we compute these newly defined neighborhood augmented Sombor indices for certain nanostructures of chemical importance like nanocones and dendrimers.

Keywords: neighborhood augmented Sombor index; reciprocal neighborhood augmented Sombor index; nanocones; dendrimers.

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1. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u . We refer [1] for undefined notations and terminologies. A graph index is a numerical parameter mathematically derived from the graph structure. Several graph indices have been considered in Theoretical Chemistry and many graph indices were defined by using vertex degree concept [2]. Graph indices have their applications in various disciplines in Science and Technology [3,4].

The augmented Sombor index [5] of a graph G is defined as

$$ASO(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u)^2 + d_G(v)^2}{d_G(u) + d_G(v) - 2}}$$

Recently, some augmented indices were studied in [6–10]. We define the neighborhood augmented Sombor index of a molecular graph G as

$$NASO(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_G(u)^2 + S_G(v)^2}{S_G(u) + S_G(v) - 2}}$$

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We define the reciprocal neighborhood augmented Sombor index of a graph G as

$$RNASO(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_G(u) + S_G(v) - 2}{S_G(u)^2 + S_G(v)^2}}$$

Recently, some neighborhood indices were studied in [11–17]. In this work, we determine the neighborhood augmented Sombor and reciprocal neighborhood augmented Sombor indices for certain families of nanocones and dendrimers.

2. Results for Nanocones $C_n[k]$

In this section, we consider nanocones $C_n[k]$. The molecular structure of $C_4[2]$ is shown in Figure 1.

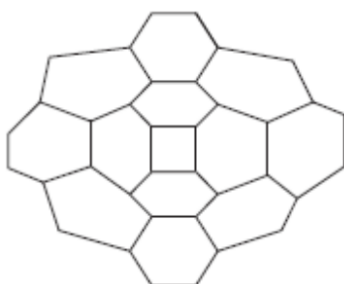


Figure 1: The molecular structure of $C_4[2]$

Let G be the molecular structure of $C_n[k]$. By calculation, G has $n(k + 1)^2$ vertices and $\frac{n}{2}(k + 1)(3k + 2)$ edges. Also by calculation, we obtain that G has five types of edges based on $S_G(u)$ and $S_G(v)$ the degrees of end vertices of each edge as given in Table 1.

$S_G(u), S_G(v) \setminus uv \in E(G)$	Number of edges
(5, 5)	n
(5, 7)	$2n$
(6, 7)	$2(k - 1)n$
(7, 9)	nk
(9, 9)	$\frac{nk}{2}(3k - 1)$

Table 1: Edge partition of $C_n[k]$ based on $S_G(u), S_G(v)$

We compute the neighborhood augmented Sombor index of $C_n[k]$.

Theorem 2.1. *Let $C_n[k]$ be the family of nanocones. Then*

$$NASO(G) = \frac{27}{4\sqrt{2}}nk^2 + \left(2\sqrt{\frac{85}{11}} + \sqrt{\frac{65}{7}} - \frac{9}{4\sqrt{2}}\right)nk + \left(\frac{5}{2} + 2\sqrt{\frac{37}{5}} - 2\sqrt{\frac{85}{11}}\right)n.$$

Proof. Let G be the molecular graph of $C_n[k]$. By using the definitions and Table 1, we deduce

$$NASO(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_G(u)^2 + S_G(v)^2}{S_G(u) + S_G(v) - 2}}$$

$$\begin{aligned}
 &= n\sqrt{\frac{5^2 + 5^2}{5 + 5 - 2}} + 2n\sqrt{\frac{5^2 + 7^2}{5 + 7 - 2}} + 2(k - 1)n\sqrt{\frac{6^2 + 7^2}{6 + 7 - 2}} \\
 &+ nk\sqrt{\frac{7^2 + 9^2}{7 + 9 - 2}} + \frac{nk}{2}(3k - 1)\sqrt{\frac{9^2 + 9^2}{9 + 9 - 2}}
 \end{aligned}$$

After simplification, we get the desired result. □

We compute the reciprocal neighborhood augmented Sombor index of $C_n[k]$.

Theorem 2.2. *Let $C_n[k]$ be the family of nanocones. Then*

$$RNASO(G) = \frac{\sqrt{2}}{3}nk^2 + \left(2\sqrt{\frac{11}{85}} + \sqrt{\frac{7}{65}} - \frac{\sqrt{2}}{9}\right)nk + \left(\frac{2}{5} + 2\sqrt{\frac{5}{37}} - 2\sqrt{\frac{11}{85}}\right)n.$$

Proof. Let G be the molecular graph of $C_n[k]$. By using the definitions and Table 1, we deduce

$$\begin{aligned}
 RNASO(G) &= \sum_{uv \in E(G)} \sqrt{\frac{S_G(u) + S_G(v) - 2}{S_G(u)^2 + S_G(v)^2}} \\
 &= n\sqrt{\frac{5 + 5 - 2}{5^2 + 5^2}} + 2n\sqrt{\frac{5 + 7 - 2}{5^2 + 7^2}} + 2(k - 1)n\sqrt{\frac{6 + 7 - 2}{6^2 + 7^2}} \\
 &+ nk\sqrt{\frac{7 + 9 - 2}{7^2 + 9^2}} + \frac{nk}{2}(3k - 1)\sqrt{\frac{9 + 9 - 2}{9^2 + 9^2}}
 \end{aligned}$$

After simplification, we get the desired result. □

3. Results for $NS_2[n]$ Dendrimers

In this section, we focus on the class of $NS_2[n]$ dendrimers with $n \geq 1$. The graph of $NS_2[3]$ is shown in Figure 2.

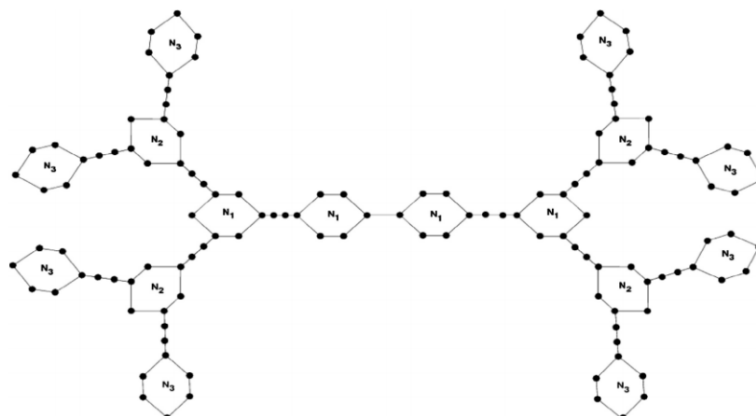


Figure 2: The graph of $NS_2[3]$

Let G be the graph of $NS_2[n]$. By calculation, G has $16 \times 2^n - 4$ vertices and $18 \times 2^n - 5$ edges. Also by calculation, we obtain that G has seven types of edges based on $S_G(u), S_G(v)$ the degrees of end vertices of each edge as given in Table 2.

$S_G(u), S_G(v) \setminus uv \in E(G)$	(4, 4)	(5, 4)	(5, 5)	(5, 6)	(7, 7)	(5, 7)	(6, 6)
Number of edges	2×2^n	2×2^n	$2 \times 2^n + 2$	6×2^n	1	4	$6 \times 2^n - 12$

Table 2: Edge partition of $NS_2[n]$ based on $S_G(u)$ and $S_G(v)$

We compute the neighborhood augmented Sombor index of $NS_2[n]$.

Theorem 3.1. *Let $NS_2[n]$ be the family of dendrimers. Then*

$$NASO(G) = \left(\frac{4}{\sqrt{3}} + \sqrt{\frac{41}{7}} + \frac{5}{2} \right) 2 \times 2^n + \left(\frac{\sqrt{61}}{3} + \frac{6}{\sqrt{5}} \right) 6 \times 2^n + 5 + \frac{7}{\sqrt{6}} + 4 \cdot \sqrt{\frac{37}{5}} - 72\sqrt{\frac{1}{5}}.$$

Proof. Let G be the molecular graph of $NS_2[n]$. By using the definitions and Table 2, we deduce

$$\begin{aligned} NASO(G) &= \sum_{uv \in E(G)} \sqrt{\frac{S_G(u)^2 + S_G(v)^2}{S_G(u) + S_G(v) - 2}} \\ &= 2 \times 2^n \sqrt{\frac{4^2 + 4^2}{4 + 4 - 2}} + 2 \times 2^n \sqrt{\frac{5^2 + 4^2}{5 + 4 - 2}} + (2 \times 2^n + 2) \sqrt{\frac{5^2 + 5^2}{5 + 5 - 2}} + 6 \times 2^n \sqrt{\frac{5^2 + 6^2}{5 + 6 - 2}} \\ &\quad + 1 \sqrt{\frac{7^2 + 7^2}{7 + 7 - 2}} + 4 \sqrt{\frac{5^2 + 7^2}{5 + 7 - 2}} + (6 \times 2^n - 12) \sqrt{\frac{6^2 + 6^2}{6 + 6 - 2}}. \end{aligned}$$

After simplification, we obtain the desired result. □

We compute the reciprocal neighborhood augmented Sombor index of $NS_2[n]$.

Theorem 3.2. *Let $NS_2[n]$ be the family of dendrimers. Then*

$$RNASO(G) = \left(\frac{\sqrt{3}}{4} + \sqrt{\frac{7}{41}} + \frac{2}{5} \right) 2 \times 2^n + \left(\frac{3}{\sqrt{61}} + \frac{\sqrt{5}}{6} \right) 6 \times 2^n + \frac{4}{5} + \frac{\sqrt{6}}{7} + 4 \cdot \sqrt{\frac{5}{37}} - 2\sqrt{5}.$$

Proof. Let G be the molecular graph of $NS_2[n]$. By using the definitions and Table 2, we deduce

$$\begin{aligned} RNASO(G) &= \sum_{uv \in E(G)} \sqrt{\frac{S_G(u) + S_G(v) - 2}{S_G(u)^2 + S_G(v)^2}} \\ &= 2 \times 2^n \sqrt{\frac{4 + 4 - 2}{4^2 + 4^2}} + 2 \times 2^n \sqrt{\frac{5 + 4 - 2}{5^2 + 4^2}} + (2 \times 2^n + 2) \sqrt{\frac{5 + 5 - 2}{5^2 + 5^2}} + 6 \times 2^n \sqrt{\frac{5 + 6 - 2}{5^2 + 6^2}} \\ &\quad + 1 \sqrt{\frac{7 + 7 - 2}{7^2 + 7^2}} + 4 \sqrt{\frac{5 + 7 - 2}{5^2 + 7^2}} + (6 \times 2^n - 12) \sqrt{\frac{6 + 6 - 2}{6^2 + 6^2}}. \end{aligned}$$

After simplification, we obtain the desired result. □

4. Results for $NS_3[n]$ Dendrimers

In this section, we focus on another type of dendrimers $NS_3[n]$ with $n \geq 1$. The molecular structure of $NS_3[2]$ is presented in Figure 3.

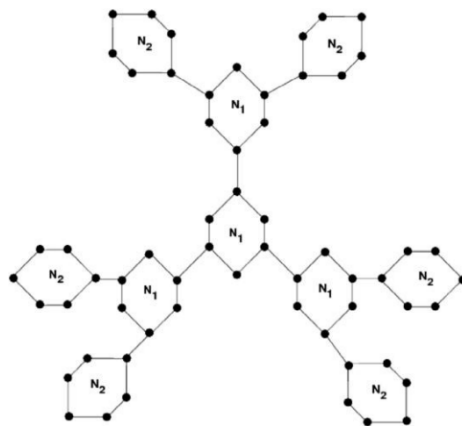


Figure 3: The structure of $NS_3[2]$

Let G be the molecular graph of $NS_3[n]$. By calculation, we obtain that G has $18 \times 2^n - 12$ vertices and $21 \times 2^n - 15$ edges. Also by calculation, we get that G has five types of edges based on $S_G(u)$ and $S_G(v)$ the degrees of end vertices of each edge as given in Table 3.

$S_G(u), S_G(v) \setminus uv \in E(G)$	(4, 4)	(5, 4)	(5, 7)	(6, 7)	(7, 7)
Number of edges	3×2^n	3×2^n	3×2^n	$9 \times 2^n - 12$	$3 \times 2^n - 3$

Table 3: Edge partition of $NS_3[n]$ based on $S_G(u)$ and $S_G(v)$

We compute the neighborhood augmented Sombor index of $NS_3[n]$.

Theorem 4.1. *Let $NS_3[n]$ be the family of dendrimers. Then*

$$NASO(G) = \left(\frac{4}{\sqrt{3}} + \sqrt{\frac{41}{7}} + \sqrt{\frac{37}{5}} + \frac{7}{\sqrt{6}} \right) 3 \times 2^n + \sqrt{\frac{85}{11}} 9 \times 2^n - 12\sqrt{\frac{85}{11}} - \frac{21}{\sqrt{6}}.$$

Proof. Let G be the molecular graph of $NS_3[n]$. By using the definitions and Table 3, we deduce

$$\begin{aligned} NASO(G) &= \sum_{uv \in E(G)} \sqrt{\frac{S_G(u)^2 + S_G(v)^2}{S_G(u) + S_G(v) - 2}} \\ &= 3 \times 2^n \sqrt{\frac{4^2 + 4^2}{4 + 4 - 2}} + 3 \times 2^n \sqrt{\frac{5^2 + 4^2}{5 + 4 - 2}} + 3 \times 2^n \sqrt{\frac{5^2 + 7^2}{5 + 7 - 2}} \\ &\quad + (9 \times 2^n - 12) \sqrt{\frac{6^2 + 7^2}{6 + 7 - 2}} + (3 \times 2^n - 3) \sqrt{\frac{7^2 + 7^2}{7 + 7 - 2}}. \end{aligned}$$

After simplification, we obtain the desired result. □

We compute the reciprocal neighborhood augmented Sombor index of $NS_3[n]$.

Theorem 4.2. *Let $NS_3[n]$ be the family of dendrimers. Then*

$$RNASO(G) = \left(\frac{\sqrt{3}}{4} + \sqrt{\frac{7}{41}} + \sqrt{\frac{5}{37}} + \frac{\sqrt{6}}{7} \right) 3 \times 2^n + \sqrt{\frac{11}{85}} 9 \times 2^n - 12\sqrt{\frac{11}{85}} - \frac{3\sqrt{6}}{7}.$$

Proof. Let G be the molecular graph of $NS_3[n]$. By using the definitions and Table 3, we deduce

$$\begin{aligned} RNASO(G) &= \sum_{uv \in E(G)} \sqrt{\frac{S_G(u) + S_G(v) - 2}{S_G(u)^2 + S_G(v)^2}} \\ &= 3 \times 2^n \sqrt{\frac{4 + 4 - 2}{4^2 + 4^2}} + 3 \times 2^n \sqrt{\frac{5 + 4 - 2}{5^2 + 4^2}} + 3 \times 2^n \sqrt{\frac{5 + 7 - 2}{5^2 + 7^2}} \\ &\quad + (9 \times 2^n - 12) \sqrt{\frac{6 + 7 - 2}{6^2 + 7^2}} + (3 \times 2^n - 3) \sqrt{\frac{7 + 7 - 2}{7^2 + 7^2}}. \end{aligned}$$

After simplification, we obtain the desired result. □

5. Conclusion

In this work, we have determined the neighborhood augmented Sombor and reciprocal neighborhood augmented Sombor indices for certain families of nanocones and dendrimers.

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