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A New Hybrid Fuzzy Aggregation Approach

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Abstract

Fuzzy graph theory is useful in the analysis of systems with uncertainty, ambiguity and imprecision. Classical fuzzy graph models usually rely on simple aggregation operators that may not be sufficient to describe complex interactions in uncertain situations. We propose a novel Hybrid Sugeno-Dombi operator by combining the Sugeno intrinsic with Dombi t-norm in this paper. The proposed hybrid framework integrates the Sugeno integral for global information aggregation with the Dombi t-norm for local edge evaluation, thus providing enhanced versatility and flexibility for uncertainty modelling. The suggested approach enhances the efficiency of decision-making and system modeling in complicated real-world applications and delivers a consistent and efficient framework for the analysis of uncertain network systems.

Keywords: t-norm; t-conorm; Dombi t-norm; Sugeno t-norm; Hybrid Sugeno-Dombi t-norm.

2020 Mathematics Subject Classification: 03E72, 94D0.

1. Introduction

A crucial method in modeling systems with vagueness, imprecision, and uncertainty is fuzzy graph theory. Lotfi A. Zadeh initially introduced the concept of fuzzy set in 1965, establishing a theoretical foundation for handling ambiguous data. In 1975, Azriel Rosenfeld merged fuzzy set theory with graph theory to establish the concept of fuzzy graphs.. To clarify uncertain relationships in networks, Rosenfeld's model provides membership values to the corresponding vertices and edges in the interval [0,1].Furthermore, a variety of generalized fuzzy graph versions have also been suggested by researchers to strengthen the modeling effectiveness of complex systems. Intuitionistic fuzzy graphs, interval-valued fuzzy graphs, bipolar fuzzy graphs, regular fuzzy graphs, generalized fuzzy graphs, and Dombi fuzzy graphs are a few of them. This improvement provides greater adaptability in depicting uncertain interactions and multiple levels of vertex relationship. In traditional fuzzy graph models, classic t-norm operators such algebraic product, minimum, and Sugeno (Kasiewicz) t-norms are typically employed to compute edge memberships. These operators may not accurately reflect the

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unpredictable behavior and dynamic interaction strengths observed in real-world systems, despite their mathematical elegance and efficient computation. Although the Dombi operator presents a parameterized model that enables more adaptive oversight of the degree of intersection, the Sugeno t-norm delivers linearity and simplicity.Inspired by those findings, this research merges the Sugeno and Dombi t-norms to demonstrate a Hybrid Sugeno–Dombi Fuzzy Graph. The beneficial effects of both operators—the parametric versatility provided by the Dombi operator and the linear attributes of the Sugeno operators—are integrated in the proposed hybrid approach. A more customizable framework for generating uncertainty in complicated network structures can be obtained under this hybridization.The key concepts and attributes of the proposed fuzzy graph are clarified in the current research.

2. Preliminaries

Definition 2.1 (T-norm (Triangular Norm)). *A T-norm is a binary operation used in Fuzzy Logic to represent the logical AND (intersection) operation for fuzzy sets. A function* $T : [0,1] \times [0,1] \to [0,1]$ *is called a T-norm if it satisfies:*

- *Commutativity:* $T(a,b) = T(b,a)$;
- *Associativity:* $T(a,T(b,c)) = T(T(a,b),c)$;
- *Monotonicity: If* $a \leq c$ *and* $b \leq d$*, then* $T(a,b) \leq T(c,d)$;
- *Boundary Condition:* $T(a,1) = a$.

Example 2.2. *Minimum T-norm:* $T(a,b) = \min(a,b)$.

Definition 2.3 (T-conorm (Triangular Conorm / S-norm)). *A T-conorm is a binary operation used in Fuzzy Logic to represent the logical OR (union) operation for fuzzy sets. A function* $S : [0,1] \times [0,1] \to [0,1]$ *is called a T-conorm if it satisfies:*

- *Commutativity:* $S(a,b) = S(b,a)$;
- *Associativity:* $S(a,S(b,c)) = S(S(a,b),c)$;
- *Monotonicity: If* $a \leq c$ *and* $b \leq d$*, then* $S(a,b) \leq S(c,d)$;
- *Boundary Condition:* $S(a,0) = a$.

Example 2.4. *Maximum T-conorm:* $S(a,b) = \max(a,b)$.

Definition 2.5 (Sugeno T-norm). *The Sugeno T-norm is a parameterized T-norm used in Fuzzy Logic for modeling fuzzy intersection (AND operation). For a parameter* $\lambda > -1$,

$$T_S(a,b) = \max\left(0, \frac{a+b-1+\lambda ab}{1+\lambda}\right)$$

where $a, b \in [0,1]$ *and* λ *is the parameter controlling the operation. When* $\lambda = 0$, $T(a,b) = \max(0, a+b-1)$ *which is the Łukasiewicz T-norm.*

Definition 2.6 (Sugeno T-conorm)**.** *The Sugeno T-conorm is the dual operation of the Sugeno T-norm and represents fuzzy union (OR operation). For* $\lambda > -1$, $S_S(a,b) = \min(1, a+b+\lambda ab)$, *where* $a, b \in [0,1]$. *When* $\lambda = 0$, $S(a,b) = \min(1, a+b)$ *which is the Łukasiewicz T-conorm.*

Definition 2.7 (Dombi T-norm)**.** *The Dombi T-norm is a parameterized T-norm used in Fuzzy Logic to model the fuzzy AND (intersection) operation. For* $\lambda > 0$,

$$T_D(a,b) = \frac{1}{1 + \left[\left(\frac{1-a}{a}\right)^{\lambda} + \left(\frac{1-b}{b}\right)^{\lambda} \right]^{1/\lambda}}$$

where $a, b \in (0,1)$ *and* λ *is a parameter controlling the behavior of the operator.*

Definition 2.8 (Dombi T-conorm)**.** *The Dombi T-conorm is the dual operator of the Dombi T-norm and represents the fuzzy OR (union) operation. For* $\lambda > 0$,

$$S_D(a,b) = 1 - \frac{1}{1 + \left[\left(\frac{a}{1-a}\right)^{\lambda} + \left(\frac{b}{1-b}\right)^{\lambda} \right]^{1/\lambda}}$$

3. Main Results

Hybrid Sugeno-Dombi t-norm: Let $a, b \in [0,1]$, $\theta \in [0,1]$ and $\lambda > 0$. Then the Hybrid Sugeno-Dombi t-norm is defined as

$$T_{HSD}(a,b) = \theta \min(a,b) + (1-\theta) \left[\frac{1}{1 + \left[\left(\frac{1-a}{a}\right)^{\lambda} + \left(\frac{1-b}{b}\right)^{\lambda} \right]^{1/\lambda}} \right]$$

where:

- $\min(a,b)$ is the Sugeno t-norm component,
- the second term is the Dombi t-norm component,
- θ controls the hybridization,
- λ controls the Dombi flexibility parameter.

Special Cases:

- When $\theta = 1$, $T_{HSD}(a,b) = \min(a,b)$ which becomes the Sugeno t-norm.
- When $\theta = 0$, $T_{HSD}(a,b) = T_D(a,b)$ which becomes the Dombi t-norm.

Hybrid Sugeno-Dombi t-Conorm: Let $a, b \in [0,1]$, $\theta \in [0,1]$ and $\lambda > 0$, then the Hybrid Sugeno-Dombi t-conorm is defined as

$$S_{HSD}(a,b) = \theta \max(a,b) + (1-\theta)\left[\frac{1}{1+\left[\left(\frac{1-a}{a}\right)^{\lambda} + \left(\frac{1-b}{b}\right)^{\lambda}\right]^{-1/\lambda}}\right]$$

where:

- $\max(a,b)$ is the Sugeno t-conorm component,
- the second term is the Dombi t-conorm component,
- θ is the hybrid parameter,
- λ is the Dombi parameter.

Special Cases:

- When $\theta = 1$, $S_{HSD}(a,b) = \max(a,b)$ which becomes the Sugeno t-conorm.
- When $\theta = 0$, $S_{HSD}(a,b) = S_D(a,b)$ which becomes the Dombi t-conorm.

Properties of Hybrid Sugeno-Dombi t-Norm: The operator

$$T_{HSD}(a,b) = \theta \min(a,b) + (1-\theta)\left[\frac{1}{1+\left[\left(\frac{1-a}{a}\right)^{\lambda} + \left(\frac{1-b}{b}\right)^{\lambda}\lambda\right]^{1/\lambda}}\right]$$

must satisfy the following conditions.

- **Domain Condition:** $a, b \in [0,1]$, Membership values must lie in the unit interval.
- **Hybrid Parameter Condition:** $0 \leq \theta \leq 1$, this ensures proper weighting between Sugeno and Dombi parts.
- **Dombi Parameter Condition:** The Dombi parameter must be positive $\lambda > 0$.
- **Boundary Conditions:** $T_{HSD}(a,1) = a$, $T_{HSD}(a,0) = 0$.

These are essential t-norm properties.

- **Commutativity:** $T_{HSD}(a,b) = T_{HSD}(b,a)$.
- **Associativity:** $T_{HSD}(a, T_{HSD}(b,c)) = T_{HSD}(T_{HSD}(a,b), c)$.
- **Monotonicity:** If $a_1 \leq a_2$, then $T_{HSD}(a_1,b) \leq T_{HSD}(a_2,b)$

- **Continuity:** The operator should be continuous on $[0,1]^2$.

Properties of Hybrid Sugeno–Dombi t-Conorm: The operator

$$S_{HSD}(a,b) = \theta \max(a,b) + (1-\theta) \left[\frac{1}{1 + \left[(\frac{1-a}{a})^{\lambda} + \left(\frac{1-b}{b}\right)^{\lambda} \right]^{-1/\lambda}} \right]$$

must satisfy the following conditions.

- **Domain Condition:** $a, b \in [0,1]$, Membership values must lie in the unit interval.
- **Hybrid Parameter Condition:** $0 \leq \theta \leq 1$, this ensures proper weighting between Sugeno and Dombi parts.
- **Dombi Parameter Condition:** The Dombi parameter must be positive $\lambda > 0$.
- **Boundary Conditions:** $S_{HSD}(a,0) = a$, $S_{HSD}(a,1) = 1$.

These are essential t-conorm properties.

- **Commutativity:** $S_{HSD}(a,b) = S_{HSD}(b,a)$
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- **Monotonicity:** If $a_1 \leq a_2$, then $S_{HSD}(a_1,b) \leq S_{HSD}(a_2,b)$
- **Continuity:** The operator must be continuous over $[0,1]^2$.

Duality condition: The operators should satisfy the duality relation:

$$S_{HSD}(a,b) = 1 - T_{HSD}(1-a, 1-b)$$

This ensures logical consistency between AND and OR operations.
Limiting conditions:

- **Sugeno Case:** When $\theta = 1$, then t-norm becomes $T_{HSD}(a,b) = \min(a,b)$ and t-conorm becomes $S_{HSD}(a,b) = \max(a,b)$.
- **Dombi Case:** When $\theta = 0$, then t-norm becomes $T_{HSD}(a,b) = T_D(a,b)$ and t-conorm becomes $S_{HSD}(a,b) = S_D(a,b)$.

4. Conclusion

This paper proposed a novel Sugeno–Dombi Hybrid operator for modeling uncertainty in complex systems.The model combines the Sugeno integral and Dombi t-norm to improve flexibility and accuracy. Several structural properties of the hybrid operator was studied. The proposed framework provides an efficient method for handling vague and imprecise information in network systems.The model can be applied in decision-making, communication networks, and intelligent systems. Future research may extend this work to advanced hybrid fuzzy graph structures and real-world applications.

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