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Further Study on ω-closed Sets

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Abstract: The aim of this paper is to prove the notion called semi-ω_α-open sets which is weaker than $\alpha - \omega_\alpha$-open sets and stronger than $\beta - \omega_\alpha$-open sets. Also we introduce and investigate some new generalized classes of τ_ω.

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1. Introduction

In this paper, we prove the notion called semi-ω_α-open sets which is weaker than α-ω_α-open sets and stronger than β-ω_α-open sets and investigate some new generalized classes of τ_ω.

2. Preliminaries

By a space (X, τ), we always mean a topological space (X, τ) with no separation properties assumed. If $H \subset X$, $cl(H)$ and $int(H)$ will, respectively, denote the closure and interior of H in (X, τ).

Definition 2.1 ([14]). *A subset H of a space (X, τ) is called*

(1). α-closed if $cl(int(cl(H))) \subset H$,

(2). α-open if $X \backslash H$ is α-closed, or equivalently, if $H \subset int(cl(int(H)))$.

For a subset H of (X, τ), the intersection of all α-closed subsets of X containing H is called the α-closure of H and is denoted by $cl_\alpha(H)$. It is known that $cl_\alpha(H) = H \cup cl(int(cl(H)))$ and $cl_\alpha(H) \subset cl(H)$. The union of all α-open subsets of X contained in H is called the α-interior of H and is denoted by $int_\alpha(H)$.
In 1982, the notions of ω-closed sets and ω-open sets were introduced and studied by Hdeib [8]. In 2009, Noiri et al [15] introduced some generalizations of ω-open sets and investigated some properties of the sets. Moreover, they used them to obtain decompositions of continuity. Throughout this paper, R (resp. N, Q, Q*, Z) denotes the set of all real numbers (resp. the set of all natural numbers, the set of all rational numbers, the set of all irrational numbers, the set of all integers). τu denotes the usual topology.

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Definition 2.2 ([7]). *A space (X, τ) is called submaximal if every dense subset is open.*

Definition 2.3 ([17]). *Let H be a subset of a space (X, τ), a point p in X is called a condensation point of H if for each open set U containing p, $U \cap H$ is uncountable.*

Definition 2.4 ([8]). *A subset H of a space (X, τ) is called ω-closed if it contains all its condensation points. The complement of an ω-closed set is called ω-open.*

It is well known that a subset W of a space (X, τ) is ω-open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and U-W is countable. The family of all ω-open sets, denoted by $\tau\omega$, is a topology on X, which is finer than τ. The interior and closure operator in $(X, \tau\omega)$ are denoted by $int\ \omega$ and $cl\ \omega$ respectively.

Definition 2.5 ([16]). *A subset H of a space (X, τ) is said to be semi-ω-open if $H \subset cl(int\omega(H))$.*

Definition 2.6 ([16]). *A subset H of a space (X, τ) is said to be*

(1). semi*-ω-open if $H \subset cl\ \omega(int(H))$.

(2). semi*-ω-closed if $int\ \omega(cl(H)) \subset H$.

Definition 2.7. *A subset H of a space (X, τ) is said to be*

(1). α-$\omega\alpha$-open if $H \subset int\ \omega(cl\ \alpha(int\ \omega(H)))$,

(2). semi-$\omega\alpha$-open if $H \subset cl\ \alpha(int\ \omega(H))$,

(3). pre-$\omega\alpha$-open if $H \subset int\ \omega(cl\ \alpha(H))$,

(4). β-$\omega\alpha$-open if $H \subset cl\ \alpha(int\ \omega(cl\ \alpha(H)))$,

(5). b-$\omega\alpha$-open if $H \subset int\ \omega(cl\ \alpha(H)) \cup cl\ \alpha(int\ \omega(H))$.

An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies

(1). $A \in I$ and $B \subseteq A \Rightarrow B \in I$ and

(2). $A \in I$ and $B \in I \Rightarrow A \cup B \in I$.

If I is an ideal on X and $X \notin I$, then $F = X : G \in I$ is a filter [11]. Given a topological space (X, τ) with an ideal I on X and if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^* : \wp(X) \to \wp(X)$, called a local function [?] of A with respect to τ and I, is defined as follows: for $A \subseteq X$, $A^*(I, \tau) = x \in X | U \cap A \notin I$ for every $U \in \tau(x)$ where $\tau(x) = U \in \tau | x \in U$. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I, \tau)$, called the *-topology, finer than τ is defined by $cl^*(A) = A \cup A^*(I, \tau)$ [11]. When there is no chance for confusion, we will simply write A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$. $int^*(A)$ will denote the interior of A in (X, τ^*). (X, τ, I) is called an ideal topological space or an ideal space.

Definition 2.8 ([11]). *A subset H of an ideal topological space (X, τ, I) is said to be *-closed if $H^\star \subset H$ or $cl^\star(H) = H$. The complement of an *-closed set is called *-open.*

Proposition 2.9 ([4]). *If (X, τ) is a door space, then every pre-ω-α-open set in (X, τ, I) is ω-open.*

Lemma 2.10 ([11]). *Let (X, τ, I) be an ideal topological space and A, B subsets of X. Then the following properties hold:*

(1). $A \subset B \Rightarrow A^\star \subset B^\star$,

(2). $A^\star = cl(A^\star) \subset cl(A)$,

(3). $A^\star \cup B^\star = (A \cup B)^\star$,

(4). $(A^\star)^\star \subset A^\star$,

(5). $A^\star$ *is closed in* (X, τ),

(6). *If* $H \in \tau$, *then* $H \cap A^\star \subset (H \cap A)^\star$.

3. Properties of Semi-$\omega\alpha$-open Sets

Definition 3.1. *A subset* H *of a space* (X, τ) *is said to be*

(1). *semi-$\omega\alpha$-open if* $H \subset cl\ \alpha(int\omega(H))$.

(2). *semi-$\omega\alpha$-closed if* $int\ \alpha(cl\ \omega(H)) \subset H$.

The complement of a semi-$\omega\alpha$-open set is called semi-$\omega\alpha$-closed.

Example 3.2. *In* $(R, \tau u), H = Q^\star$ *is semi-$\omega\alpha$-open for* $cl\ \alpha(int\ \omega(H)) = cl\ \alpha(H) = R \supset H$.

Example 3.3. *In* $(R, \tau u)$, $H = Q$ *is not semi-$\omega\alpha$-open for* $cl\alpha(int\ \omega(H)) = cl\ \alpha(\phi) = \phi \not\supseteq H$.

Proposition 3.4. *In a space* (X, τ), *every semi-$\omega\alpha$-open subset is semi-ω-open. Let* H *be semi-$\omega\alpha$-open in* (X, τ). *Then* $H \subset cl\ \alpha(int\ \omega(H)) \subset cl(int\ \omega(H))$. *This proves that* H *is semi-ω-open.*

Remark 3.5. *The converse of Proposition 3.4 is not true.*

Example 3.6. *In* R *with the topology* $\tau = \phi, R, Q$, $H = Q \cup \sqrt{2}$ *is semi-ω-open for* $cl(int\ \omega(H)) = cl(Q) = R \supset H$. *But* H *is not semi-$\omega\alpha$-open for* $cl\ \alpha(int\ \omega(H)) = cl\ \alpha(Q) = R \not\supseteq H$.

Proposition 3.7. *Let* H *be a subset of an ideal topological space* (X, τ, I). *Then* H *is α-$I\omega$-open if and only if it is semi-$I\omega$-open and pre-$I\omega$-open.*

Theorem 3.8. *For a subset of a space* (X, τ), *the following properties hold:*

(1). *Every ω-open set is semi-$\omega\alpha$-open.*

(2). *Every open set is semi-$\omega\alpha$-open.*

(3). *Every α-$\omega\alpha$-open set is semi-$\omega\alpha$-open.*

(4). *Every semi-$\omega\alpha$-open set is β-$\omega\alpha$-open.*

(5). *Every semi-$\omega\alpha$-open set is b-$\omega\alpha$-open.*

Proof.

(1). If H is ω-open, then $H \subset cl\ \alpha(H) = cl\ \alpha(int\ \omega(H))$. Therefore H is semi-$\omega\alpha$-open.

(2). If H is open, then $H \subset cl\ \alpha(H) = cl\ \alpha(int(H)) \subset cl\ \alpha(int\ \omega(H))$. Therefore H is semi-$\omega\alpha$-open.

(3). If H is α-$\omega\alpha$-open, then $H \subset int\ \omega(cl\ \alpha(int\ \omega(H))) \subset cl\ \alpha(int\ \omega(H))$. Therefore H is semi-$\omega\alpha$-open.

(4). If H is semi-$\omega\alpha$-open, then $H \subset cl\ \alpha(int\ \omega(H)) \subset cl\ \alpha(int\ \omega(cl\ \alpha(H)))$. Therefore H is β-$\omega\alpha$-open.

(5). If H is semi-$\omega\alpha$-open, then $H \subset cl\ \alpha(int\ \omega(H)) \subset int\ \omega(cl\ \alpha(H)) \cup cl\ \alpha(int\ \omega(H))$. Therefore H is b-$\omega\alpha$-open. The following Examples support that the separate converses of Theorem 3.8 are not true in general. □

Example 3.9. *In R with the topology $\tau = \phi, R, N, Q^{\star}$, $Q^{\star} \cup N$,*

(1). $H = Q$ is semi-$\omega\alpha$-open, since $cl\ \alpha(int\ \omega(H)) = cl\ \alpha(N) = Q \supset H$. But $H = Q$ is not ω-open, since $int\ \omega(H) = N \neq H$.

(2). $H = Q$ is semi-$\omega\alpha$-open by (1), but not open.

Example 3.10. *By (1) of Example 3.9, $H = Q$ is semi-$\omega\alpha$-open. But $int\ \omega(cl\ \alpha(int\ \omega(H))) = int\ \omega(cl\ \alpha(N)) = int\ \omega(Q) = N \not\supseteq Q = H$.*

Example 3.11. *In $(R, \tau u)$,*

(1). $H = Q$ is β-$\omega\alpha$-open, since $cl\ \alpha(int\ \omega(cl\ \alpha(H))) = cl\ \alpha(int\ \omega(R)) = cl\ \alpha(R) = R \supset Q = H$. But $H = Q$ is not semi-$\omega\alpha$-open, since $cl\ \alpha(int\ \omega(H)) = cl\ \alpha(\phi) = \phi \not\supseteq Q = H$.

(2). $H = Q$ is b-$\omega\alpha$-open, since $int\ \omega(cl\ \alpha(H)) \cup cl\ \alpha(int\ \omega(H)) = int\ \omega(R) \cup cl\ \alpha(\phi) = R \cup \phi = R \supset H$. But $H = Q$ is not semi-$\omega\alpha$-open by (1).

Proposition 3.12. *The intersection of a semi-$\omega\alpha$-open set and an open set is semi-$\omega\alpha$-open.*

Proof. Let H be semi-$\omega\alpha$-open and U be open in X. Then $H \subset cl\ \alpha(int\ \omega(H))$ and $int\ \omega(U) = U$. By Lemma 2.10, we have $U \cap H \subset U \cap cl\ \alpha(int\ \omega(H)) = cl\ \alpha(U \cap int\ \omega(H)) = cl\ \alpha(int\ \omega(U) \cap int\ \omega(H)) = cl\ \alpha(int\ \omega(U \cap H))$ which proves that $U \cap H$ is semi-$\omega\alpha$-open □

Remark 3.13. *The intersection of two semi-$\omega\alpha$-open sets need not be semi-$\omega\alpha$-open as can be seen from the following Example.*

Example 3.14. *In $(R, \tau u)$, $A = (0, 1]$ is semi-$\omega\alpha$-open for $cl\ \alpha(int\ \omega(A)) = cl\ \alpha((0, 1)) = [0, 1] \supset A$. Similarly $B = [1, 2)$ is also semi-$\omega\alpha$-open. But $A \cap B = 1$ is not semi-$\omega\alpha$-open for $cl\ \alpha(int\ \omega(A \cap B)) = cl\ \alpha(int\ \omega(1)) = cl\ \alpha(\phi) = \phi \not\supseteq 1 = A \cap B$.*

Theorem 3.15. *If a subset H of a space (X, τ) is both α-closed and β-$\omega\alpha$-open, then H is semi-$\omega\alpha$-open.*

Proof. Since H is β-$\omega\alpha$-open, $H \subset cl\ \alpha(int\ \omega(cl\ \alpha(H))) = cl\alpha(int\omega(H))$, H being α-closed. Therefore H is semi-$\omega\alpha$-open. □

Theorem 3.16. *If a subset H of a space (X, τ) is both β-$\omega\alpha$-open and a t-$\omega\alpha$-set, then H is semi-$\omega\alpha$-open.*

Proof. Since H is a t-$\omega\alpha$-set, $int(H) = int\ \omega(cl\ \alpha(H))$. Also H is β-$\omega\alpha$-open implies $H \subset cl\ \alpha(int\ \omega(cl\ \alpha(H))) \subset cl\ \alpha(int(H)) \subset cl\ \alpha(int\ \omega(H))$. Therefore H is semi-$\omega\alpha$-open. □

Theorem 3.17. *If a subset H of a space (X, τ) is both b-$\omega\alpha$-open and a t-$\omega\alpha$-set, then H is semi-$\omega\alpha$-open.*

Proof. Since H is a t-$\omega\alpha$-set, $int\ \omega(cl\ \alpha(H)) = int(H) \subset int\ \omega(H)$. Also H is b-$\omega\alpha$-open implies $H \subset int\ \omega(cl\ \alpha(H)) \cup cl\ \alpha(int\ \omega(H)) \subset int\ \omega(H) \cup cl\ \alpha(int\ \omega(H)) = cl\alpha(int\ \omega(H))$. Therefore H is semi-$\omega\alpha$-open. □

Proposition 3.18. *A subset H of a space (X, τ) is semi-$\omega\alpha$-open if and only if $cl\alpha(H) = cl\alpha(int\ \omega(H))$.*

Proof. Let H be semi-$\omega\alpha$-open. Then $H \subset cl\ \alpha(int\ \omega(H))$ and $cl\ \alpha(H) \subset cl\alpha(int\ \omega(H))$. But $cl\ \alpha(int\ \omega(H)) \subset cl\ \alpha(H)$. Thus $cl\ \alpha(H) = cl\ \alpha(int\omega(H))$. Conversely, let the condition hold. We have $H \subset cl\ \alpha(H) = cl\ \alpha(int\ \omega(H))$, by assumption. Thus $H \subset cl\ \alpha(int\ \omega(H))$ and hence H is semi-$\omega\alpha$-open. □

Proposition 3.19. *In (X, τ) if H is a b-$\omega\alpha$-open set such that $cl\ \alpha(H) = \phi$, then H is semi-$\omega\alpha$-open.*

Definition 3.20. *A subset H of a space (X, τ) is called α-dense if $cl\ \alpha(H) = X$.*

Definition 3.21. *A space (X, τ) is called α-submaximal if every α-dense subset of X is open.*

Theorem 3.22. *For a subset H of an α-submaximal space (X, τ), the following are equivalent.*

(1). H is semi-$\omega\alpha$-open,

(2). H is β-$\omega\alpha$-open.

Proof. (1) $\Rightarrow$ (2): From (4) of Theorem 3.8.
(2) $\Rightarrow$ (1): Let H be a β-$\omega\alpha$-open set in X. Then $H \subset cl\ \alpha(int\ \omega(cl\ \alpha(H)))$ and $cl\ \alpha(H) \subset cl\ \alpha(int\ \omega(cl\ \alpha(H)))$. Thus, $cl\alpha(H)$ is semi-$\omega\alpha$-open. Put $A = cl\ \alpha(H)$ and $K = H \cup (X\alpha(H))$. We have $H = cl\ \alpha(H) \cap K$ and $cl\ \alpha(K) = X$. This implies that $H = A \cap K$, where A is semi-$\omega\alpha$-open and K is α-dense. Since X is α-submaximal, K is open. By Proposition 3.12, $H = A \cap K$ is semi-$\omega\alpha$-open. □

Theorem 3.23. *A subset H of a space (X, τ) is semi-$\omega\alpha$-open if and only if there exists $U \in \tau\omega$ such that $U \subset H \subset cl\alpha(U)$.*

Proof. Let H be semi-$\omega\alpha$-open. Then $H \subset cl\ \alpha(int\ \omega(H))$. Take $int\ \omega(H) = U$. Then $U \subset H \subset cl\ \alpha(U)$. Conversely, let $U \subset H \subset cl\ \alpha(U)$ for some $U \in \tau\omega$. Since $U \subset H$, $U \subset int\ \omega(H)$ and $H \subset cl\ \alpha(U) \subset cl\alpha(int\ \omega(H))$ which implies H is semi-$\omega\alpha$-open. □

Proposition 3.24. *If A is a semi-$\omega\alpha$-open set in a space (X, τ) and $A \subset B \subset cl\ \alpha(A)$, then B is semi-$\omega\alpha$-open.*

Proof. By assumption $B \subset cl\ \alpha(A) \subset cl\ \alpha(cl\ \alpha(int\ \omega(A)))$ (for A is semi-$\omega\alpha$-open) $= cl\ \alpha(int\ \omega(A)) \subset cl\ \alpha(int\ \omega(B))$ by assumption. This implies B is semi-$\omega\alpha$-open. □

4. Properties of δ-$\omega\alpha$-open Sets

Definition 4.1. *A subset H of a space (X, τ) is said to be*

(1). δ-$\omega\alpha$-open if $int\ \omega(cl\ \alpha(H)) \subset cl\ \alpha(int\ \omega(H))$.

(2). δ-$\omega\alpha$-closed if $int\ \alpha(cl\ \omega(H)) \subset cl\ \omega(int\ \alpha(H))$. The complement of a δ-$\omega\alpha$-open set is called δ-$\omega\alpha$-closed.

Example 4.2. *In $(R, \tau u)$, for the subset Q, $int\ \omega(cl\ \alpha(Q)) = int\ \omega(R) = R$ and $cl\ \alpha(int\ \omega(Q)) = cl\ \alpha(\phi) = \phi$. Thus $int\ \omega(cl\ \alpha(Q) \nsubseteq cl\ \alpha(int\ \omega(Q))$ which proves that Q is not δ-$\omega\alpha$-open.*

Example 4.3. *In $(R, \tau u)$, for the subset $H = 1$, $int\ \omega(cl\ \alpha(H)) = int\ \omega(H) = \phi$ and $cl\ \alpha(int\ \omega(H)) = cl\ \alpha(\phi) = \phi$. Thus $int\ \omega(cl\ \alpha(H)) \subset cl\ \alpha(int\ \omega(H))$ which proves that H is δ-$\omega\alpha$-open.*

Proposition 4.4. *For a subset H of a space (X, τ), the following properties hold:*

(1). Every α-$\omega\alpha$-open set is δ-$\omega\alpha$-open.

(2). Every t-$\omega\alpha$-set is δ-$\omega\alpha$-open.

Proof.

(1). Since H is α-$\omega\alpha$-open,$H \subset int\ \omega(cl\ \alpha(int\ \omega(H))) \subset cl\ \alpha(int\ \omega(H))$. So $cl\ \alpha(H) \subset cl\ \alpha(int\ \omega(H))$ and $int\ \omega(cl\ \alpha(H)) \subset cl\ \alpha(H) \subset cl\ \alpha(int\ \omega(H))$. Therefore H is δ-$\omega\alpha$-open.

(2). Since H is an t-$\omega\alpha$-set, $int\ \omega(cl\ \alpha(H)) = int(H) \subset H$. Then $int\ \omega(cl\ \alpha(H)) \subset int\ \omega(H) \subset cl\ \alpha(int\ \omega(H))$. Therefore H is δ-$\omega\alpha$-open. The converses of (1) and (2) in Proposition 4.4 are not true in general as seen from the following Example. □

Example 4.5.

(1). In $(R, \tau u)$, the subset $H = 1$ is δ-$\omega\alpha$-open by Example 4.3. But H is not α-$\omega\alpha$-open for $int\ \omega(cl\ \alpha(int\ \omega(H))) = int\ \omega(cl\ \alpha(\phi)) = int\ \omega(\phi) = \phi \not\supseteq 1 = H$.

(2). In $(R, \tau u)$, for the subset $H = Q^\star$, $int\ \omega(cl\ \alpha(H)) = int\ \omega(R) = R$ and $cl\ \alpha(int\ \omega(H)) = cl\ \alpha(H) = R$. Thus $int\ \omega(cl\ \alpha(H)) \subset cl\ \alpha(int\ \omega(H))$ and hence H is δ-$\omega\alpha$-open. But H is not a t-$\omega\alpha$-set for $int\ \omega(cl\ \alpha(H)) = int\ \omega(R) = R \neq \phi = int(H)$.

Definition 4.6. *A subset H of a space (X, τ) is said to be β-$\omega\alpha$-closed if $int\ \alpha(cl\ \omega(int\ \alpha(H))) \subset H$. The complement of a β-$\omega\alpha$-open set is called β-$\omega\alpha$-closed.*

Proposition 4.7. *A subset H of a space (X, τ) is β-$\omega\alpha$-closed if and only if $int\ \alpha(cl\ \omega(int\ \alpha(H))) = int\ \alpha(H)$.*

Proof. Since H is β-$\omega\alpha$-closed set, $int\ \alpha(cl\ \omega(int\ \alpha(H))) \subset H$ and hence $int\ \alpha(cl\ \omega(int\ \alpha(H))) \subset int\ \alpha(H)$. Also $int\ \alpha(H) \subset cl\ \omega(int\ \alpha(H))$ and hence $int\ \alpha(H) \subset int\ \alpha(cl\ \omega(int\ \alpha(H)))$. Thus $int\ \alpha(cl\ \omega(int\ \alpha(H))) = int\ \alpha(H)$. Conversely, let the condition hold. We have $int\ \alpha(cl\ \omega(int\ \alpha(H))) = int\ \alpha(H) \subset H$. Therefore H is β-$\omega\alpha$-closed. □

Theorem 4.8. *For a subset H of a space (X, τ), the following properties are equivalent.*

(1). H is semi-$\omega\alpha$-closed.

(2). H is β-$\omega\alpha$-closed and δ-$\omega\alpha$-closed.

Proof. (1) $\Rightarrow$ (2): Let H be semi-$\omega\alpha$-closed. By (4) of Theorem 3.8, H is β-$\omega\alpha$-closed. Since H is semi-$\omega\alpha$-closed, $int\ \alpha(cl\ \omega(H)) \subset H$ and so $int\ \alpha(cl\ \omega(H)) \subset int\ \alpha(H)$ which implies $cl\ \omega(int\ \alpha(cl\ \omega(H))) \subset cl\ \omega(int\ \alpha(H))$. Thus $int\ \alpha(cl\ \omega(H)) \subset cl\ \omega(int\ \alpha(cl\ \omega(H))) \subset cl\ \omega(int\ \alpha(H))$ and so H is δ-$\omega\alpha$-closed.
(2) $\Rightarrow$ (1): Since H is δ-$\omega\alpha$-closed, $int\ \alpha(cl\ \omega(H)) \subset cl\ \omega(int\ \alpha(H))$ and so $int\ \alpha(cl\ \omega(H)) \subset int\ \alpha(cl\ \omega(int\ \alpha(H))) \subset H$ since H is β-$\omega\alpha$-closed. Thus H is semi-$\omega\alpha$-closed. □

Remark 4.9. *The following Examples show that the concepts of β-$\omega\alpha$-closedness and δ-$\omega\alpha$-closedness are independent.*

Example 4.10. *In $(R, \tau u)$, the subset $H = R - 1$ is δ-$\omega\alpha$-closed by Example 4.3. But $int\ \alpha(cl\ \omega(int\ \alpha(H))) = int\ \alpha(cl\ \omega(int(H))) = int\ \alpha(cl\ \omega(H)) = int\ \alpha(R) = int(R) = R \not\subseteq R - 1 = H$ which proves that H is not β-$\omega\alpha$-closed.*

Example 4.11. *In $(R, \tau u)$, the subset $H = Q^\star$ is not δ-$\omega\alpha$-closed for Q is not δ-$\omega\alpha$-open by Example 4.2. But $int\ \alpha(cl\ \omega(int\ \alpha(H))) = int\ \alpha(cl\ \omega(int(H))) = int\ \alpha(cl\ \omega(\phi)) = int\ \alpha(\phi) = \phi \subset H$. Thus $H = Q^\star$ is β-$\omega\alpha$-closed.*

Theorem 4.12. *Let (X, τ) be a space. Then a subset of X is α-$\omega\alpha$-open if and only if it is both δ-$\omega\alpha$-open and pre-$\omega\alpha$-open.*

Proof. **Necessity:** Let H be an α-$\omega\alpha$-open set. Then $H \subset int\ \omega(cl\ \alpha(int\ \omega(H)))$. It implies that $cl\ \alpha(H) \subset cl\ \alpha(int\ \omega(H))$ and $int\ \omega(cl\ \alpha(H)) \subset int\ \omega(cl\ \alpha(int\ \omega(H))) \subset cl\ \alpha(int\ \omega(H))$. Hence, H is a δ-$\omega\alpha$-open set. On the other hand, since H is an α-$\omega\alpha$-open set, H is a pre-$\omega\alpha$-open set by Proposition 3.7.

Sufficiency: Let H be both δ-$\omega\alpha$-open and pre-$\omega\alpha$-open. Since H is δ-$\omega\alpha$-open, $int\ \omega(cl\ \alpha(H)) \subset cl\ \alpha(int\ \omega(H))$ and hence $int\ \omega(cl\ \alpha(H)) \subset int\ \omega(cl\ \alpha(int\ \omega(H)))$. Since H is pre-$\omega\alpha$-open, $H \subset int\ \omega(cl\ \alpha(H)) \subset int\ \omega(cl\ \alpha(int\ \omega(H)))$ which proves that H is an α-$\omega\alpha$-open set. $\square$

Remark 4.13. *The following Examples show that the concepts of δ-$\omega\alpha$-open-ness and pre-$\omega\alpha$-openness are independent.*

Example 4.14. *In $(R, \tau u)$, $H = (0, 1]$ is δ-$\omega\alpha$-open, since $int\ \omega(cl\ \alpha(H)) = int\ \omega([0, 1]) = (0, 1)$ and $cl\ \alpha(int\ \omega(H)) = cl\ \alpha((0, 1)) = [0, 1]$. But $H = (0, 1]$ is not pre-$\omega\alpha$-open, since $int\ \omega(cl\ \alpha(H)) = int\ \omega([0, 1]) = (0, 1) \not\supseteq H$.*

Example 4.15. *In $(R, \tau u)$, $H = Q$ is pre-$\omega\alpha$-open, since $int\ \omega(cl\ \alpha(H)) = int\ \omega(R) = R \supset Q = H$. But $cl\ \alpha(int\ \omega(H)) = cl\ \alpha(\phi) = \phi$ and $int\ \omega(cl\ \alpha(H)) = R$ implies $int\ \omega(cl\ \alpha(H)) \not\subseteq cl\ \alpha(int\ \omega(H))$. Thus H is not δ-$\omega\alpha$-open.*

Proposition 4.16. *Let A and B be subsets of a space (X, τ). If $A \subset B \subset cl\ \alpha(A)$ and A is δ-$\omega\alpha$-open in X, then B is δ-$\omega\alpha$-open in X.*

Proof. Suppose that $A \subset B \subset cl\ \alpha(A)$ and A is δ-$\omega\alpha$-open in X. Then $int\ \omega(cl\ \alpha(A)) \subset cl\ \alpha(int\ \omega(A)) \subset cl\ \alpha(int\ \omega(B))$. Since $B \subset cl\ \alpha(A)$, $cl\ \alpha(B) \subset cl\ \alpha(cl\ \alpha(A)) = cl\ \alpha(A)$ and $int\ \omega(cl\ \alpha(B)) \subset int\ \omega(cl\ \alpha(A))$. Therefore $int\ \omega(cl\ \alpha(B)) \subset cl\ \alpha(int\ \omega(B))$. This shows that B is a δ-$\omega\alpha$-open set. $\square$

Corollary 4.17. *Let (X, τ) be a space. If $A \subset X$ is δ-$\omega\alpha$-open and α-dense in (X, τ), then every subset of X containing A is δ-$\omega\alpha$-open.*

Proof. It is obvious by Proposition 4.16. $\square$

5. Properties of Semi*-$\omega\alpha$-open Sets

Definition 5.1. *A subset H of a space (X, τ) is said to be*

(1). semi*-$\omega\alpha$-open if $H \subset cl\ \omega(int\ \alpha(H))$.

(2). semi*-$\omega\alpha$-closed if $int\ \omega(cl\ \alpha(H)) \subset H$. The complement of a semi*-$\omega\alpha$-open set is called semi*-$\omega\alpha$-closed.

Example 5.2. *In $(R, \tau u)$, $H = R\{0$ is not semi*-$\omega\alpha$-closed, since $int\ \omega(cl\ \alpha(H)) = int\ \omega(R) = R \not\subseteq H\}$.*

Example 5.3. *In R with the topology $\tau = \phi, R, Q, Q^\star$, $H = Q$ is semi*-$\omega\alpha$-closed, since $int\ \omega(cl\ \alpha(H)) = int\ \omega(H) = H \subset H$.*

Proposition 5.4. *For a subset of a space (X, τ), every semi*-ω-open set is semi*-$\omega\alpha$-open. If H is semi*-ω-open, then $H \subset cl\ \omega(int(H)) \subset cl\ \omega(int\ \alpha(H))$. Therefore H is semi*-$\omega\alpha$-open.*

Remark 5.5. *The converse of Proposition 5.4 is not true.*

Example 5.6. *In R with usual topology τu and ideal $I = P(R)$, $H = Q$ is semi*-$\omega\alpha$-closed, since $int\ \omega(cl\ \alpha(H)) = int\ \omega(H) = \phi \subset H$. But $H = Q$ is not semi*-ω-closed, since $int\ \omega(cl(H)) = int\ \omega(R) = R \not\subseteq H$.*

Proposition 5.7. *A subset H of a space (X, τ) is semi*-$\omega\alpha$-open if and only if $cl\ \omega(H) = cl\ \omega(int\ \alpha(H))$.*

Proof. If H is semi*-$\omega\alpha$-open set, then $H \subset cl\ \omega(int\ \alpha(H))$ and $cl\ \omega(H) \subset cl\ \omega(int\ \alpha(H))$. But $cl\ \omega(int\ \alpha(H)) \subset cl\ \omega(H)$. Hence $cl\ \omega(H) = cl\ \omega(int\ \alpha(H))$. Conversely, $H \subset cl\ \omega(H) = cl\ \omega(int\ \alpha(H))$ by assumption. Therefore H is semi*-$\omega\alpha$-open. □

Definition 5.8. *A subset H of a space (X, τ) is said to be a t-$\omega(\alpha^{\star})$-set if $int\ \omega(cl\ \alpha(H)) = int\ \omega(H)$.*

Example 5.9. *In R with usual topology τu and ideal $I = \phi$,*

(1). $H = (0, 1]$ is a t-$\omega(\alpha^{\star})$-set, since $int\ \omega(H) = (0, 1)$ and $int\ \omega(cl\ \alpha(H)) = int\ \omega([0, 1]) = (0, 1)$.

(2). $H = Q^{\star}$ is not a t-$\omega(\alpha^{\star})$-set, since $int\ \omega(H) = H$ and $int\ \omega(cl\ \alpha(H)) = int\ \omega(R) = R$.

Proposition 5.10. *In a space (X, τ), every α-closed set is a t-$\omega(\alpha^{\star})$-set.*

Let H be an α-closed set. Then $H = cl\ \alpha(H)$ and $int\ \omega(cl\ \alpha(H)) = int\ \omega(H)$ which proves that H is a t-$\omega(\alpha^{\star})$-set.

Remark 5.11. *The converse of Proposition 5.10 is not true.*

Example 5.12. *In $(R, \tau u)$, $H = (0, 1]$ is t-$\omega(\alpha^{\star})$-set by (1) of Example 5.9. But $H = (0, 1]$ is not α-closed, since $cl\ \alpha(H) = [0, 1] \neq H$.*

Proposition 5.13. *In a space (X, τ), every t-$\omega\alpha$-set is t-$\omega(\alpha^{\star})$-set.*

If H is a t-$\omega\alpha$-set, then $int\ \omega(cl\ \alpha(H)) = int(H) \subset int\ \omega(H) \subset int\ \omega(cl\ \alpha(H))$. Thus $int\ \omega(cl\ \alpha(H)) = int\ \omega(H)$ and hence H is a t-$\omega(\alpha^{\star})$-set.

Remark 5.14. *The converse of Proposition 5.13 is not true.*

Example 5.15. *In R with usual topology τu and ideal $I = P(R)$, $H = (0, 1) \cap Q^{\star}$ is a t-$\omega(\alpha^{\star})$-set since $int\ \omega(cl\ \alpha(H)) = int\ \omega(H)$. But H is not a t-$\omega\alpha$-set since $int\ \omega(cl\ \alpha(H)) = int\ \omega(H) = H \neq \phi = int(H)$.*

Theorem 5.16. *A subset H of a space (X, τ) is semi*-$\omega\alpha$-closed if and only if H is a t-$\omega(\alpha^{\star})$-set.*

H is a semi*-$\omega\alpha$-closed in $X \Leftrightarrow X'$ is semi*-$\omega\alpha$-open $\Leftrightarrow cl\ \omega(X'') = cl\ \omega(int\ \alpha(X''))$ by Proposition 5.7 $\Leftrightarrow X\omega(H) = X\omega(cl\ \alpha(H)) \Leftrightarrow int\ \omega(H) = int\ \omega(cl\ \alpha(H)) \Leftrightarrow H$ is a t-$\omega(\alpha^{\star})$-set.

Proposition 5.17. *If A and B are t-$\omega(\alpha^{\star})$-sets of a space (X, τ), then $A \cap B$ is a t-$\omega(\alpha^{\star})$-set.*

Let A and B be t-$\omega(\alpha^{\star})$-sets. Then $int\ \omega(A \cap B) \subset int\ \omega(cl\ \alpha(A \cap B)) \subset int\ \omega(cl\ \alpha(A) \cap cl\ \alpha(B)) = int\ \omega(cl\ \alpha(A)) \cap int\ \omega(cl\ \alpha(B)) = int\omega(A) \cap int\omega(B) = int\omega(A \cap B)$. Thus $int\ \omega(A \cap B) = int\ \omega(cl\ \alpha(A \cap B))$ and hence $A \cap B$ is a t-$\omega(\alpha^{\star})$-set.

Definition 5.18. *A subset H of a space (X, τ) is said to be semi-$\omega\alpha$-regular if H is semi-$\omega\alpha$-open and a t-$\omega(\alpha^{\star})$-set.*

Example 5.19. *In R with usual topology τu and ideal I=ϕ,*

(1). $H = (0, 1]$ is a t-$\omega(\alpha^{\star})$-set by (1) of Example 5.9. Also $cl\ \alpha(int\ \omega(H)) = cl\ \alpha((0, 1)) = [0, 1] \supset H$. Thus H is semi-$\omega\alpha$-open. Hence $(0, 1]$ is semi-$\omega\alpha$-regular.

(2). $H = Q^{\star}$ is not a t-$\omega(\alpha^{\star})$-set by (2) of Example 5.9. Hence $Q^{\star}$ is not semi-$\omega\alpha$-regular.

Remark 5.20. *In a space (X, τ),*

(1). Every semi-$\omega\alpha$-regular set is semi-$\omega\alpha$-open.

(2). Every semi-$\omega\alpha$-regular set is t-$\omega(\alpha^\star)$-set.

The converses of (1) and (2) in Remark 5.20 are not true in general as illustrated in the following Examples.

Example 5.21. *In R with usual topology τu and ideal $I = \phi$, the subset $H = Q^\star$ is semi-$\omega\alpha$-open by Example 3.2. But $H = Q^\star$ is not semi-$\omega\alpha$-regular by (2) of Example 5.19.*

Example 5.22. *In $(R, \tau u)$, $N =$ the set of all natural numbers is a t-$\omega(\alpha^\star)$-set for $int\ \omega(cl\ \alpha(N)) = int\ \omega(N)$. But N is not semi-$\omega\alpha$-regular for N is not semi-$\omega\alpha$-open since $cl\ \alpha(int\ \omega(N)) = cl\ \alpha(\phi) = \phi \not\supseteq N$.*

Theorem 5.23. *A subset of a space (X, τ) is semi-$\omega\alpha$-regular if and only if it is both β-$\omega\alpha$-open and semi*-$\omega\alpha$-closed.*

Proof. If H is semi-$\omega\alpha$-regular, then H is both semi-$\omega\alpha$-open and a t-$\omega(\alpha^\star)$-set. Since H is semi-$\omega\alpha$-open, H is β-$\omega\alpha$-open by (4) of Theorem 3.8. Also H is a t-$\omega(\alpha^\star)$-set by assumption. Hence by Example 7.7, H is semi*-$\omega\alpha$-closed. Conversely, let H be semi*-$\omega\alpha$-closed and β-$\omega\alpha$-open. Since H is semi*-$\omega\alpha$-closed, by Theorem 5.16, H is a t-$\omega(\alpha^\star)$-set. Since H is β-$\omega\alpha$-open, $H \subset cl\ \alpha(int\ \omega(cl\ \alpha(H))) = cl\ \alpha(int\ \omega(H))$. Therefore H is semi-$\omega\alpha$-open. Since H is both semi-$\omega\alpha$-open and a t-$\omega(\alpha^\star)$-set, H is semi-$\omega\alpha$-regular. □

Remark 5.24. *The following Example shows that the concepts of β-$\omega\alpha$-open -ness and semi*-$\omega\alpha$-closedness are independent.*

Example 5.25.

(1). In R with the topology $\tau = \phi, R, Q^\star$, $H = Q$ is semi*-$\omega\alpha$-closed, since $int\ \omega(cl\ \alpha(H)) = int\ \omega(H) = \phi \subset H$. But $H = Q$ is not β-$\omega\alpha$-open, since $cl\ \alpha(int\ \omega(cl\ \alpha(H))) = cl\ \alpha(int\ \omega(H)) = cl\ \alpha(\phi) = \phi \not\supseteq H$.

(2). In $(R, \tau u)$, $H = Q$ is β-$\omega\alpha$-open, since $cl\ \alpha(int\ \omega(cl\ \alpha(H))) = cl\ \alpha(int\ \omega(R)) = R \supset H$. But $H = Q$ is not semi*-$\omega\alpha$-closed, since $int\ \omega(cl\ \alpha(H)) = int\ \omega(R) = R \not\subseteq H$.

6. Properties of $\omega\alpha$-R-closed Sets

Definition 6.1. *A subset H of a space (X, τ) is called $\omega\alpha$-R-closed if $H = cl\ \alpha(int\ \omega(H))$.*

Example 6.2. *In R with the topology $\tau = \phi, R, Q^\star$, $H = Q$ is not $\omega\alpha$-R-closed, since $cl\ \alpha(int\ \omega(H)) = cl\ \alpha(\phi) = \phi \neq H$.*

Example 6.3. *In $(R, \tau u)$, $H = [0, 1]$ is $\omega\alpha$-R-closed for $cl\ \alpha(int\ \omega(H)) = cl\ \alpha((0, 1)) = H$.*

Theorem 6.4. *For a subset H of a space (X, τ), the following properties are equivalent.*

(1). H $(\neq \phi)$ is $\omega\alpha$-R-closed.

(2). There exists a non-empty ω-open set G such that $G \subset H = cl\ \alpha(G)$.

(3). There exists a non-empty ω-open set G such that $H = G \cup (cl\ \alpha(G) - G)$.

Proof. (1) $\Rightarrow$ (2): Suppose H $(\neq \phi)$ is an $\omega\alpha$-R-closed set. Then $H = cl\ \alpha(int\ \omega(H))$. Let $G = int\ \omega(H)$. G is the required ω-open set such that $G \subset H = cl\ \alpha(G)$.

(2) $\Rightarrow$ (3): Since $H = cl\ \alpha(G) = G \cup (cl\ \alpha(G) - G)$ where G is a nonempty ω-open set, (3) follows.

(3) $\Rightarrow$ (1): $H = G \cup (cl\ \alpha(G) - G)$ implies that $H = cl\ \alpha(G) = cl\ \alpha(int\ \omega(G)) \subset cl\ \alpha(int\ \omega(H))$, since G is ω-open and $G \subset H$. Also $cl\ \alpha(int\ \omega(H)) \subset cl\ \alpha(H) = cl\ \alpha(G) = H$. Therefore $H = cl\ \alpha(int\ \omega(H))$ which implies that H is $\omega\alpha$-R-closed. □

Theorem 6.5. *For each β-$\omega\alpha$-open subset H of a space (X, τ), $cl\ \alpha(H)$ is $\omega\alpha$-R-closed.*

Proof. Suppose H is β-$\omega\alpha$-open. Then $H \subset cl\ \alpha(int\ \omega(cl\ \alpha(H)))$ and so $cl\ \alpha(H) \subset cl\ \alpha(int\ \omega(cl\ \alpha(H))) \subset cl\ \alpha(H)$ which implies that $cl\ \alpha(H) = cl\ \alpha(int\ \omega(cl\ \alpha(H)))$. Therefore $cl\ \alpha(H)$ is $\omega\alpha$-R-closed. □

Theorem 6.6. *For a subset H of a space (X, τ), the following properties are equivalent.*

(1). H is $\omega\alpha$-R-closed.

(2). H is semi-$\omega\alpha$-open and α-closed.

(3). H is β-$\omega\alpha$-open and α-closed.

Proof. (1) $\Rightarrow$ (2): If H is $\omega\alpha$-R-closed, then $H = cl\ \alpha(int\ \omega(H))$. Since $H \subset cl\ \alpha(int\ \omega(H))$, H is semi-$\omega\alpha$-open. Also, $H = cl\ \alpha(H)$ and thus H is α-closed.

(2) $\Rightarrow$ (3): It follows from the fact that every semi-$\omega\alpha$-open set is a β-$\omega\alpha$-open.

(3) $\Rightarrow$ (1): Suppose H is β-$\omega\alpha$-open and α-closed. Then $H \subset cl\ \alpha(int\ \omega(cl\ \alpha(H)))$ and $H = cl\ \alpha(H)$. Now $cl\ \alpha(int\ \omega(H)) \subset cl\ \alpha(H) = H$. Also, $H \subset cl\ \alpha(int\ \omega(H))$. Therefore $H = cl\ \alpha(int\ \omega(H))$ which implies that H is $\omega\alpha$-R-closed. □

Remark 6.7. *The following Examples show that*

(1). the concepts of semi-$\omega\alpha$-openness and α-closedness are independent.

(2). the concepts of β-$\omega\alpha$-openness and α-closedness are independent.

Example 6.8. *In $(R, \tau u)$,*

(1). $H = Q^\star$ is semi-$\omega\alpha$-open by Example 5.21. But H is not α-closed for $cl\ \alpha(H) = R \neq H$.

(2). $N =$ the set of all natural numbers is not semi-$\omega\alpha$-open by Example 5.22. But N is α-closed for $cl\ \alpha(N) = N$.

Example 6.9. *In $(R, \tau u)$,*

(1). the subset $H = Q^\star$ is semi-$\omega\alpha$-open by Example 5.21 and hence β-$\omega\alpha$-open by (4) of Theorem 3.8. But $H = Q^\star$ is not α-closed for $cl\ \alpha(H) = R \neq H$.

(2). $N =$ the set of all natural numbers is α-closed by (2) of Example 6.8. But N is not β-$\omega\alpha$-open for $cl\ \alpha(int\ \omega(cl\ \alpha(N))) = cl\ \alpha(int\ \omega(N)) = cl\ \alpha(\phi) = \phi \not\supseteq N$.

7. Further Properties

Definition 7.1. *A space (X, τ) is called $\omega\alpha$-submaximal if every α-dense subset of X is ω-open.*

Proposition 7.2.

(1). Every submaximal space is α-submaximal.

(2). Every α-submaximal space is $\omega\alpha$-submaximal.

Proof.

(1). If (X, τ) is submaximal and H is α-dense in the space (X, τ), then $cl\ \alpha(H) = X$. But $X = cl\ \alpha(H) \subset cl(H)$ implies $cl(H) = X$. Thus H is dense in X and by assumption H is open in X. This shows that (X, τ) is α-submaximal.

(2). Proof follows directly since any open set is ω-open. The converses of (1) and (2) in Proposition 7.2 are not true in general as illustrated below. □

Example 7.3.

(1). In $(R, \tau u)$, if H is any α-dense subset, then $cl\ \alpha(H) = R$ and so $H = R$ which is open. Thus $(R, \tau u)$ is α-submaximal. But in $(R, \tau u)$, Q is dense in R since $cl(Q) = R$. But Q is not open. This shows that $(R, \tau u)$ is not submaximal.

(2). In the space (N, τ), where N is the set of all natural numbers, $\tau = \phi, N$, if H is any α-dense subset in N, then $H \subset N$. Since N is countable, H is ω-open. Thus (N, τ) is $\omega\alpha$-submaximal. The subset $A = 1$ is α-dense in N for $cl\ \alpha(A) = N$. But $A = 1$ is not open in N. This shows that (N, τ) is not α-submaximal.

Definition 7.4. *A subset H of a space (X, τ) is called α-co-dense if X' is α-dense.*

Theorem 7.5. *For a space (X, τ), the following are equivalent.*

(1). X is $\omega\alpha$-submaximal,

(2). Every α-co-dense subset of X is ω-closed.

X is $\omega\alpha$-submaximal $\Leftrightarrow$ every α-dense subset of X is ω-open $\Leftrightarrow$ every α-co-dense subset of X is ω-closed since a subset A is α-dense in X if and only if $X - A$ is α-co-dense in X.

Example 7.6.

(1). In R with usual topology τu and ideal $I = P(R)$, Z is a $t\alpha$-$I(\omega^{\#})$-set, since $cl^{\star}(int\ \omega(cl^{\star}(Z))) = cl^{\star}(int\ \omega(Z)) = cl^{\star}(\phi) = \phi = int(Z)$.

(2). In R with usual topology τu and ideal $I = \phi$, $H = (0, 1)$ is not a $t\alpha$-$I(\omega^{\#})$-set, since $int(H) = (0, 1)$ and $cl^{\star}(int\ \omega(cl^{\star}(H))) = cl^{\star}(int\ \omega([0, 1])) = cl^{\star}((0, 1)) = [0, 1]$ which implies $int(H) \neq cl^{\star}(int\ \omega(cl^{\star}(H)))$.

(3). In R with usual topology τu and ideal $I = \phi$, $H = Q$ is not a $t\alpha$-$I(\omega^{\#})$-set, since $int(Q) = \phi$ and $cl^{\star}(int\ \omega(cl^{\star}(Q))) = cl^{\star}(int\ \omega(cl(Q))) = cl^{\star}(int\ \omega(R)) = cl^{\star}(R) = R$ which implies $int(Q) \neq cl^{\star}(int\ \omega(cl(Q)))$.

Example 7.7.

(1). In R with usual topology τu and ideal $I = P(R)$, Z is a $t\alpha$-$I(\omega^{\#})$-set by (1) of Example 7.6 and hence a $B\alpha$-$I(\omega^{\#})$-set by (2) of Remark 7.8.

(2). In R with usual topology τu and ideal $I = \phi$, $H = Q$ is not a $B\alpha$-$I(\omega^{\#})$-set. If $H = U \cap V$ where U is open and V is a $t\alpha$-$I(\omega^{\#})$-set, then $H \subset U$. But R is the only open set containing H. Hence $U = R$ and $H = R \cap V = V$ which means that H is a $t\alpha$-$I(\omega^{\#})$-set which is a contradiction by (3) of Example 7.6. Thus H is not a $B\alpha$-$I(\omega^{\#})$-set.

Remark 7.8. *In an ideal topological space (X, τ, I),*

(1). Every open set is a $B\alpha$-$I(\omega^{\#})$-set.

(2). Every $t\alpha$-$I(\omega^{\#})$-set is a $B\alpha$-$I(\omega^{\#})$-set.

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