

# Antimagic Labeling of Arrow Graph, Double Arrow Graph and Globe Graph

Shanti S. Khunti<sup>1,\*</sup>, Jekil A. Gadhiya<sup>2</sup>, Mehul A. Chaurasiya<sup>3</sup> and Mehul P. Rupani<sup>3</sup>

<sup>1</sup> Department of Mathematics, Saurashtra University, Rajkot, Gujarat, India.

<sup>2</sup> Department of Mathematics, Marwadi University, Rajkot, Gujarat, India.

<sup>3</sup> Department of Mathematics, Shree H.N.Shukla Group of College, Rajkot, Gujarat, India.

**Abstract:** All the Graphs consider in this article are finite, simple and undirected. In this article we investigate antimagic labeling of Arrow Graph ( $A_n^2$ ), Double Arrow Graph ( $DA_n^2$ ) and Globe graph ( $Gl(n)$ ).

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## 1. Introduction

We begin with a finite, connected and undirected graph  $G = (V(G), E(G))$  without loops and multiple edges. Throughout this paper  $|V(G)|$  and  $|E(G)|$  denote the number of vertices and number of edges respectively. We denote the edge  $e$  with end vertices  $u$  and  $v$  by  $e = uv$ . For any graph theoretic notation and terminology, we rely upon Balakrishnan and Ranganathan [1]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

According to Beineke and Hegde [2] labeling of discrete structure is a frontier between graph theory and theory of numbers. For an extensive survey of graph labeling as well as bibliographic references we refer to Gallian [6].

An antimagic labeling of a graph  $G$  is a bijection from  $E(G)$  to the set  $\{1, 2, \dots, |E(G)|\}$  such that for any two distinct vertices  $u$  and  $v$ , the sum of the labels on edges incident to  $u$  is different from the sum of the labels on edges incident to  $v$ . Hartsfield and Ringel [7] have introduced the concept of an antimagic graph in 1990. They proved that paths  $P_n$  ( $n \leq 3$ ), cycles, wheels, and complete graphs  $K_n$  ( $n \leq 3$ ) admit antimagic labeling. They have also conjectured that;

1. all trees except  $K_2$  are antimagic.
2. all connected graphs except  $K_2$  are antimagic.

These two conjectures are still not settled.

In grid graph on  $mn$  vertices, vertices  $v_{1,1}, v_{2,1}, v_{3,1}, \dots, v_{m,1}$  and  $v_{1,n}, v_{2,n}, v_{3,n}, \dots, v_{m,n}$  are known as superior vertices from both ends shown in Figure 1.

\* E-mail: [shantikhunti@yahoo.com](mailto:shantikhunti@yahoo.com)

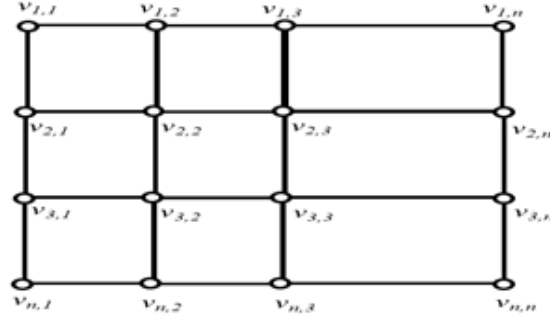


Figure 1.

An arrow graph  $A_n^t$  with width  $t$  and Length  $n$  is obtained by joining a vertex  $v$  with superior vertices of grid graph by  $m$  new edges [8].

A double arrow graph  $DA_n^t$  with width  $t$  and length  $n$  is obtained by joining a vertex  $v$  and  $w$  with superior vertices of grid graph by  $2m$  new edges from both the ends.

A globe graph  $Gl(n)$  is a graph obtained from two isolated vertex are joined by  $n$  paths of length two.

## 2. Main Results

**Theorem 2.1.**  $A_n^2$  is an antimagic graph ( $n \geq 2$ ).

*Proof.* Consider the graph  $A_n^2$ . Note that  $A_n^2$  is finite, simple and undirected graph with  $2n + 1$  vertices and  $3n$  edges as shown in the following Figure 2

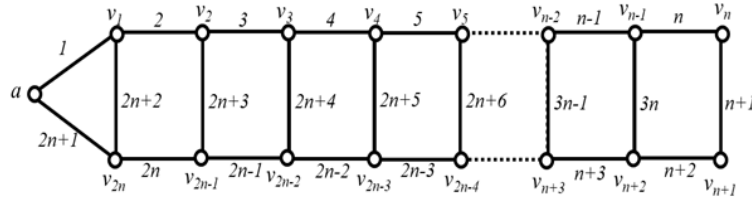


Figure 2.

Now we define function  $f : E(G) \rightarrow \{1, 2, 3, \dots, |E(G)|\}$ , as follows

$$f(v_i v_{i+1}) = i + 1, \quad \forall \quad i = 1, 2, 3, \dots, 2n - 1.$$

$$f(av_1) = 1,$$

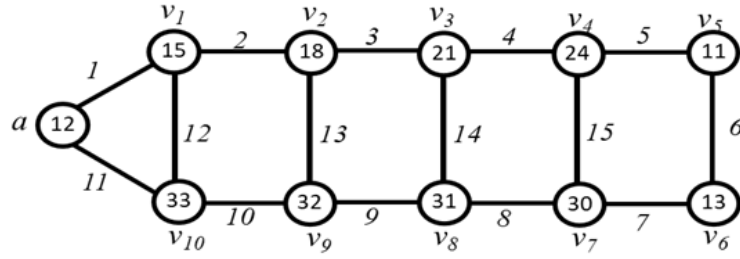
$$f(av_{2n}) = 2n + 1,$$

$$f(v_i v_j) = (2n + 1) + i, \quad \text{where } i < j, i \neq n \text{ and } i + j = 2n + 1$$

□

Note that above defined edge labeling function is well defined and satisfies the condition for antimagic labeling. So,  $A_n^2$  is antimagic.

**Illustration 2.2.** Arrow graph  $A_5^2$  and its antimagic labeling is shown in below Figure 3.



**Remark 2.5.** It is important to note that above defined labelling function is not working for equal to 2 i.e., above defined labeling pattern is not working for  $DA_2^2$ . So, in the following Figure 6 we have shown the graph of  $DA_2^2$  and its antimagic labeling.

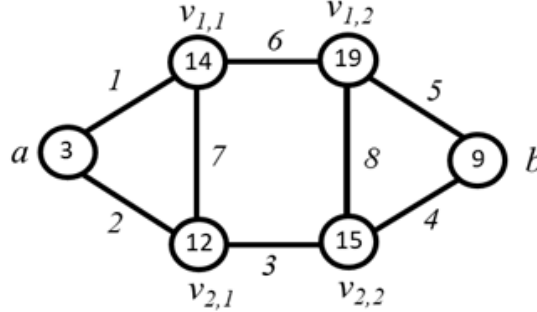


Figure 6.

**Theorem 2.6.**  $DA_n^2$  is an antimagic graph ( $n \geq 2$ ).

*Proof.* Proof follows from Theorem 2.4 and Remark 2.5. □

**Illustration 2.7.** Double arrow graph  $DA_5^2$  and its antimagic labeling is shown in following Figure 7.

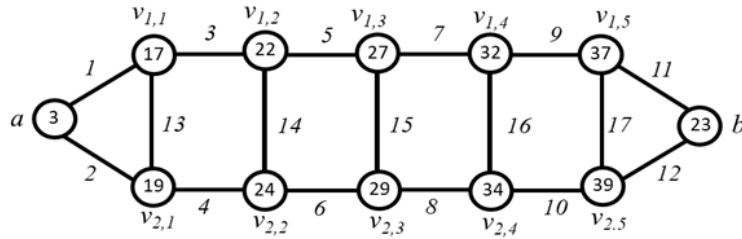


Figure 7.

**Illustration 2.8.** Double arrow graph  $DA_6^2$  and its antimagic labeling is shown in following Figure 8.

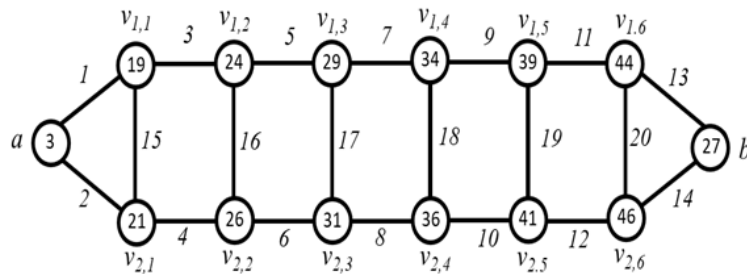


Figure 8.

**Theorem 2.9.**  $Gl(n)$  is an antimagic graph ( $n \geq 2$ ).

*Proof.* Consider the graph  $Gl(n)$ . Note that globe graph  $Gl(n)$  is a finite, simple and undirected graph with  $n + 2$  vertices and  $2n$  edges as shown in the following Figure 9.

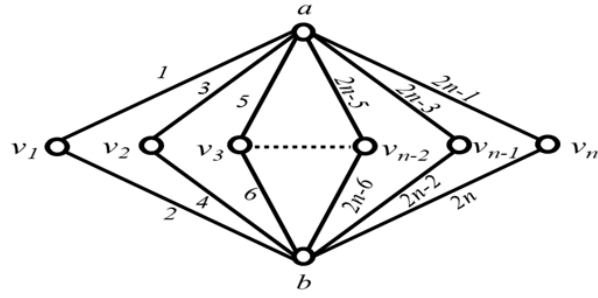


Figure 9.

Now we define function :  $E(G) \rightarrow \{1, 2, 3, \dots, |E(G)|\}$ , as follows

$$f(av_i) = 2i - 1, \quad \forall i = 1, 2, 3, \dots, n.$$

$$f(bv_i) = 2i, \quad \forall i = 1, 2, 3, \dots, n.$$

Above defined edge labeling function will generate all the distinct vertex labels satisfying the condition for antimagic labeling. Hence  $Gl(n)$  is antimagic.  $\square$

**Illustration 2.10.** Globe graph  $Gl(5)$  and its antimagic labeling is shown in following Figure 10.

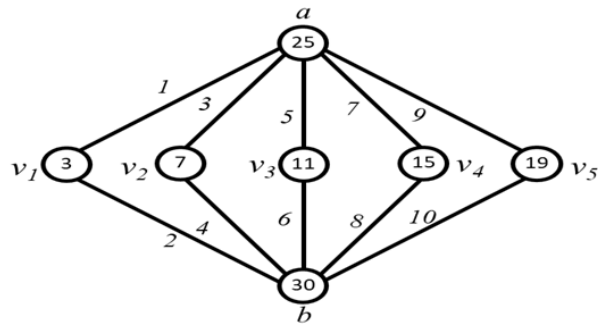


Figure 10.

**Illustration 2.11.** Globe graph  $Gl(6)$  and its antimagic labeling is shown in following Figure 11.

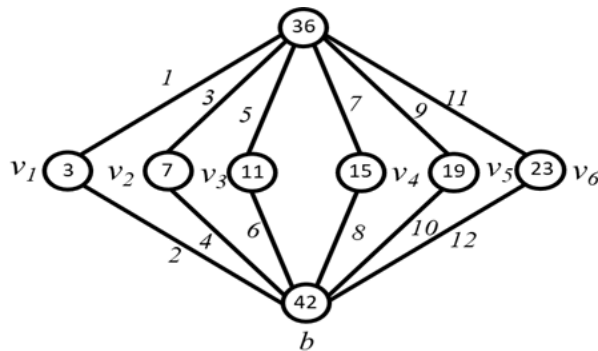


Figure 11.

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