Int. J. Math. And Appl., 8(1)(2020), 217–230
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Soft $\tau_1\tau_2$ g^*s Closed Sets and Their Mappings in Bi Soft Topological Spaces

K. Sowbarnia[1,*]

1 Research Scholar, PG & Research Department of Mathematics, Nallamuthu Gounder Mahalingam College, Tamil Nadu, India.

Abstract: In this paper we introduce and study soft $\tau_1\tau_2$ g^*s closed sets, soft $\tau_1\tau_2$ g^*s continuous mappings and soft $\tau_1\tau_2$ g^*s irresolute mappings soft $\tau_1\tau_2$ g^*s homeomorphisms in bi soft topological spaces.

MSC: 54A40, 06D72, 54E55, 54C05, 57S05.

Keywords: soft $\tau_1\tau_2$ g^*s closed sets, soft $\tau_1\tau_2$ g^*s continuous mappings, soft $\tau_1\tau_2$ g^*s irresolute mappings, soft $\tau_1\tau_2$ g^*s homeomorphisms.

1. Introduction

In real life condition, we cannot beautifully use the traditional classical methods because of different types of uncertainties presented in the problems in economics, engineering, social sciences, medical science etc.. To overcome these difficulties, some kinds of theories were put forwarded like theory of fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, in which we can safely use a mathematical technique for the businessing with uncertainties. But, all these theories have their inherent difficulties. In 1999, Russian scientist Molodtsov [11], originated the notion of soft set as a new mathematical technique for uncertainties, which is free from the above complications. Application of soft set theory in many disciplines and real life problems, have established their role in scientific literature. Many researchers are working in this very important area. Molodtsov [11] and Ahmad [1] successfully applied the soft set theory in the different directions, such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, probability, theory of measurement and so on. Concept of soft topological spaces is introduced in [15], where soft separation axioms have been studied as well. Kelly [10] introduced the concept of bi topological spaces and studied the separation properties for bi topological spaces. The concept of soft topological spaces have been generalized to initiate the study of the bi soft topological spaces. The concept of bi soft topological spaces has been introduced in [12] and studied the separation axioms for bi soft topological spaces. In this paper we introduce the soft $\tau_1\tau_2$ g^*s closed sets in bi soft topological space. Also we investigate related properties of these sets and compared their properties with other existing soft closed sets in bi soft topological spaces. Also we introduce soft $\tau_1\tau_2$ g^*s continuous mappings, soft $\tau_1\tau_2$ g^*s irresolute mappings and soft $\tau_1\tau_2$ g^*s homeomorphism and a detailed study of some of its properties in bi soft topological spaces.

* *E-mail: sowbarnia333@mail.com*

2. Preliminaries

In this section some basic concepts which are pre-requisites for present study. Throughout this paper $(\widetilde{X}, \tau_1, \tau_2, E)$ represents a non-empty bi soft topological space. For a subset (F, E) of $\widetilde{X}$, the closure, the interior, the complement of (F, E) are denoted by $cl(F, E)$, $int(F, E)$ and $(F, E)^c$ respectively.

Definition 2.1 ([10]). *A bi topological space is the triplet(X, P, Q) where X is a non empty set, P and Q are two topologies on X.*

Definition 2.2 ([11]). *Let $\widetilde{X}$ be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of $\widetilde{X}$ and A be a non empty subset of E. A pair (F, A) is called a soft set over $\widetilde{X}$, where F is a mapping given by $F : A \to P(X)$.*

Definition 2.3 ([2]). *A soft set (F, E) over $\widetilde{X}$ is said to be i). A null soft set, denoted by ϕ, if $\forall e \in E$, $F(e) = \phi$. ii). An absolute soft set, denoted by $\widetilde{X}$, if $\forall e \in E$, $F(e) = \widetilde{X}$. The soft set (F, E) over an universe $\widetilde{X}$ in which all the parameters of the set E are same is a family of soft sets, denoted by $SS(\widetilde{X})_E$.*

Definition 2.4 ([15]). *Let τ be the collection of soft sets over $\widetilde{X}$. Then τ is said to be a soft topology on $\widetilde{X}$ if i). ϕ and $\widetilde{X}$ belong to τ. ii). The union of any number of soft sets in τ belongs to τ. iii). The intersection of any two soft sets in τ belongs to τ.*

The triplet $(\widetilde{X}, \tau, E)$ is called a soft topological space over $\widetilde{X}$ and any member of τ is known as soft open set in $\widetilde{X}$. The complement of a soft open set is called as soft closed set over $\widetilde{X}$.

Definition 2.5 ([15]). *Let $(\widetilde{X}, \tau, E)$ be a soft topological space over $\widetilde{X}$ and (F, E) be a soft set over $\widetilde{X}$. Then i). A soft interior of a soft set (F, E) is defined as the union of all soft open sets contained in (F, E). Thus $int(F, E)$ is the largest soft open set contained in (F, E). ii). A soft closure of a soft set (F, E) is the intersection of all soft closed super sets of (F, E). Thus $cl(F, E)$ be the smallest soft closed set over $\widetilde{X}$ which contains (F, E).*

Definition 2.6. *A soft subset (A, E) of a soft topological space (X, τ, E) is called as i) soft semi open [13] if $(A, E) \widetilde{\subseteq} cl(int(A, E))$. ii) soft semi closed [13] if $int(cl(A, E)) \widetilde{\subseteq} (A, E)$. iii) regular open [4] if $(A, E) = int(cl(A, E))$. iv) regular closed [4] if $(A, E) = cl(int(A, E))$. v)α open [14] if $(A, E) \widetilde{\subseteq} int(cl(int(A, E)))$. vi) α closed [14] if $cl(int(cl(A, E))) \widetilde{\subseteq} (A, E)$.*

Definition 2.7. *A subset A of a bi topological space (X, τ_1, τ_2) is called a i) $\tau_1\tau_2$ g closed set [9] if τ_2 $cl(A) \subseteq U$, whenever $A \subseteq U$, and U is τ_1 open. ii) $\tau_1\tau_2$ g^* closed set [5] if τ_2 $cl(A) \subseteq U$, whenever $A \subseteq U$, and U is $\tau_1 g$ open. iii) $\tau_1\tau_2$ gs closed set [9] if τ_2 $scl(A) \subseteq U$ whenever $A \subseteq U$, and U is τ_1 open. iv) $\tau_1\tau_2$ g^*s closed set [16] if τ_2 $scl(A) \subseteq U$ whenever $A \subseteq U$, and U is τ_1 gs open. v) $\tau_1\tau_2$ sg closed set [9] if τ_2 $scl(A) \subseteq U$ whenever $A \subseteq U$, and U is τ_1 semi open.*

Definition 2.8 ([6]). *The intersection of all soft semi closed sets containing a soft subset (A, E) of $(\widetilde{X}, \tau, E)$ is called as soft semi closure of (A, E) and is denoted by $scl(A, E)$. The soft semi interior of (A, E) is the largest soft semi open set contained in (A, E) and is denoted by $sint(A, E)$.*

Definition 2.9 ([8]). *A subset (F, E) of a soft topological space $(\widetilde{X}, \tau, E)$ is called soft generalized semi closed(soft gs closed) if $scl(F, E) \widetilde{\subseteq} (U, E)$, whenever $(F, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft open in $\widetilde{X}$.*

Definition 2.10 ([16]). *Let $(\widetilde{X}, \tau, E)$ be a soft topological space and (F, E) be a subset of $\widetilde{X}$. The set (F, E) is said to be soft g^*s closed if $scl(A, E) \widetilde{\subseteq} (U, E)$ whenever $(F, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft gs open set.*

Definition 2.11 ([12]). *Let τ_1 and τ_2 be two soft topologies on $\widetilde{X}$. Then the quadruple $(\widetilde{X}, \tau_1, \tau_2, E)$ is said to be a bi soft topological space over $\widetilde{X}$. The members of τ_1 are called τ_1 soft open. And the complement of τ_1 soft open set is called τ_1 soft closed set. Similarly, the members of τ_2 are called τ_2 soft open sets and the complement of τ_2 soft open sets are called τ_2 closed sets.*

3. Soft $\tau_1\tau_2$ g^*s Closed Sets

In this section, we define and investigate some basic properties of soft $\tau_1\tau_2$ g^*s closed sets in bi soft topological spaces.

Definition 3.1. *A soft subset (A, E) of a bi soft topological space $(\widetilde{X}, \tau_1, \tau_2, E)$ is called a soft $\tau_1\tau_2$ g^*s closed set if τ_2 $scl(A, E)\widetilde{\subseteq}(U, E)$, whenever $(A, E)\widetilde{\subseteq}(U, E)$ and (U, E) is a soft τ_1 gs open set. The complement of soft $\tau_1\tau_2$ g^*s closed set is called as soft $\tau_1\tau_2$ g^*s open set.*

Definition 3.2. *Let (S, E) be a soft set in bi soft topological space. Then $\tau_1\tau_2$ g^*s closure and $\tau_1\tau_2$ g^*s interior of (S, E) are defined as follows: i).$\tau_1\tau_2$ g^*s $cl(S, E) = \widetilde{\cap}\{(B, E) : (B, E)$ is soft $\tau_1\tau_2$ g^*s closed set and $(S, E)\widetilde{\subseteq}(B, E)\}$. ii). $\tau_1\tau_2$ g^*s $int(S, E) = \widetilde{\cup}\{(C, E) : (C, E)$ is soft $\tau_1\tau_2$ g^*s open set and $(C, E)\widetilde{\subseteq}(S, E)\}$*

Theorem 3.3. *Union of two soft $\tau_1\tau_2$ g^*s closed sets in a bi soft topological space is a soft $\tau_1\tau_2$ g^*s closed set.*

Proof. Assume (A, E) and (B, E) are two soft $\tau_1\tau_2$ g^*s closed sets and (U, E) be a soft τ_1 gs open set in $(\widetilde{X}, \tau_1, \tau_2, E)$ such that $(A, E)\widetilde{\cup}(B, E)\widetilde{\subseteq}(U, E)$. This implies $(A, E)\widetilde{\subseteq}(U, E)$ as well $(B, E)\widetilde{\subseteq}(U, E)$. Then τ_2 $scl(A, E)\widetilde{\subseteq}(U, E)$ and τ_2 $scl(B, E)\widetilde{\subseteq}(U, E) \Rightarrow \tau_2$ $scl(A, E)$ $\widetilde{\cup}$ τ_2 $scl(B, E)\widetilde{\subseteq}(U, E) \Rightarrow \tau_2$ $scl((A, E)\widetilde{\cup}(B, E))\widetilde{\subseteq}(U, E)$ Thus $(A, E)\widetilde{\cup}(B, E)$ is a soft $\tau_1\tau_2$ g^*s closed set. □

Remark 3.4. *Union of two soft $\tau_1\tau_2$ g^*s open sets in a bi soft topological space is not a soft $\tau_1\tau_2$ g^*s open set.*

Example 3.5. *Let $X = \{k_1, k_2, k_3\}, E = \{e_1, e_2\}$ with $\tau_1 = \{\phi, \widetilde{X}, \{(e_1, \{k_1, k_2\}), (e_2, \phi)\}, \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}\}$ and $\tau_2 = \{\phi, \widetilde{X}, \{(e_1, \{k_2\}), (e_2, \{k_1\})\}\}$ are two soft topologies of the bi soft topological space $(\widetilde{X}, \tau_1, \tau_2, E)$. Consider the soft sets $(A, E) = \{(e_1, \{k_2, k_3\}), (e_2, \phi)\}$ and $(B, E) = \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}$. These sets are soft $\tau_1\tau_2$ g^*s open sets. But the union, $(A, E)\widetilde{\cup}(B, E) = \{(e_1, \widetilde{X}), (e_2, \{k_2, k_3\})\}$ is not a soft $\tau_1\tau_2$ g^*s open set.*

Theorem 3.6. *Intersection of two soft $\tau_1\tau_2$ g^*s open sets in a bi soft topological space is soft $\tau_1\tau_2$ g^*s open.*

Proof. Assume that (C, E) and (D, E) are two soft $\tau_1\tau_2$ g^*s open sets in $(\widetilde{X}, \tau_1, \tau_2, E)$. Then $(C, E)^c$ and $(D, E)^c$ are soft $\tau_1\tau_2$ g^*s closed sets. This gives $(C, E)^c\widetilde{\cup}(D, E)^c$ is a soft $\tau_1\tau_2$ g^*s closed set which implies $((C, E)\widetilde{\cap}(D, E))^c$ is soft $\tau_1\tau_2$ g^*s closed. Hence $((C, E)\widetilde{\cap}(D, E))$ is soft $\tau_1\tau_2$ g^*s open set. □

Remark 3.7. *Intersection of two soft $\tau_1\tau_2$ g^*s closed sets in a bi soft topological space need not be soft $\tau_1\tau_2$ g^*s closed.*

Example 3.8. *Let $X = \{k_1, k_2, k_3\}, E = \{e_1, e_2\}$ with $\tau_1 = \{\phi, \widetilde{X}, \{(e_1, \{k_1, k_2\}), (e_2, \phi)\}, \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}\}$ and $\tau_2 = \{\phi, \widetilde{X}, \{(e_1, \{k_2\}), (e_2, \{k_1\})\}\}$ are two soft topologies of the bi soft topological space $(\widetilde{X}, \tau_1, \tau_2, E)$. Consider the soft sets $(C, E) = \{(e_1, \{k_1, k_3\}), (e_2, \widetilde{X})\}$ and $(D, E) = \{(e_1, \{k_2, k_3\}), (e_2, \widetilde{X})\}$. These sets are soft $\tau_1\tau_2$ g^*s closed sets. But the intersection, $(C, E)\widetilde{\cup}(D, E) = \{(e_1, \{k_3\}), (e_2, \widetilde{X})\}$ is not a soft $\tau_1\tau_2$ g^*s open set.*

Theorem 3.9. *i). Every soft τ_2 closed set in a bi soft topological space is soft $\tau_1\tau_2$ g^*s closed set. ii). Every soft τ_2 semi closed set in a bi soft topological space is soft $\tau_1\tau_2$ g^*s closed.*

Proof. $i)$. Assume that (F,E) be a soft τ_2 closed set and (U,E) be a soft τ_1 gs open set such that $(F,E)\widetilde{\subseteq}(U,E)$. Since (F,E) be a soft τ_2 closed set, we have $\tau_2\ cl(F,E)=(F,E)$. Also every soft τ_2 closed set is soft τ_2 semi closed. Therefore $\tau_2\ scl(F,E)\ \widetilde{\subseteq}\ \tau_2\ cl(F,E)=(F,E)\ \widetilde{\subseteq}\ (U,E)\Rightarrow\tau_2\ scl(F,E)\widetilde{\subseteq}(U,E)$, where (U,E) is τ_1 gs open. Hence (F,E) is soft $\tau_1\tau_2$ g^*s closed. $ii)$. Assume that (H,E) be a soft τ_2 semi closed in $(\widetilde{X},\tau_1,\tau_2,E)$ and (U,E) be a soft τ_1 gs open set such that $(H,E)\widetilde{\subseteq}(U,E)$. Since (H,E) be a soft τ_2 semi closed set, $\tau_2\ scl(H,E)\widetilde{\subseteq}(H,E)$. Now the soft set (H,E) is a soft $\tau_1\tau_2$ g^*s closed set. □

Remark 3.10. *Converse of the above theorem need not be true. Which is shown in the following example.*

Example 3.11. *$i)$. Let $\widetilde{X}=\{k_1,k_2,k_3\}$ be the universal set and $E=\{e_1,e_2\}$ be the parameter set with $\tau_1=\{\phi,\widetilde{X},\{(e_1,\{k_1,k_2\}),(e_2,\phi)\},\{(e_1,\{k_1,k_2\}),(e_2,\{k_2,k_3\})\}\}$ and $\tau_2=\{\phi,\widetilde{X},\{(e_1,\{k_2\}),(e_2,\{k_1\})\}\}$ are the two soft topologies of the bi soft topological space $(\widetilde{X},\tau_1,\tau_2,E)$. Consider the soft set $(F,E)=\{(e_1,\{k_3\}),(e_2,\{k_1,k_3\})\}$, this is a soft $\tau_1\tau_2$ g^*s closed set in $(\widetilde{X},\tau_1,\tau_2,E)$. But (F,E) is not a soft τ_2 closed set. $ii)$. Let $\widetilde{X}=\{k_1,k_2,k_3\}$ be the universal set and $E=\{e_1,e_2\}$ be the parameter set. Let $\tau_1=\{\phi,\widetilde{X},\{(e_1,\{k_1,k_2\}),(e_2,\phi)\},\{(e_1,\{k_1,k_2\}),(e_2,\{k_2,k_3\})\}\}$, $\tau_2=\{\phi,\widetilde{X},\{(e_1,\{k_2\}),(e_2,\{k_1\})\}\}$ are two soft topologies on $\widetilde{X}$. Then $(\widetilde{X},\tau_1,\tau_2,E)$ be a bi soft topological space. Consider the soft set $(H,E)=\{(e_1,\{k_1\}),(e_2,\widetilde{X})\}$ in $(\widetilde{X},\tau_1,\tau_2,E)$. Which is a soft $\tau_1\tau_2$ g^*s closed set, but not a soft τ_2 semi closed set.*

Definition 3.12. *A soft subset (B,E) of a bi soft topological space $(\widetilde{X},\tau_1,\tau_2,E)$ is called a soft $\tau_1\tau_2$ gs closed set if $\tau_2\ scl(B,E)\widetilde{\subseteq}(U,E)$ whenever $(B,E)\widetilde{\subseteq}(U,E)$, and (U,E) is soft τ_1 open.*

Definition 3.13. *A soft subset (C,E) of a bi soft topological space $(\widetilde{X},\tau_1,\tau_2,E)$ is called a soft $\tau_1\tau_2$ sg closed set if $\tau_2\ scl(C,E)\widetilde{\subseteq}(U,E)$ whenever $(C,E)\widetilde{\subseteq}(U,E)$, and (U,E) is soft τ_1 semi open.*

Theorem 3.14. *$i)$. Every soft $\tau_1\tau_2$ g^*s closed set in a bi soft topological space is soft $\tau_1\tau_2$ gs closed. $ii)$. Every soft $\tau_1\tau_2$ g^*s closed set in a bi soft topological is soft $\tau_1\tau_2$ sg closed.*

Proof. i). Assume that (G,E) be a soft $\tau_1\tau_2$ g^*s closed set in the bi soft topological space $(\widetilde{X},\tau_1,\tau_2,E)$ and (U,E) be a soft τ_1 open set such that $(G,E)\widetilde{\subseteq}(U,E)$. Since every soft τ_1 open set is soft τ_1 gs open, we have $\tau_2\ scl(G,E)\widetilde{\subseteq}(U,E)$. Therefore (G,E) is a soft $\tau_1\tau_2$ gs closed set. ii) the proof is similar. □

Remark 3.15. *Converse of the above theorem need not be true. This shown in the following example.*

Example 3.16. *Let $(\widetilde{X},\tau_1,\tau_2,E)$ be a bi soft topological space on $\widetilde{X}$ and the soft topologies defined as Example 3.11i). Consider the soft set $(G,E)=\{(e_1,\phi),(e_2,\{k_1\})\}$ is a soft $\tau_1\tau_2$ gs closed set. But it is not a soft $\tau_1\tau_2$ g^*s closed set. ii) Let $\widetilde{X}=\{k_1,k_2\}$, $E=\{e_1,e_2\}$ and the soft topologies defined on the bi soft topological space $(\widetilde{X},\tau_1,\tau_2,E)$ are $\tau_1=\{\phi,\widetilde{X},\{(e_1,\{k_1\}),(e_2,\phi)\},\{(e_1,\phi),(e_2,\{k_1\})\},\{(e_1,\{k_1\}),(e_2,\{k_1\})\},\{(e_1,\{k_2\}),(e_2,\{k_1\})\},\{(e_1,\widetilde{X}),(e_2,\{k_1\})\}\}$ and $\tau_2=\{\phi,\widetilde{X},\{(e_1,\phi),(e_2,\{k_1\})\},\{(e_1,\widetilde{X}),(e_2,\phi)\},\{(e_1,\widetilde{X}),(e_2,\{k_1\})\}\}$. Define the soft set $(G,E)=\{(e_1,\{k_1\}),(e_2,\{k_1\})\}$. Then the soft set (G,E) is a soft $\tau_1\tau_2$ g^*s closed set but it is not a soft $\tau_1\tau_2$ sg closed set.*

Theorem 3.17. *Every soft τ_2 α closed sets in a bi soft topological space is soft $\tau_1\tau_2$ g^*s closed.*

Proof. Assume that (D,E) be a soft τ_2 α closed set and (U,E) be a soft τ_1 gs open set such that $(D,E)\widetilde{\subseteq}(U,E)$. Since (D,E) is soft τ_2 α closed $\tau_2\ cl(\tau_2\ int(\tau_2\ cl(D,E)))\widetilde{\subseteq}(D,E)$ and $(D,E)\widetilde{\subseteq}(U,E)\Rightarrow\tau_2\ cl(\tau_2\ int(\tau_2\ cl(D,E)))\widetilde{\subseteq}(U,E)$ $\Rightarrow(U,E)^c\widetilde{\subseteq}\tau_2\ int(\tau_2\ cl(\tau_2\ int(D,E)^c))=\tau_2\ int(\tau_2\ cl(A,E))^c\Rightarrow\tau_2\ cl(\tau_2\ int(D,E))\widetilde{\subseteq}(U,E)\Rightarrow\tau_2\ cl(D,E)\widetilde{\subseteq}(U,E)$. But $\tau_2\ scl(D,E)\widetilde{\subseteq}\tau_2\ cl(D,E)\Rightarrow\tau_2\ scl(D,E)\widetilde{\subseteq}(U,E)$. Thus (D,E) is a soft $\tau_1\tau_2$ g^*s closed. □

Remark 3.18. *Converse of the above theorem need not be true. This shown in the following example.*

Example 3.19. *Let $(\widetilde{X}, \tau_1, \tau_2, E)$ be a bi soft topological space over $\widetilde{X}$ defined as in Example 3.11 i). Consider the soft set $(A, E) = \{(e_1, \widetilde{X}), (e_2, \phi)\}$ in $(\widetilde{X}, \tau_1, \tau_2, E)$. Then the soft set (A, E) is soft $\tau_1\tau_2$ g^*s closed, but it is not a soft τ_2 α closed .*

Definition 3.20. *Let (A, E) be a soft set in bi soft topological space $(\widetilde{X}, \tau_1, \tau_2, E)$. i) (A, E) is called a soft $\tau_1\tau_2$ g^*s^* closed set if τ_2 $scl(A, E)\widetilde{\subseteq}(U, E)$ whenever $(A, E)\widetilde{\subseteq}(U, E)$, and (U, E) is soft τ_1 g^* open. ii). (A, E) is called a soft $\tau_1\tau_2$ g^{**} closed set if τ_2 $cl(A, E)\widetilde{\subseteq}(U, E)$ whenever $(A, E)\widetilde{\subseteq}(U, E)$, and (U, E) is soft τ_1 g^* open.*

Theorem 3.21. *i). Every soft $\tau_1\tau_2$ g^*s closed set in bi soft topological space is soft $\tau_1\tau_2$ g^*s^* closed. ii). Every soft $\tau_1\tau_2$ g^*s closed set in bi soft topological space is soft $\tau_1\tau_2$ g^{**} closed set.*

Proof. i) Let (F, E) be a soft $\tau_1\tau_2$ g^*s closed set and (U, E) be a soft τ_1 gs open set such that $(F, E)\widetilde{\subseteq}(U, E)$. From the definition, τ_2 $scl(F, E)\widetilde{\subseteq}(U, E)$. Also since every soft τ_1 gs open set is soft τ_1 g^* open. Thus (F, E) is soft $\tau_1\tau_2$ g^*s^* closed set. ii) Assume that (G, E) be a soft $\tau_1\tau_2$ g^*s closed set and (U, E) be a τ_1 gs open set. Since every soft $\tau_1\tau_2$ g^*s closed set is soft $\tau_1\tau_2$ g^*s^* closed set and also every soft $\tau_1\tau_2$ g^*s^* closed set is soft $\tau_1\tau_2$ g^{**} closed set. Hence the soft set (G, E) is soft $\tau_1\tau_2$ g^{**} closed set. □

Remark 3.22. *The converse of the above theorem need not be true. This shows in the following example.*

Example 3.23. *Let $\widetilde{X} = \{k_1, k_2\}$ be the universal set and $\{e_1, e_2\}$ be parameter set with $\tau_1 = \{\phi, \widetilde{X}, \{(e_1, \{k_1\}), (e_2, \phi)\}\}$ and $\tau_2 = \{\phi, \widetilde{X}, \{(e_1, \{k_1\}), (e_2, \phi)\}, \{(e_1, \{k_1\}), (e_2, \{k_2\})\}\}$ are the two soft topologies on the bi soft topological space $(\widetilde{X}, \tau_1, \tau_2, E)$. i) Consider the soft set $(F, E) = \{(e_1, \{k_1\}), (e_2, \{k_2\})\}$ on $(\widetilde{X}, \tau_1, \tau_2, E)$. The soft set (F, E) is soft $\tau_1\tau_2$ g^*s^* closed set but it is not a soft $\tau_1\tau_2$ g^*s closed. ii) Consider the soft set $(G, E) = \{(e_1, \{k_1\}), (e_2, \{k_1\})\}$. Then (G, E) is soft $\tau_1\tau_2$ g^{**} closed set but it is not soft $\tau_1\tau_2$ g^*s closed set.*

Theorem 3.24. *Let (K, E) ba a soft $\tau_1\tau_2$ g^*s closed set in $(\widetilde{X}, \tau_1, \tau_2, E)$ if and only if τ_2 $scl(K, E)\backslash(K, E)$ does not contain any non-empty soft τ_1 gs closed set.*

Proof. Suppose that (V, E) is a soft τ_1 gs closed subset of τ_2 $scl(K, E)\backslash(A, E)$ i.e., $(V, E)\widetilde{\subseteq}\tau_2$ $scl(K, E)\backslash(K, E)$ and $(K, E)\widetilde{\subseteq}\widetilde{X}\backslash(V, E)$. Since $\widetilde{X}\backslash(V, E)$ is a soft τ_1 gs open set, (K, E) is soft $\tau_1\tau_2$ g^*s closed such that τ_2 $scl(K, E)\widetilde{\subseteq}\widetilde{X}\backslash(V, E)$. Therefore $(V, E)\widetilde{\subseteq}\tau_2$ $scl(K, E)\widetilde{\cap}(\widetilde{X}\backslash\tau_2$ $scl(K, E)) = \phi$. Hence τ_2 $scl(K, E)\backslash(K, E)$ does not contain any non-empty soft τ_1 gs closed set. Conversely, assume that τ_2 $scl(K, E)\backslash(K, E)$ does not contain any non-empty soft τ_1 gs closed set. Let $(K, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft τ_1 gs open. Suppose that τ_2 $scl(K, E)$ is not contained in (U, E), τ_2 $scl(K, E)\widetilde{\cap}(U, E)^c$ is a non-empty soft τ_1 gs closed set of τ_2 $scl(K, E)\backslash(K, E)$. Which arrives a contradiction. Therefore τ_2 $scl(K, E)\widetilde{\subseteq}(U, E)$ and hence (K, E) is a soft $\tau_1\tau_2$ g^*s closed set. □

Theorem 3.25. *If (K, E) be a soft $\tau_1\tau_2$ g^*s closed set $(\widetilde{X}, \tau_1, \tau_2, E)$ such that $(K, E)\widetilde{\subseteq}(L, E)\widetilde{\subseteq}\tau_2$ $scl(K, E)$ then (L, E) is soft $\tau_1\tau_2$ g^*s closed in $(\widetilde{X}, \tau_1, \tau_2, E)$.*

Proof. Assume that (K, E) is soft $\tau_1\tau_2$ g^*s closed in $(\widetilde{X}, \tau_1, \tau_2, E)$ such that $(K, E)\widetilde{\subseteq}(L, E)\widetilde{\subseteq}\tau_2$ $scl(K, E)$. Let (U, E) be a soft τ_1 gs open set in $(\widetilde{X}, \tau_1, \tau_2, E)$ such that $(L, E)\widetilde{\subseteq}(U, E)$. From the hypothesis, we have τ_2 $scl(K, E)\widetilde{\subseteq}(U, E)$. Now, τ_2 $scl(L, E)\widetilde{\subseteq}\tau_2$ $scl(\tau_2$ $scl(K, E)) = \tau_2$ $scl(K, E)\widetilde{\subseteq}(U, E)$ i.e., τ_2 $scl(L, E)\widetilde{\subseteq}(U, E)$, where (U, E) is soft τ_1 gs open. Thus (L, E) is soft $\tau_1\tau_2$ g^*s closed in $(\widetilde{X}, \tau_1, \tau_2, E)$. □

Theorem 3.26. *If $(K, E)\widetilde{\subseteq}\widetilde{Y}\widetilde{\subseteq}\widetilde{X}$ and (K, E) is soft $\tau_1\tau_2$ g^*s closed in $(\widetilde{X}, \tau_1, \tau_2, E)$, then (K, E) is soft $\tau_1\tau_2$ g^*s closed relative to $\widetilde{Y}$.*

Proof. Assume that $(K,E)\tilde{\subseteq}\tilde{Y}\tilde{\subseteq}\tilde{X}$ and (K,E) is soft $\tau_1\tau_2$ g^*s closed in $(\tilde{X},\tau_1,\tau_2,E)$. Let $(K,E)\tilde{\subseteq}\tilde{Y}\tilde{\subseteq}(U,E)$, where (U,E) is soft $\tau_1\tau_2$ gs open in $(\tilde{X},\tau_1,\tau_2,E)$. Given (K,E) is soft $\tau_1\tau_2$ g^*s closed, $(K,E)\tilde{\subseteq}(U,E)$ $\Rightarrow \tau_2$ $scl(K,E)\tilde{\subseteq}(U,E)$ $\Rightarrow \tilde{Y}\tilde{\cap}\tau_2$ $scl(K,E)\tilde{\subseteq}\tilde{Y}\tilde{\cap}(U,E)$. Hence (K,E) is a soft $\tau_1\tau_2$ g^*s closed set relative to $\tilde{Y}$. □

Theorem 3.27. *Let (A,E) be a subset of a bi soft topological space. Then the following are equivalent: i). (A,E) is soft τ_2 regular open. ii). (A,E) is soft τ_2 open and soft $\tau_1\tau_2$ g^*s closed.*

Proof. i)$\Rightarrow$ ii) Let (H,E) be a soft τ_1 gs open set in $(\tilde{X},\tau_1,\tau_2,E)$ containing (A,E). Since every soft τ_2 regular open set is τ_2 open, $(A,E)\tilde{\cup}\tau_2$ $int(\tau_2$ $cl(A,E))\tilde{\subseteq}(A,E)\tilde{\subseteq}(H,E)$. Hence τ_2 $scl(A,E)\tilde{\subseteq}(H,E)$, where (H,E) is soft τ_1 gs open. Therefore (A,E) is soft $\tau_1\tau_2$ g^*s closed. ii)$\Rightarrow$ i) Let (A,E) is soft τ_2 open and soft $\tau_1\tau_2$ g^*s closed set. i.e., τ_2 $scl(A,E)\tilde{\subseteq}(A,E)$ and so $(A,E)\tilde{\cup}\tau_2$ $int(\tau_2$ $cl(A,E))$ $\tilde{\subseteq}$ (A,E). But (A,E) is a soft τ_2 open in $(\tilde{X},\tau_1,\tau_2,E)$. Therefore τ_2 $int(\tau_2$ $cl(A,E))\tilde{\subseteq}(A,E)\cdots(1)$. Since every soft τ_2 open set is soft τ_2 pre open, we have, $(A,E)\tilde{\subseteq}\tau_2$ $int(\tau_2$ $cl(A,E))\cdots(2)$. Thus (1) and (2) implies that $(A,E)=\tau_2$ $int(\tau_2$ $cl(A,E))$ Hence (A,E) is soft τ_2 regular open. □

Definition 3.28. *A subset (A,E) of a bi soft topological space $(\tilde{X},\tau_1,\tau_2,E)$ is said to be a soft $\tau_1\tau_2$ Q set if τ_2 $int(\tau_1$ $cl(A,E))=\tau_2$ $cl(\tau_1$ $int(A,E))$.*

Theorem 3.29. *If (D,E) is a subset of a bi soft topological space, the following are equivalent: i). (D,E) is a soft τ_2 clopen. ii).(D,E) is soft τ_2 open, a soft τ_2 Q set and $\tau_1\tau_2$ g^*s closed.*

Proof. i)$\Rightarrow$ ii) Since (D,E) is a soft τ_2 clopen, then (D,E) is both soft τ_2 open and soft τ_2 Q set. Assume (F,E) be a soft τ_1 gs open set and $(D,E)\tilde{\subseteq}(F,E)$. We have that $(D,E)\tilde{\cup}\tau_2$ $int(\tau_2$ $cl(D,E))\tilde{\subseteq}(F,E)$ and so τ_2 $scl(D,E)\tilde{\subseteq}(F,E)$, where (F,E) is soft τ_1 *gs open*. Hence (D,E) is soft $\tau_1\tau_2$ g^*s closed in $(\tilde{X},\tau_1,\tau_2,E)$. ii)$\Rightarrow$ i) By the Theorem 3.27, the soft set (D,E) is a soft τ_2 regular open. Since every soft τ_2 regular open set is soft τ_2 open, we have (D,E) is soft τ_2 open. Also (D,E) is a soft τ_2 Q set, then (D,E) is soft τ_2 closed. Therefore (D,E) is soft τ_2 clopen set. □

4. Soft $\tau_1\tau_2$ g^*s Continuous and Soft $\tau_1\tau_2$ g^*s Irresolute Mappings

In this section we introduce soft $\tau_1\tau_2$ g^*s continuous and soft $\tau_1\tau_2$ g^*s irresolute mappings in bi soft topological spaces and we also investigate some of their properties.

Definition 4.1. *A soft mapping $f:(\tilde{X},\tau_1,\tau_2,E)\to(\tilde{Y},\tau_1',\tau_2',E)$ is said to be a soft $\tau_1\tau_2$ g^*s continuous if the inverse image of every soft τ_2' closed sets in $\tilde{Y}$ is soft $\tau_1\tau_2$ g^*s closed set in $\tilde{X}$.*

Theorem 4.2. *Every soft τ_2 continuous mapping in bi soft topological space is a soft $\tau_1\tau_2$ g^*s continuous.*

Proof. Assume that $f:(\tilde{X},\tau_1,\tau_2,E)\to(\tilde{Y},\tau_1',\tau_2',E)$ is a soft τ_2 continuous mapping. Let (A,E) be a soft τ_2' closed set in $\tilde{Y}$. Then $f^{-1}(A,E)$ is soft τ_2 closed set in $\tilde{X}$ as f is soft τ_2 continuous. Since every soft τ_2 closed set in $\tilde{X}$ is soft $\tau_1\tau_2$ g^*s closed set, the set $f^{-1}(A,E)$ is soft $\tau_1\tau_2$ g^*s closed set in $\tilde{X}$. Hence f is soft $\tau_1\tau_2$ g^*s continuous. □

Remark 4.3. *Converse of the above theorem need not be true. This is seen in the following example.*

Example 4.4. *Let $\tilde{X}=\{k_1,k_2,k_3\}=\tilde{Y}$ and the parameter $E=\{e_1,e_2\}$ with $\tau_1=\{\phi,\ \tilde{X},\ \{(e_1,\{k_1,k_2\}),(e_2,\phi)\},\ \{(e_1,\{k_1,k_2\}),\ (e_2,\{k_2,k_3\})\}\}$ and $\tau_2=\{\phi,\tilde{X},\{(e_1,\{k_2\}),(e_2,\{k_1\})\}\}$ are two soft topologies on $\tilde{X}$. Also $\tau_1'=\{\phi,\tilde{Y},\{(e_1,\{k_1,k_2\})\}$ and $\tau_2'=\{\phi,\tilde{Y},\{(e_1,\{k_1,k_3\}),(e_2,\phi)\},\{(e_1,\phi),(e_2,\{k_2,k_3\})\},\{(e_1,\{k_1,k_3\}),(e_2,\{k_2,k_3\})\}\}$ are soft topologies on $\tilde{Y}$. Define the soft mapping $f:(\tilde{X},\tau_1,\tau_2,E)\to(\tilde{Y},\tau_1',\tau_2',E)$ on bi soft topological spaces as $f(k_1)=k_1$, $f(k_2)=k_2$ and $f(k_3)=k_3$. Consider the soft set $(A,E)=\{(e_1,\{k_1,k_3\}),(e_2,\{k_2,k_3\})\}$. This set is a soft*

$\tau_1\tau_2$ g^*s *closed set but its inverse not a soft* τ_2 *closed set. Hence the soft mapping* f *is soft* $\tau_1\tau_2$ g^*s *continuous but it is not a soft* τ_2 *continuous mapping.*

Definition 4.5. *A soft mapping* $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ *is said to be a soft* $\tau_1\tau_2$ gs *continuous if the inverse image of every soft* τ_2' *closed sets in* $\widetilde{Y}$ *is soft* $\tau_1\tau_2$ gs *closed set in* $\widetilde{X}$.

Theorem 4.6. *Every soft* $\tau_1\tau_2$ g^*s *continuous mapping in bi soft topological space is a soft* $\tau_1\tau_2$ gs *continuous.*

Proof. Assume that $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is a soft $\tau_1\tau_2$ g^*s continuous mapping. Let (A, E) be a soft $\tau_1'\tau_2'$ closed set in $\widetilde{Y}$. Then $f^{-1}(A, E)$ is soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$. Since every soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$ is soft $\tau_1\tau_2$ gs closed, the soft set $f^{-1}(A, E)$ is soft $\tau_1\tau_2$ gs closed set in $\widetilde{X}$. Hence f is soft $\tau_1\tau_2$ gs continuous. □

Remark 4.7. *Converse of the above theorem need not be true. This is seen in the following example.*

Example 4.8. *Let* $\widetilde{X} = \{k_1, k_2, k_3\} = \widetilde{Y}$ *and* $E = \{e_1, e_2\}$ *with* $\tau_1 = \{\phi, \widetilde{X}, \{(e_1, \{k_1, k_2\}), (e_2, \phi)\}, \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}\}$ *and* $\tau_2 = \{\phi, \widetilde{X}, \{(e_1, \{k_2\}), (e_2, \{k_1\})\}\}$ *are the two soft topologies on* $\widetilde{X}$. *Also* $\tau_1' = \{\phi, \widetilde{Y}, \{(e_1, \{k_1\}), (e_2, \{k_2, k_3\})\}\}$ *and* $\tau_2' = \{\phi, \widetilde{Y}, \{(e_1, \widetilde{Y}), (e_2, k_2, k_3)\}\}$ *are two soft topologies on* $\widetilde{Y}$. *Define the soft mapping* $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ *on bi soft topological spaces as* $f(k_1) = k_1,\ f(k_2) = k_2$ *and* $f(k_3) = k_3$. *Consider the soft set* $(A, E) = \{(e_1, \widetilde{Y}), (e_2, \{k_1\})\}$. *Then the mapping* f *is a soft* $\tau_1\tau_2$ gs *continuous but it is not a soft* $\tau_1\tau_2$ g^*s *continuous mapping.*

Theorem 4.9. *Every soft* τ_2 *semi continuous mapping in bi soft topological space is soft* $\tau_1\tau_2$ g^*s *continuous mapping.*

Proof. Let $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is soft τ_2 semi continuous mapping. Let (B, E) be a soft τ_2' closed set in $\widetilde{Y}$. Then $f^{-1}(B, E)$ is soft τ_2 semi closed set $\widetilde{X}$. Since every soft τ_2 semi closed set is soft $\tau_1\tau_2$ g^*s closed, $f^{-1}(B, E)$ is soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$. Therefore, f is soft $\tau_1\tau_2$ g^*s continuous mapping. □

Remark 4.10. *Converse of the above theorem need not be true. This is seen in the following example.*

Example 4.11. *Let* $\widetilde{X} = \{k_1, k_2, k_3\} = \widetilde{Y}$ *and* $E = \{e_1, e_2\}$ *with* $\tau_1 = \{\phi, \widetilde{X}, \{(e_1, \{k_1, k_2\}), (e_2, \phi)\}, \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}\}$ *and* $\tau_2 = \{\phi, \widetilde{X}, \{(e_1, \{k_2\}), (e_2, \{k_1\})\}\}$ *are soft topologies on* $\widetilde{X}$. *Also* $\tau_1' = \{\phi, \widetilde{Y}, \{(e_1, \{k_1, k_3\}), (e_2, \phi)\}, \{(e_1, \phi), (e_2, \{k_2, k_3\})\}, \{(e_1, \{k_1, k_3\}), (e_2, \{k_2, k_3\})\}\}$ *and* $\tau_2' = \{\phi, \widetilde{Y}, \{(e_1, \{k_2, k_3\})\}\}$ *are soft topologies on* $\widetilde{Y}$. *Define the soft mapping* $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ *on bi soft topological spaces as* $f(k_1) = k_1,\ f(k_2) = k_2$ *and* $f(k_3) = k_3$. *Consider the soft set* $(C, E) = \{(e_1, \widetilde{Y}), (e_2, \{k_2\})\}$. *This set is a soft* $\tau_1\tau_2$ g^*s *closed set but its inverse is not a soft* τ_2 *semi closed set. Hence the mapping is soft* $\tau_1\tau_2$ g^*s *continuous but it is not a soft* τ_2 *semi continuous mapping.*

Definition 4.12. *A soft mapping* $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ *is said to be a soft* $\tau_1\tau_2$ g^{**} *continuous if the inverse image of every soft* τ_2' *closed sets in* $\widetilde{Y}$ *is soft* $\tau_1\tau_2$ g^{**} *closed set in* $\widetilde{X}$.

Theorem 4.13. *Every soft* $\tau_1\tau_2$ g^{**} *continuous mapping in bi soft topological space is soft* $\tau_1\tau_2$ g^*s *continuous mapping.*

Proof. Let $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is soft $\tau_1\tau_2$ g^{**} continuous mapping. Let (B, E) be a soft τ_2' closed set in $\widetilde{Y}$. Then $f^{-1}(B, E)$ is soft $\tau_1\tau_2$ g^{**} closed set in $(\widetilde{X}, \tau_1, \tau_2, E)$. Since every soft $\tau_1\tau_2$ g^{**} closed set is soft $\tau_1\tau_2$ g^*s closed, $f^{-1}(B, E)$ is soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$. Therefore f is soft $\tau_1\tau_2$ g^*s continuous mapping. □

Definition 4.14. *A mapping* $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ *is said to be a soft* $\tau_1\tau_2$ g^*s^* *continuous if the inverse image of every soft* τ_2' *closed sets in* $\widetilde{Y}$ *is soft* $\tau_1\tau_2$ g^*s^* *closed set in* $\widetilde{X}$.

Theorem 4.15. *Every soft* $\tau_1\tau_2$ g^*s^* *continuous mapping in bi soft topological space is soft* $\tau_1\tau_2$ g^*s *continuous mapping.*

Proof. Let $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is soft $\tau_1\tau_2$ g^*s^* continuous mapping. Let (D, E) be a soft τ_2' closed set in $\widetilde{Y}$. Then $f^{-1}(D, E)$ is soft $\tau_1\tau_2$ g^*s^* closed set in $\widetilde{X}$. Since every soft $\tau_1\tau_2$ g^*s^* closed set is soft $\tau_1\tau_2$ g^*s closed, $f^{-1}(D, E)$ is soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$. Hence f is soft $\tau_1\tau_2$ g^*s continuous mapping. □

Remark 4.16. *Converse of the above Theorems 4.13 and 4.15 need not be true. This is seen in the following example.*

Example 4.17. *Let* $\widetilde{X} = \{k_1, k_2\} = \widetilde{Y}$ *and* $E = \{e_1, e_2\}$ *with* $\tau_1 = \{\phi, \widetilde{X}, \{(e_1, \{k_1\}), (e_2, \phi)\}, \{(e_1, \phi), (e_2, \{k_2\})\}, \{(e_1, \{k_1\}), (e_2, \{k_2\})\}\}$ *and* $\tau_2 = \{\phi, \widetilde{X}, \{(e_1, \{k_1\}), (e_2, \{k_1\})\}\}$ *are soft topologies on* $\widetilde{X}$. *Also* $\tau_1' = \{\phi, \widetilde{Y}, \{(e_1, \{k_1\}), (e_2, \phi)\}\}$ *and* $\tau_2' = \{\phi, \widetilde{Y}, \{(e_1, \{k_1\}), (e_2, \phi)\}, \{(e_1, \{k_1\}), (e_2, \{k_2\})\}\}$ *are two soft topologies on* $\widetilde{Y}$. *Define* $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ *be a soft mapping on bi soft topological space as* $f(k_1) = k_2$ *and* $f(k_2) = k_1$. *Consider the soft set* $(B, E) = \{(e_1, \{k_1\}), (e_2, k_1)\}$. *Then this mapping* f *is soft* $\tau_1\tau_2$ g^*s^* *continuous and soft* $\tau_1\tau_2$ g^{**} *continuous but it is not a soft* $\tau_1\tau_2$ g^*s *continuous mapping.*

Definition 4.18. *A soft mapping* $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ *is said to be a soft* $\tau_1\tau_2$ *sg continuous if the inverse image of every soft* τ_2' *closed sets in* $\widetilde{Y}$ *is soft* $\tau_1\tau_2$ *sg closed set in* $\widetilde{X}$.

Theorem 4.19. *Every soft* $\tau_1\tau_2$ g^*s *continuous mapping is a soft* $\tau_1\tau_2$ *sg continuous mapping.*

Proof. Suppose that $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is a soft $\tau_1\tau_2$ g^*s continuous mapping. Assume that (A, E) be a soft $\tau_1'\tau_2'$ closed set in $\widetilde{Y}$. Since the map f is soft $\tau_1\tau_2$ g^*s continuous, the inverse image of every soft $\tau_1'\tau_2'$ closed set in $\widetilde{Y}$ is soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$, i.e., $f^{-1}(A, E)$ is soft $\tau_1\tau_2$ g^*s closed in $\widetilde{X}$. Since every soft $\tau_1\tau_2$ g^*s closed set is soft $\tau_1\tau_2$ *sg* closed, the soft set $f^{-1}(A, E)$ is soft $\tau_1\tau_2$ *sg* closed in $\widetilde{X}$. Thus the mapping f is soft $\tau_1\tau_2$ *sg* continuous. □

Definition 4.20. *A mapping* $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ *is said to be a soft* $\tau_1\tau_2$ *gs continuous if the inverse image of every soft* τ_2' *closed sets in* $\widetilde{Y}$ *is soft* $\tau_1\tau_2$ *gs closed set in* $\widetilde{X}$.

Theorem 4.21. *Every soft* $\tau_1\tau_2$ g^*s *continuous mapping is a soft* $\tau_1\tau_2$ *gs continuous mapping.*

Proof. Suppose that $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is a soft $\tau_1\tau_2$ g^*s continuous mapping. Assume that the soft set (A, E) is a soft $\tau_1'\tau_2'$ closed set in $\widetilde{Y}$. Since the map f is soft $\tau_1\tau_2$ g^*s continuous, the inverse image of every soft $\tau_1'\tau_2'$ closed set in $\widetilde{Y}$ is soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$, i.e., $f^{-1}(A, E)$ is soft $\tau_1\tau_2$ g^*s closed in $\widetilde{X}$. Since every soft $\tau_1\tau_2$ g^*s closed set is soft $\tau_1\tau_2$ *gs* closed, the soft set $f^{-1}(A, E)$ is soft $\tau_1\tau_2$ *sg* closed in $\widetilde{X}$. Thus the mapping f is soft $\tau_1\tau_2$ *gs* continuous. □

Remark 4.22. *Converse of the above theorems need not be true. This is seen in the following example.*

Example 4.23. *Let* $\widetilde{X} = \{k_1, k_2, k_3\} = \widetilde{Y}$ *and the parameter* $E = \{e_1, e_2\}$ *with* $\tau_1 = \{\phi, \widetilde{X}, \{(e_1, \{k_1\}), (e_2, \{k_2, k_3\})\}$ *and* $\tau_2 = \{\phi, \widetilde{X}, \{(e_1, \widetilde{X}), (e_2, \{k_2, k_3\})\}\}$ *are two soft topologies on* $\widetilde{X}$. *Also* $\tau_1' = \{\phi, \widetilde{Y}, \{(e_1, \{k_1, k_2\}), (e_2, \phi)\}, \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}\}$ *and* $\tau_2' = \{\phi, \widetilde{Y}, \{(e_1, \{k_2\}), (e_2, \{k_1\})\}\}$ *are soft topologies on* $\widetilde{Y}$. *Define the soft mapping* $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ *on bi soft topological spaces as* $f(k_1) = k_1$, $f(k_2) = k_2$ *and* $f(k_3) = k_3$. *Consider the soft set* $(A, E) = \{(e_1, \phi), (e_2, \{k_1\})\}$. *This set is a soft* $\tau_1\tau_2$ *gs closed set but its inverse is not a soft* $\tau_1\tau_2$ g^*s *closed set. Hence the soft mapping is soft* $\tau_1\tau_2$ *gs continuous but not a soft* $\tau_1\tau_2$ g^*s *continuous mapping. Also for the soft set* $(B, E) = \{(e_1, \{k_1, k_2\}), (e_2, \{k_1, k_3\})\}$, *we see that it is soft* $\tau_1\tau_2$ *sg closed set but its inverse is not a soft* $\tau_1\tau_2$ g^*s *closed set.*

Theorem 4.24. *Every soft* τ_2 α *continuous mapping in bi soft topological space is a soft* $\tau_1\tau_2$ g^*s *continuous.*

Proof. Assume that $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is a soft τ_2 α continuous mapping. Let (A, E) be a soft τ_2' closed set in $\widetilde{Y}$. Then $f^{-1}(A, E)$ is soft τ_2 α closed set in $\widetilde{X}$ as f is soft τ_2 α continuous. Since every soft τ_2 α closed set in $\widetilde{X}$ is soft $\tau_1\tau_2$ g^*s closed set, the set $f^{-1}(A, E)$ is soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$. Hence f is a soft $\tau_1\tau_2$ g^*s continuous mapping. □

Remark 4.25. *Converse of the above theorem need not be true. This is seen in the following example.*

Example 4.26. *Let $\widetilde{X} = \{k_1, k_2, k_3\} = \widetilde{Y}$ and the parameter $E = \{e_1, e_2\}$ with $\tau_1 = \{\phi,\ \widetilde{X},\ \{(e_1, \{k_1, k_2\}),\ (e_2, \phi)\}, \{(e_1, \{k_3\}),\ (e_2, \{k_1, k_2\})\}, \{(e_1, \widetilde{Y}),\ (e_2, \{k_1, k_2\})\}\}$ and also $\tau_2 = \{\phi, \widetilde{X}, \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}\}$ are two soft topologies on $\widetilde{X}$. Also $\tau_1' = \{\phi, \widetilde{Y}, \{(e_1, \{k_1, k_2\}), (e_2, \phi)\}, \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}\}$ and $\tau_2' = \{\phi, \widetilde{Y}, \{(e_1, \{k_2\}), (e_2, \{k_1\})\}\}$ are soft topologies on $\widetilde{Y}$. Define the soft mapping $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ on bi soft topological spaces as $f(k_1) = k_1,\ f(k_2) = k_2$ and $f(k_3) = k_3$. Consider the soft set $(A, E) = \{(e_1, \{k_3\}), (e_2, \{k_1\})\}$. Then this set is a soft $\tau_1\tau_2\ g^*s$ closed set but its inverse is not a soft $\tau_2\ \alpha$ closed set. Hence the mapping is soft $\tau_1\tau_2\ g^*s$ continuous but it is not a soft $\tau_2\ \alpha$ continuous mapping.*

Theorem 4.27. *Let $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is a soft $\tau_1\tau_2\ g^*s$ continuous mapping. Then i). $f(\tau_1\tau_1\ g^*s\ cl(F, E)) \widetilde{\subseteq} \tau_1'\tau_2'\ cl(f(F, E))$. ii). $\tau_1\tau_2\ g^*s\ cl(f^{-1}(G, E)) \widetilde{\subseteq} f^{-1}(\tau_1'\tau_2'\ cl(G, E))$.*

Proof. Assume that the soft mapping $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ be a soft $\tau_1\tau_2\ g^*s$ continuous function, where $\widetilde{X}$ and $\widetilde{Y}$ are bi soft topological spaces.

i). Since $\tau_1'\tau_2'\ cl(f(F, E))$ is soft τ_2' closed set in $\widetilde{Y} \Rightarrow f^{-1}(\tau_1'\tau_2'\ cl(f(F, E)))$ is soft $\tau_1\tau_1\ g^*s$ closed set in $\widetilde{X}$. Then $(F, E)\ \widetilde{\subseteq}\ f^{-1}(\tau_1'\tau_2'\ cl(f(F, E))$ which implies $\tau_1\tau_2\ g^*s\ cl(F, E)\ \widetilde{\subseteq}\ f^{-1}(\tau_1'\tau_2'\ cl(f(F, E))) \Rightarrow f(\tau_1\tau_2\ g^*s\ cl(F, E)) \widetilde{\subseteq} \tau_1'\tau_2'\ cl(f(F, E))$. ii). From first part we have that, $f(\tau_1\tau_1\ g^*s\ cl(F, E)) \widetilde{\subseteq} \tau_1'\tau_2'\ cl(f(F, E))$. Now we replace (F, E) by $f^{-1}(G, E)$, we get, $f(\tau_1\tau_1\ g^*s\ cl(f^{-1}(G, E))) \widetilde{\subseteq} \tau_1'\tau_2'\ cl(f(f^{-1}(G, E))) \widetilde{\subseteq} \tau_1'\tau_2'\ cl(G, E)$. This implies that, $\tau_1\tau_1\ g^*s\ cl(f^{-1}(G, E)) \widetilde{\subseteq} f^{-1}(\tau_1'\tau_2'\ cl(G, E))$. Thus the results proved. □

Theorem 4.28. *Let $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is a soft mapping. Then the following are equivalent. i). f is soft $\tau_1\tau_2\ g^*s$ continuous. ii). The inverse image of each soft τ_2' open set in $\widetilde{Y}$ is soft $\tau_1\tau_2\ g^*s$ open in $\widetilde{X}$.*

Proof. Assume that $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ be a soft $\tau_1\tau_2\ g^*s$ continuous. Let (G, E) be a soft τ_2' open in $\widetilde{Y}$. Then $(G, E)^c$ is soft $\tau_1'\tau_2'$ closed in $\widetilde{Y}$. Since f is a soft $\tau_1\tau_2\ g^*s$ continuous, $f^{-1}(G, E)^c$ is a soft $\tau_1\tau_2\ g^*s$ closed in $\widetilde{X}$. But $f^{-1}(G, E)^c = \widetilde{X} - f^{-1}(G, E)$. Thus $f^{-1}(G, E)$ is soft $\tau_1\tau_2\ g^*s$ open in $\widetilde{X}$. Conversely, assume that the inverse image of each soft $\tau_1'\tau_2'\ g^*s$ open set in $\widetilde{Y}$ is soft $\tau_1\tau_2\ g^*s$ open in $\widetilde{X}$. Let (F, E) be any soft $\tau_1'\tau_2'$ closed set in $\widetilde{Y}$. But $f^{-1}(F, E)$ is soft $\tau_1\tau_2\ g^*s$ open in $\widetilde{X}$, $f^{-1}(F, E)^c = \widetilde{X} - f^{-1}(F, E)$. Thus $\widetilde{X} - f^{-1}(F, E)$ is a soft $\tau_1\tau_2\ g^*s$ open in $\widetilde{X}$ and so $f^{-1}(F, E)$ is soft $\tau_1\tau_2\ g^*s$ closed set in $\widetilde{X}$. Thus f is a soft $\tau_1\tau_2\ g^*s$ continuous mapping. □

Theorem 4.29. *If $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is a soft $\tau_1\tau_2\ g^*s$ continuous mapping and $g : (\widetilde{Y}, \tau_1', \tau_2', E) \to (\widetilde{Z}, \tau_1'', \tau_2'', E)$ is a soft $\tau_1'\tau_2'$ continuous mapping then $g \circ f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Z}, \tau_1'', \tau_2'', E)$ is a soft $\tau_1\tau_2\ g^*s$ continuous mapping.*

Proof. Let (F, E) be a soft $\tau_1''\tau_2''$ closed set in $(\widetilde{Z}, \tau_1'', \tau_2'', E)$. Since g is a soft $\tau_1'\tau_2'$ continuous, $g^{-1}(F, E)$ is soft $\tau_1'\tau_2'$ closed in $(\widetilde{Y}, \tau_1', \tau_2', E)$ and since f is a soft $\tau_1\tau_2\ g^*s$ continuous mapping, $f^{-1}(g^{-1}(F, E))$ is soft $\tau_1\tau_2\ g^*s$ closed set in $(\widetilde{X}, \tau_1, \tau_2, E)$. That is $(g \circ f)^{-1}(F, E)$ is a soft $\tau_1\tau_2\ g^*s$ closed set in $(\widetilde{X}, \tau_1, \tau_2, E)$. Thus $g \circ f$ is a soft $\tau_1\tau_2\ g^*s$ continuous mapping. □

Definition 4.30. *A soft mapping $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is said to be soft $\tau_1\tau_2\ g^*s$ irresolute if the inverse image of every soft $\tau_1'\tau_2'\ g^*s$ closed set in $(\widetilde{Y}, \tau_1', \tau_2', E)$ is soft $\tau_1\tau_2\ g^*s$ closed set in $(\widetilde{X}, \tau_1, \tau_2, E)$.*

Theorem 4.31. *A mapping $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is the soft $\tau_1\tau_2\ g^*s$ irresolute if and only if the inverse image of every soft $\tau_1'\tau_2'\ g^*s$ open set in $(\widetilde{Y}, \tau_1', \tau_2', E)$ is soft $\tau_1\tau_2\ g^*s$ open set in $(\widetilde{X}, \tau_1, \tau_2, E)$.*

Proof. Assume that f is soft $\tau_1\tau_2\ g^*s$ irresolute mapping. Let (A, E) be any soft $\tau_1'\tau_2'\ g^*s$ open set in $\widetilde{Y}$. Then $(A, E)^c$ is soft $\tau_1'\tau_2'\ g^*s$ closed set in $\widetilde{Y}$. Since the mapping f is soft $\tau_1\tau_2\ g^*s$ irresolute, $f^{-1}((A, E)^c)$ is soft $\tau_1\tau_2\ g^*s$ closed set in

$\widetilde{X}$. But $f^{-1}((A,E)^c) = \widetilde{X} - f^{-1}(A,E)$ and so $f^{-1}(A,E)$ is a soft $\tau_1\tau_2$ g^*s open set in $\widetilde{X}$. Hence the inverse image of every soft $\tau_1'\tau_2'$ g^*s open set in $\widetilde{Y}$ is soft $\tau_1\tau_2$ g^*s open in $\widetilde{X}$. Conversely, assume that the inverse image of every soft $\tau_1'\tau_2'$ g^*s open set in $\widetilde{Y}$. Let (A,E) be any soft $\tau_1'\tau_2'$ g^*s closed set in $\widetilde{Y}$. Then $(A,E)^c$ is soft $\tau_1'\tau_2'$ g^*s open set in $\widetilde{Y}$. By assumption $f^{-1}((A,E)^c)$ is soft $\tau_1\tau_2$ g^*s open in $\widetilde{X}$. But $f^{-1}((A,E)^c) = \widetilde{X} - f^{-1}(A,E)$ and so $f^{-1}(A,E)$ is soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$. Therefore f is a soft $\tau_1\tau_2$ g^*s irresolute mapping. □

Theorem 4.32. *A mapping $f : (\widetilde{X},\tau_1,\tau_2,E) \to (\widetilde{Y},\tau_1',\tau_2',E)$ is the soft $\tau_1\tau_2$ g^*s irresolute if $f(\tau_1\tau_2\ g^*s\ cl(A,E)) \widetilde{\subseteq} \tau_1\tau_2\ g^*s\ cl(f(A,E))$.*

Proof. Let us assume that $f(\tau_1\tau_2\ g^*s\ cl(A,E)) \widetilde{\subseteq} \tau_1\tau_2\ g^*s\ cl(f(A,E))$, for every $(A,E) \in \widetilde{X}$. Suppose (F,E) be a soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{Y}$. Then we have $(F,E) = \tau_1'\tau_2'\ g^*s\ cl(F,E) \Rightarrow f^{-1}(F,E) \in \widetilde{X}$ By hypothesis, $f(\tau_1\tau_2\ g^*s\ cl(f^{-1}(F,E))) \widetilde{\subseteq} \tau_1'\tau_2'\ g^*s\ cl(f(f^{-1}(\tau_1'\tau_2'\ g^*s\ cl(F,E)))) \Rightarrow f(\tau_1\tau_2\ g^*s\ cl(f^{-1}(F,E))) \widetilde{\subseteq} \tau_1'\tau_2'\ g^*s\ cl(f(f^{-1}(F,E))) = (F,E)$ Which implies $f^{-1}(F,E)$ is soft $\tau_1\tau_2$ g^*s closed set in $(\widetilde{X},\tau_1,\tau_2,E)$. Thus the mapping is a soft $\tau_1\tau_2$ g^*s irresolute mapping. □

Theorem 4.33. *If $f : (\widetilde{X},\tau_1,\tau_2,E) \to (\widetilde{Y},\tau_1',\tau_2',E)$ is a soft $\tau_1\tau_2$ g^*s irresolute then it is $\tau_1\tau_2$ g^*s continuous.*

Proof. Assume f is a soft $\tau_1\tau_2$ g^*s irresolute. Let (F,E) be any soft τ_2 closed set in $\widetilde{Y}$. Since every soft τ_2 closed set is soft $\tau_1\tau_2$ g^*s closed set, (F,E) is a soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{Y}$. Since f is soft $\tau_1\tau_2$ g^*s irresolute, $f^{-1}(F,E)$ is a soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$. Therefore f is a soft $\tau_1\tau_2$ g^*s continuous mapping. □

Theorem 4.34. *If $f : (\widetilde{X},\tau_1,\tau_2,E) \to (\widetilde{Y},\tau_1',\tau_2',E)$ is a soft $\tau_1\tau_2$ g^*s irresolute and $g : (\widetilde{Y},\tau_1',\tau_2',E) \to (\widetilde{Z},\tau_1'',\tau_2'',E)$ is a soft $\tau_1'\tau_2'$ g^*s irresolute. Then $g\circ f : (\widetilde{X},\tau_1,\tau_2,E) \to (\widetilde{Z},\tau_1'',\tau_2'',E)$ is soft $\tau_1\tau_2$ g^*s irresolute.*

Proof. Let (G,E) be a soft $\tau_1''\tau_2''$ closed set in $(\widetilde{Z},\tau_1'',\tau_2'',E)$. Since every soft $\tau_1''\tau_2''$ closed set is soft $\tau_1''\tau_2''$ g^*s closed set. Also since g is a soft $\tau_1'\tau_2'$ g^*s irresolute, $g^{-1}(G,E)$ is a soft $\tau_1'\tau_2'$ g^*s closed set in $(\widetilde{Y},\tau_1',\tau_2',E)$. Since f is a soft $\tau_1\tau_2$ g^*s irresolute, the soft set $f^{-1}(g^{-1}(G,E))$ is a soft $\tau_1\tau_2$ g^*s closed set in $(\widetilde{X},\tau_1,\tau_2,E)$. i.e., $f^{-1}(g^{-1}(G,E)) = (g\circ f)^{-1}(G,E)$ is a soft $\tau_1\tau_2$ g^*s closed set in $(\widetilde{X},\tau_1,\tau_2,E)$. Hence $g\circ f$ is a soft $\tau_1\tau_2$ g^*s irresolute mapping. □

Theorem 4.35. *For any soft $\tau_1\tau_2$ g^*s irresolute mapping $f : (\widetilde{X},\tau_1,\tau_2,E) \to (\widetilde{Y},\tau_1',\tau_2',E)$ and any soft $\tau_1'\tau_2'$ g^*s continuous mapping $g : (\widetilde{Y},\tau_1',\tau_2',E) \to (\widetilde{Z},\tau_1'',\tau_2'',E)$, the composition $g\circ f : (\widetilde{X},\tau_1,\tau_2,E) \to (\widetilde{Z},\tau_1'',\tau_2'',E)$ is a soft $\tau_1\tau_2$ g^*s continuous mapping.*

Proof. Let (F,E) be any soft $\tau_1''\tau_2''$ closed set in $(\widetilde{Z},\tau_1'',\tau_2'',E)$. Since g is a soft $\tau_1'\tau_2'$ g^*s continuous , $g^{-1}(F,E)$ is a soft $\tau_1'\tau_2'$ g^*s closed set in $(\widetilde{Y},\tau_1',\tau_2',E)$. Since f is soft $\tau_1\tau_2$ g^*s irresolute, $f^{-1}(g^{-1}(F,E))$ is a soft $\tau_1\tau_2$ g^*s closed set in $(\widetilde{X},\tau_1,\tau_2,E)$. But $f^{-1}(g^{-1}(F,E)) = (g\circ f)^{-1}(F,E)$. Thus $g\circ f : (\widetilde{X},\tau_1,\tau_2,E) \to (\widetilde{Z},\tau_1'',\tau_2'',E)$ is soft $\tau_1\tau_2$ g^*s continuous on $(\widetilde{X},\tau_1,\tau_2,E)$. □

5. Soft $\tau_1\tau_2$ g^*s Homeomorphism

Soft $\tau_1\tau_2$ g^*s homeomorphism and soft $\tau_1\tau_2$ g^*s^* homeomorphism in bi soft topological spaces are introduced in this section and some basic properties of these homeomorphism are also studied.

Definition 5.1. *A bijection $f : (\widetilde{X},\tau_1,\tau_2,E) \to (\widetilde{Y},\tau_1',\tau_2',E)$ is called soft $\tau_1\tau_2$ g^*s homeomorphism if f is both soft $\tau_1\tau_2$ g^*s continuous and soft $\tau_1\tau_2$ g^*s open.*

Proposition 5.2. *Every soft τ_2 homeomorphism is soft $\tau_1\tau_2$ g^*s homeomorphism.*

Proof. Let the bijection map $f:(\widetilde{X},\tau_1,\tau_2,E)\rightarrow(\widetilde{Y},\tau_1',\tau_2',E)$ be a soft τ_2 homeomorphism. We have that f is soft τ_2 continuous soft τ_2 open. Since every soft τ_2 continuous is soft $\tau_1\tau_2$ g^*s continuous, and every soft τ_2 open mapping is soft $\tau_1\tau_2$ g^*s open. Thus the bijection map f is soft $\tau_1\tau_2$ g^*s homeomorphism. □

Remark 5.3. *Converse of the above theorem need not be true. This shown in the following example.*

Example 5.4. *Let* $\widetilde{X}=\{k_1,k_2,k_3\}=\widetilde{Y}$ *and* $E=\{e_1,e_2\}$ *with the soft topologies on* $\widetilde{X}$ *as* $\tau_1=\{\phi,\ \widetilde{X},\ \{(e_1,\{k_1,k_2\}),(e_2,\phi)\},\ \{(e_1,\{k_1,k_2\}),\ (e_2,\{k_2,k_3\})\}\}$ *and* $\tau_2=\{\phi,\widetilde{X},\{(e_1,\{k_2\}),(e_2,\{k_1\})\}\}$ *and soft topologies on* $\widetilde{Y}$ *as,* $\tau_1'=\{\phi,\widetilde{Y},\{(e_1,\{k_1,k_2\}),(e_2,\phi)\}\}$ *and* $\tau_2'=\{\phi,\widetilde{Y},\{(e_1,\{k_1,k_3\})(e_2,\phi)\},\{(e_1,\phi),(e_2,\{k_2,e_3\})\},\{(e_1,\{k_1,k_3\}),(e_2,\{k_2,k_3\})\}$. *Define the soft mapping* $f:(\widetilde{X},\tau_1,\tau_2,E)\rightarrow(\widetilde{Y},\tau_1',\tau_2',E)$ *as* $f(k_1)=k_1,\ f(k_2)=k_2$ *and* $f(k_3)=k_3$. *Consider the soft set* $(A,E)=\{(e_1,\{k_1,k_3\}),(e_2,\{k_2,k_3\})\}$. *This set is soft* $\tau_1'\tau_2'$ g^*s *closed set in* $\widetilde{Y}$. *But* $f^{-1}(A,E)$ *is not soft* τ_2 *closed set in* $\widetilde{X}$. *Hence soft* $\tau_1\tau_2$ g^*s *continuous is not soft* τ_2 *continuous. Thus the soft* $\tau_1\tau_2$ g^*s *homeomorphism is not soft* τ_2 *homeomorphism.*

Proposition 5.5. *Every soft* τ_2 *semi homeomorphism is soft* $\tau_1\tau_2$ g^*s *homeomorphism.*

Proof. Assume that $f:(\widetilde{X},\tau_1,\tau_2,E)\rightarrow(\widetilde{Y},\tau_1',\tau_2',E)$ be a soft τ_2 semi homeomorphism. Since every soft τ_2 semi continuous is soft $\tau_1\tau_2$ g^*s continuous and every soft τ_2 semi open mapping is soft $\tau_1\tau_2$ g^*s open. Hence f is a soft $\tau_1\tau_2$ g^*s homeomorphism. □

Remark 5.6. *Converse of the above theorem need not be true. This shown in the following example.*

Example 5.7. *Let* $\widetilde{X}=\{k_1,k_2,k_3\}=\widetilde{Y}$ *and* $E=\{e_1,e_2\}$ *with* $\tau_1=\{\phi,\ \widetilde{X},\ \{(e_1,\{k_1,k_2\}),\ (e_2,\phi)\},\ \{(e_1,\{k_1,k_2\}),\ (e_2,\{k_2,k_3\})\}\}$ *and* $\tau_2=\{\phi,\widetilde{X},\{(e_1,\{k_2\}),(e_2,\{k_1\})\}\}$ *are topologies on* $\widetilde{X}$, *and the soft topologies on* $\widetilde{Y}$ *as,* $\tau_1'=\{\phi,\widetilde{Y},\{(e_1,\{k_1,k_3\}),(e_2,\phi)\},\{(e_1,\phi),(e_2,\{k_2,k_3\})\},\{(e_1,\{k_1,k_3\}),(e_2,\{k_2,k_3\})\}$ *and also* $\tau_2'=\{\phi,\widetilde{Y},\{(e_1,\{k_2,k_3\}),(e_2,\phi)\}\}$, *Define the soft mapping* $f:(\widetilde{X},\tau_1,\tau_2,E)\rightarrow(\widetilde{Y},\tau_1',\tau_2',E)$ *as* $f(k_1)=k_1,\ f(k_2)=k_2$ *and* $f(k_3)=k_3$. *Consider the soft set* $(A,E)=\{(e_1,\{k_1\}),(e_2,\widetilde{X})\}$. *This set is soft* $\tau_1'\tau_2'$ g^*s *closed set in* $\widetilde{Y}$. *But* $f^{-1}(A,E)$ *is not soft* τ_2 *semi closed set in* $\widetilde{X}$. *Thus the soft* $\tau_1\tau_2$ g^*s *homeomorphism is not soft* τ_2 *semi homeomorphism.*

Definition 5.8. *A bijection* $f:(\widetilde{X},\tau_1,\tau_2,E)\rightarrow(\widetilde{Y},\tau_1',\tau_2',E)$ *is called soft* $\tau_1\tau_2$ g^*s *homeomorphism if* f *is both soft* $\tau_1\tau_2$ *sg continuous and soft* $\tau_1\tau_2$ *sg open.*

Proposition 5.9. *Every soft* $\tau_1\tau_2$ g^*s *homeomorphism is soft* $\tau_1\tau_2$ *sg homeomorphism.*

Proof. Suppose that $f:(\widetilde{X},\tau_1,\tau_2,E)\rightarrow(\widetilde{Y},\tau_1',\tau_2',E)$ be a soft $\tau_1\tau_2$ g^*s homeomorphism. Since every soft $\tau_1\tau_2$ g^*s continuous map is soft $\tau_1\tau_2$ *sg* continuous and every soft $\tau_1\tau_2$ g^*s open mapping is soft $\tau_1\tau_2$ *sg* open. Thus the bijective map f is soft $\tau_1\tau_2$ *sg* homeomorphism. □

Remark 5.10. *Converse of the above theorem need not be true. This shown in the following example.*

Example 5.11. *Let* $\widetilde{X}=\{k_1,k_2\}=\widetilde{Y}$ *and* $E=\{e_1,e_2\}$ *with the topologies on* $\widetilde{X}$ *as,* $\tau_1=\{\phi,\ \widetilde{X},\ \{(e_1,\{k_2\}),\ (e_2,\phi)\}\}$ *and* $\tau_2=\{\phi,\widetilde{X},\{(e_1,\{k_2\}),(e_2,\{k_1\})\}\}$, *and the soft topologies on* $\widetilde{Y}$ *as,* $\tau_1'=\{\phi,\widetilde{Y},\{(e_1,\{k_1\}),(e_2,\{k_1\})\},\{(e_1,\{k_1\}),(e_2,\{k_1\})\},\{(e_1,\{k_2\}),(e_2,k_1)\},\{(e_1,\widetilde{Y}),(e_2,\{k_2\})\}\}$ *and* $\tau_2'=\{\phi,\widetilde{Y},\{(e_1,\phi),(e_2,\{k_1\})\},\{(e_1,\widetilde{Y}),(e_2,\phi)\},\{(e_1,\widetilde{Y}),(e_2,\{k_1\})\}\}$. *Define the soft mapping* $f:(\widetilde{X},\tau_1,\tau_2,E)\rightarrow(\widetilde{Y},\tau_1',\tau_2',E)$ *by* $f(k_1)=k_1,\ f(k_2)=k_2$. *Consider the soft set* $(A,E)=\{(e_1,\{k_1\}),(e_2,\widetilde{Y})\}$. *This set is soft* $\tau_1'\tau_2'$ *sg closed set in* $\widetilde{Y}$. *But* $f^{-1}(A,E)$ *is not soft* $\tau_1\tau_2$ g^*s *closed set in* $\widetilde{X}$. *Thus the soft* $\tau_1\tau_2$ *sg homeomorphism is not soft* $\tau_1\tau_2$ g^*s *homeomorphism.*

Definition 5.12. *A bijection $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is called soft $\tau_1\tau_2$ gs homeomorphism if f is both soft $\tau_1\tau_2$ g^*s continuous and soft $\tau_1\tau_2$ gs open.*

Proposition 5.13. *Every soft $\tau_1\tau_2$ g^*s homeomorphism is soft $\tau_1\tau_2$ gs homeomorphism.*

Proof. Let us assume that the soft bijective mapping $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ be a soft $\tau_1\tau_2$ g^*s homeomorphism. Since every soft $\tau_1\tau_2$ g^*s continuous map is soft $\tau_1\tau_2$ gs continuous and every soft $\tau_1\tau_2$ g^*s open mapping is soft $\tau_1\tau_2$ gs open. Therefore the map f is soft $\tau_1\tau_2$ gs homeomorphism. □

Remark 5.14. *Converse of the above theorem need not be true. This shown in the following example.*

Example 5.15. *Let $\widetilde{X} = \{k_1, k_2, k_3\} = \widetilde{Y}$ and $E = \{e_1, e_2\}$ with the topologies on $\widetilde{X}$ as $\tau_1 = \{\phi, \widetilde{X}, \{(e_1, \{k_1\})(e_2, \{k_2, k_3\})\}\}$ and $\tau_2 = \{\phi, \widetilde{X}, \{(e_1, \widetilde{Y}), (e_2, \{k_2, k_3\})\}\}$, and the soft topologies on $\widetilde{Y}$ as, $\tau_1' = \{\phi, \widetilde{Y}, \{(e_1, \{k_1, k_2\}), (e_2, \phi)\}, \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}\}$ and $\tau_2' = \{\phi, \widetilde{Y}, \{(e_1, \{k_2\}), (e_2, \{k_1\})\}\}$. Define the soft mapping $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ by $f(k_1) = k_1, f(k_2) = k_2$ and $f(k_3) = k_3$. Consider the soft set $(A, E) = \{(e_1, \phi), (e_2, \{k_1\})\}$. This set is soft $\tau_1'\tau_2'$ gs closed set in $\widetilde{Y}$. But $f^{-1}(A, E)$ is not soft $\tau_1\tau_2$ g^*s closed set in $\widetilde{X}$. Thus the soft $\tau_1\tau_2$ gs homeomorphism is not soft $\tau_1\tau_2$ g^*s homeomorphism.*

Proposition 5.16. *Every soft τ_2 α homeomorphism is soft $\tau_1\tau_2$ g^*s homeomorphism.*

Proof. Let the bijection map $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ be a soft τ_2 α homeomorphism. We have that f is soft τ_2 α continuous soft τ_2 α open. Since every soft τ_2 α continuous is soft $\tau_1\tau_2$ g^*s continuous, and every soft τ_2 α open mapping is soft $\tau_1\tau_2$ g^*s open. Thus the bijection map f is soft $\tau_1\tau_2$ g^*s homeomorphism. □

Remark 5.17. *Converse of the above theorem need not be true. This shown in the following example.*

Example 5.18. *Let $\widetilde{X} = \{k_1, k_2, k_3\} = \widetilde{Y}$ and $E = \{e_1, e_2\}$ with the soft topologies on $\widetilde{X}$ as, $\tau_1 = \{\phi, \widetilde{X}, \{(e_1, \{k_1, k_2\}), (e_2, \phi)\}, \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}\}$ and $\tau_2 = \{\phi, \widetilde{X}, \{(e_1, \{k_2\}), (e_2, \{k_1\})\}\}$ and the soft topologies on $\widetilde{Y}$ as, $\tau_1' = \{\phi, \widetilde{Y}, \{(e_1, \{k_1, k_2\})(e_2, \phi)\}, \{(e_1, \{k_3\}), (e_2, \{k_1, k_2\})\}, \{(e_1, \widetilde{X}), (e_2, \{k_1, k_2\})\}\}$ and $\tau_2' = \{\phi, \widetilde{Y}, \{(e_1, \{k_1, k_2\}), (e_2, \{k_2, k_3\})\}\}$. Define $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ by $f(k_1) = k_1$, $f(k_2) = k_2$ and $f(k_3) = k_3$. Consider the soft set $(A, E) = \{(e_1, \{k_3\}), (e_2, \{k_1\})\}$. This set is soft $\tau_1'\tau_2'$ g^*s closed set in $\widetilde{Y}$. But $f^{-1}(A, E)$ is not soft τ_2 α closed set in $\widetilde{X}$. Thus the soft $\tau_1\tau_2$ g^*s homeomorphism is not soft τ_2 α homeomorphism.*

Theorem 5.19. *Let $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ be a bijective soft $\tau_1\tau_2$ g^*s continuous map. Then the following statements are equivalent.*

i). f is soft $\tau_1\tau_2$ g^*s open mapping. ii). f is soft $\tau_1\tau_2$ g^*s homeomorphism. iii). f is soft $\tau_1\tau_2$ g^*s closed mapping.

Proof. $i) \Rightarrow ii)$ From the hypothesis the bijective map f is soft $\tau_1\tau_2$ g^*s continuous and f is a soft $\tau_1\tau_2$ g^*s open, f is a soft $\tau_1\tau_2$ g^*s homeomorphism. $ii) \Rightarrow iii)$ Let (V, E) be a soft τ_2 closed set of $(\widetilde{X}, \tau_1, \tau_2, E)$. Then $(V, E)^c$ is soft τ_2 open set in $\widetilde{X}$. Since f is soft $\tau_1\tau_2$ g^*s homeomorphism, it is soft $\tau_1\tau_2$ g^*s open, $f((V, E)^c)$ is soft $\tau_1'\tau_2'$ g^*s open set in $(\widetilde{Y}, \tau_1', \tau_2', E)$. i.e., $f((V, E)^c) = (f(V, E))^c$ is soft $\tau_1'\tau_2'$ g^*s open set in $\widetilde{Y}$. Therefore $f(V, E)$ is soft $\tau_1'\tau_2'$ g^*s closed set in $\widetilde{Y}$. Thus f is a soft $\tau_1\tau_2$ g^*s closed mapping. $ii) \Rightarrow iii)$ Let (U, E) be a soft τ_2 open set in $(\widetilde{X}, \tau_1, \tau_2, E)$. Then $(U, E)^c$ is soft τ_2 closed set in $(\widetilde{X}, \tau_1, \tau_2, E)$. By the hypothesis, $f((V, E)^c)$ is soft $\tau_1'\tau_2'$ g^*s closed set in $(\widetilde{Y}, \tau_1', \tau_2', E)$. i.e., $f((V, E)^c) = (f(V, E))^C$ is soft $\tau_1'\tau_2'$ g^*s closed set in $(\widetilde{Y}, \tau_1', \tau_2', E)$. Thus $f(V, E)$ is soft $\tau_1'\tau_2'$ g^*s open set in $(\widetilde{Y}, \tau_1', \tau_2', E)$. Hence f is soft $\tau_1\tau_2$ g^*s open mapping. □

Definition 5.20. *A bijection $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is said to be soft $\tau_1\tau_2$ g^*s* homeomorphism if f is soft $\tau_1\tau_2$ g^*s irresolute and f^{-1} is soft $\tau_1'\tau_2'$ g^*s irresolute.*

Theorem 5.21. *Every soft $\tau_1\tau_2$ g^*s^* homeomorphism is soft $\tau_1\tau_2$ g^*s homeomorphism.*

Proof. Assume that $f : (\widetilde{X}, \tau_1, \tau_2, E)$ be a soft $\tau_1\tau_2$ g^*s homeomorphism. Since every soft $\tau_1\tau_2$ g^*s irresolute map is soft $\tau_1\tau_2$ g^*s continuous and from the hypothesis the mapping f is soft $\tau_1\tau_2$ g^*s homeomorphism. □

Theorem 5.22. *If $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is soft $\tau_1\tau_2$ g^*s^* homeomorphism and $g : (\widetilde{Y}, \tau_1', \tau_2', E) \to (\widetilde{Z}, \tau_1'', \tau_2'', E)$ is soft $\tau_1'\tau_2'$ g^*s^* homeomorphism, their composition $g \circ f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Z}, \tau_1'', \tau_2'', E)$ is also soft $\tau_1\tau_2$ g^*s^* homeomorphism.*

Proof. Let (U, E) be a soft $\tau_1''\tau_2''$ g^*s open set in $(\widetilde{Z}, \tau_1'', \tau_2'', E)$. Now $(g \circ f)^{-1}(U, E) = f^{-1}(g^{-1}(U, E)) = g^{-1}(A, E)$ where $(A, E) = g^{-1}(U, E)$. By the hypothesis, (A, E) is the soft $\tau_1'\tau_2'$ g^*s open set in $(\widetilde{Y}, \tau_1', \tau_2', E)$ and so $f^{-1}(A, E)$ is soft $\tau_1\tau_2$ g^*s open set in $(\widetilde{X}, \tau_1, \tau_2, E)$. Thus $g \circ f$ is soft $\tau_1\tau_2$ g^*s irresolute. Also for a soft $\tau_1\tau_2$ g^*s open set (B, E) in $(\widetilde{X}, \tau_1, \tau_2, E)$, we have $(g \circ f)(B, E) = g(f(B, E)) = g(V, E)$, where $(V, E) = f(B, E)$. By hypothesis $f(B, E)$ is soft $\tau_1'\tau_2'$ g^*s open in $(\widetilde{Y}, \tau_1', \tau_2', E)$ and so $g(f(B, E))$ is soft $\tau_1''\tau_2''$ g^*s open in $(\widetilde{Z}, \tau_1'', \tau_2'', E)$. i.e., $(g \circ f)(B, E)$ is soft $\tau_1''\tau_2''$ g^*s open in $(\widetilde{Z}, \tau_1'', \tau_2'', E)$, the set $(g \circ f)^{-1}$ is soft $\tau_1\tau_2$ g^*s irresolute. Thus $g \circ f$ is soft $\tau_1\tau_2$ g^*s^* homeomorphism. □

Theorem 5.23. *If $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', E)$ is a soft $\tau_1\tau_2$ g^*s^* homeomorphism, then $\tau_1\tau_2$ g^*s $cl(f^{-1}(B, E)) = f^{-1}(\tau_1'\tau_2'$ $g*s$ $cl(B, E))$, for all $(B, E) \subseteq \widetilde{Y}$ is soft $\tau_1'\tau_2'$ g^*s closed.*

Proof. Let $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', \tau_2', E)$ is a soft $\tau_1\tau_2$ g^*s^* homeomorphism, we have that f is soft $\tau_1\tau_2$ g^*s irresolute and f^{-1} is soft $\tau_1'\tau_2'$ g^*s irresolute. Therefore $\tau_1'\tau_2'$ g^*s $cl(f(B, E))$ is soft $\tau_1'\tau_2'$ g^*s closed in $(\widetilde{Y}, \tau_1', \tau_2', E)$. This implies that $f^{-1}(\tau_1'\tau_2'$ g^*s $cl(f(B, E)))$ is soft $\tau_1\tau_2$ g^*s closed set in $(\widetilde{X}, \tau_1, \tau_2, E)$. Thus $\tau_1\tau_2$ g^*s $cl(f^{-1}(B, E)) \subseteq f^{-1}(\tau_1'\tau_2'$ g^*s $cl(B, E))$. Again f^{-1} is soft $\tau_1\tau_2$ g^*s irresolute, $\tau_1\tau_2$ g^*s $cl(f^{-1}(B, E))$ is soft $\tau_1\tau_2$ g^*s closed in $(\widetilde{X}, \tau_1, \tau_2, E)$. Which gives $(f^{-1})^{-1}$ $\tau_1\tau_2$ g^*s $cl(f^{-1}(B, E)) = f(\tau_1\tau_2$ g^*s $cl(f^{-1}(B, E)))$ is soft $\tau_1\tau_2$ g^*s closed in $(\widetilde{X}, \tau_1, \tau_2, E)$. Thus, $f^{-1}(\tau_1'\tau_2'$ g^*s $cl(B, E)) \subseteq \tau_1\tau_2$ g^*s $cl(f^{-1}(B, E))$. Hence $\tau_1\tau_2$ g^*s $cl(f^{-1}(B, E)) = f^{-1}(\tau_1'\tau_2'$ g^*s $cl(B, E))$. □

Theorem 5.24. *If $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', E)$ is a soft $\tau_1\tau_2$ g^*s^* homeomorphism, then $\tau_1'\tau_2'$ g^*s $cl(f(B, E)) = f(\tau_1\tau_2$ g^*s $cl(B, E))$ for all $(B, E) \subseteq \widetilde{X}$.*

Proof. Let $f : (\widetilde{X}, \tau_1, \tau_2, E) \to (\widetilde{Y}, \tau_1', E)$ is a soft $\tau_1\tau_2$ g^*s^* homeomorphism. Since f is soft $\tau_1\tau_2$ g^*s irresolute and f^{-1} is soft $\tau_1'\tau_2'$ g^*s irresolute. Then $f^{-1} : (\widetilde{Y}, \tau_1', \tau_2', E) \to (\widetilde{X}, \tau_1, \tau_2, E)$ is a soft $\tau_1'\tau_2'$ g^*s homeomorphism, we have that $\tau_1'\tau_2'$ g^*s $cl(f(B, E))$ is soft $\tau_1'\tau_2'$ g^*s closed set in $(\widetilde{Y}, \tau_1', \tau_2', E)$, which implies $f^{-1}(\tau_1'\tau_2'$ g^*s $cl(f(B, E)))$ is soft $\tau_1\tau_2$ g^*s closed set in $(\widetilde{X}, \tau_1, \tau_2, E)$. Since $(\tau_1\tau_2$ g^*s $int(B, E))^c = (\tau_1\tau_2$ g^*s $cl(B, E)^c)^c$. Then $f(\tau_1\tau_2$ g^*s $int(B, E)) = f((\tau_1\tau_2$ g^*s $cl(B, E)^c)^c) = (\tau_1'\tau_2'$ g^*s $cl(B, E)^c)^c = (\tau_1'\tau_2'$ g^*s $cl f(B, E)^c)^c = \tau_1'\tau_2'$ g^*s $int f(B, E)$. Thus $\tau_1'\tau_2'$ g^*s $cl(f(B, E)) = f(\tau_1\tau_2$ g^*s $cl(B, E))$. □

References

[1] B. Ahmad and A. Kharal, *Mapping of soft classes*, New Mathematics and Natural Computation, 7(3)(2011), 471-481.

[2] M. I. Ali, F. Feng, X. Y. Liu, W. K. Min and M. Shabir, *On some new operations in soft set theory*, Computer and Mathematics with Application, 57(2009), 1547-1553.

[3] A. Aygunoglu and H. Aygun, *Some notes on soft topological spaces*, Neural Computing and Applications, 21(1)(2012), 113-119.

[4] Gnanambal Ilanago and Mrudula Ravindran, *On soft pre open sets in soft topological spaces*, International Journal of Mathematics Research, 5(4)(2013), 399-409.

[5] T. Indira, *$\tau_1\tau_2$-$^{\#}g$ closed sets in bitopological spaces*, Annals of Pure and Applied Mathematics, 7(2)(2014), 27-34.

[6] Juthika Mahanta and Pramod Kumar Das, *On soft topological space via semi open and semi closed soft sets*, Kyungpook Mathematical Journal, 54(2014), 221-236.

[7] K. Kannan, *$\tau_1\tau_2$ semi star star generalized closed sets*, International Journal of Pure and applied Mathematics, 76(2)(2012), 277-294.

[8] K. Kannan, *Soft generalized closed sets in soft topological spaces*, Journal of Theoretical and Applied Information Technology, 37(1)(2012), 17-21.

[9] K. Kannan, D. Narasimhhan and K. Chandrasekhara Rao, *On semi star generalized closed sets in bitopological spaces*, Boletim da Sociedada Paranaense de Matematica, 28(1)(2010), 29-40.

[10] J. C. Kelly, *Bitopological spaces*, Proceedings of the London Mathematical Society, 13(1963), 71-89.

[11] D. A. Molodtsov, *Soft set theory first results*, Computers and Mathematics with Applications, 37(1999), 19-31.

[12] Munazza Naz, Muhammad Shabir and Muhammad Irfan Ali, *Separation axioms in bi soft topological spaces*, http://arxiv.org/ps/1509.00866v1 (2015).

[13] B. V. S. T Sai and V. Srinivasa kumar, *On soft semi open sets and semi topology*, International Journal of Mathematical Archive, 4(4)(2013), 114-117.

[14] D. Sivaraj and V. E. Sasikala, *A study on soft α-open sets*, IOSR Journal of Mathematics, 12(5)(2016), 70-74.

[15] M. Shabir and M. Naz, *On soft topological spaces*, Computers and Mathematics with Applications, 61(2011), 1786-1799.

[16] M. Suraiya Begum and M. Sheik John, *Soft g^*s closed sets in soft topological spaces*, International Journal of Scientific Research And Education, 4(6)(2016), 5466-5470.