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Numerical Study of Magneto-Convective Casson Fluid Flow Past an Exponentially Accelerated Vertical Porous Plate in the Presence of Radiation and Dufour Effects

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Abstract: A numerical analysis is carried out for an unsteady free convective, chemically reactive, and electrically conducting fluid past an exponentially accelerated vertical porous plate in the presence of heat source, radiation and Dufour effects. The set of non-dimensional governing equations along with boundary conditions are solved numerically. The effect of various physical parameters on flow quantities are studied with the help of graphs. For the physical interest, the variations in skin friction, Nusselt number and Sherwood number are also studied through tables.

Keywords: MHD, Casson fluid, Radiation, Chemical reaction, Heat source, Diffusion thermo effect.

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1. Introduction

Casson fluid is one type of non -Newtonian fluid. Casson fluid can be defined as a shear thinning liquid which is supposed to have an infinite viscosity at zero rates of shear and a yield stress under which no flow occurs and zero viscosity at an infinite rate of shear. Some examples of Casson fluid are Jelly, honey, tomato sauce and concentrated fruit juices. Human blood can also be treated as a Casson fluid in the presence of several substances such as fibrinogen, globulin in aqueous base plasma, protein, and human red blood cells.

Casson fluid has distinct features. This model was presented by Casson for the flow of visco-elastic fluids in 1995. This model is cast off by fuel engineers in the description of adhesive slurry and is improved for forecasting high shear-rate viscosities when only low and transitional shear-rate data are accessible. The fluid flow over a stretching surface is significant in solicitations such as extrusion, cord depiction, copper spiraling, warm progressing, and melts of high molecular weight polymers. Vajravelu and Mukhopadhyay [1] studied Diffusion of chemically reactive species in Casson fluid flow over an unsteady permeable stretching surface. Dash et al. [2] investigated on fluid flow in a pipe filled with a homogeneous porous medium. Nadeem et al. [3] considered MHD three-dimensional Casson fluid flow past a porous linearly stretching sheet. Nadeem et al. [4] analyzed MHD flow of a Casson fluid over an exponentially shrinking sheet. Pramanik [5] discussed Casson fluid flow and heat transfer past an exponentially porous stretching surface in the presence of thermal radiation. Hayat et al. [6] examined Soret and Dufour effects on magneto hydrodynamic (MHD) ?ow of Casson ?uid. Mustafa et al. [7] established an unsteady boundary layer ?ow of a Casson ?uid due to an impulsively started moving ?at plate. Makanda and Shaw [8]

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studied effects of radiation on MHD free convection of a Casson fluid from a horizontal circular cylinder with partial slip in non-Darcy porous medium with viscous dissipation. Bhattacharyya [9] analyzed MHD stagnation-point flow of Casson fluid and heat transfer over a stretching sheet with thermal radiation. Effect of Thermal Radiation on Heat and Mass Transfer in MHD Flow of a Casson Fluid over a Stretching Surface with Variable Thermal Conductivity was considered by Haritha and Sarojamma.

2. Formulation of the Problem

Influence of radiation parameter and Dufour number on unsteady MHD free convection heat and mass transfer flow of heat absorbing/generating Casson fluid past a semi-infinite oscillating vertical plate embedded in uniform porous medium with time dependent temperature and concentration is considered. Let x- axis taken towards upward direction along with the fluid and y-axis is taken normal to it. The fluid assumed hears is an electrically conducting with a uniform magnetic field of strength B_0 is applied in a direction perpendicular to the plate. The magnetic Reynold's number is assumed to be very small so that it may be ignored in comparison with magnetic field. Initially at $t = 0$, the fluid is assumed to be at rest and the plate and fluid are maintained at uniform temperature and concentration. For $t > 0$, the plate begins to oscillate in its own plane $(y = 0)$ in the form $V = u_0\, e^{a^* t^*}$. At the same time, the plate temperature is raised to $T_\infty + (T_w - T_\infty)\, e^{a^* t^*}$ which is thereafter maintained constant. The tensor of the Casson fluid can be written as

$$\tau = \tau_0 + \mu\gamma^*$$

or

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right) e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right) e_{ij}, & \pi < \pi_c \end{cases}$$

Where $\pi = e_{ij}e_{ij}$ and e_{ij} is the $(i,j)^{th}$ component of deformation rate, π is the product of the component of deformation rate with itself, π_c is the critical value of this product based on the non-Newtonian fluid, μ_B is the plastic dynamic viscosity of its fluid and is yield stress of the non-Newtonian fluid. Before forming the governing equations we have taken some assumptions that are unidirectional flow, one dimensional flow, free convection, rigid plate, incompressible flow, unsteady flow, non-Newtonian flow, oscillating vertical plate and viscous dissipation term in the energy equation is neglected. Considering the above assumptions we have formed the following set of partial differential equations.

$$\rho\frac{\partial u^*}{\partial t^*} = \mu_\beta\left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 u^*}{\partial {y^*}^2} - \sigma B_0^2 u^* - \frac{\mu\phi}{k_1}u^* + \rho g\beta\left(T^* - T_\infty\right) + \rho g\beta^*\left(C^* - C_\infty\right) \tag{1}$$

$$\rho C_p\frac{\partial T^*}{\partial t^*} = k\frac{\partial^2 T^*}{\partial {y^*}^2} + Q^*\left(T^* - T_\infty\right) - \frac{\partial q_r}{\partial y^*} + \left(\frac{D_m k\,\rho}{C_s}\right)\frac{\partial^2 C^*}{\partial {y^*}^2} + \sigma B_0^2 {u^*}^2 \tag{2}$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial {y^*}^2} + D_1\frac{\partial^2 T^*}{\partial {y^*}^2} - Kr^*(C* - C_\infty) \tag{3}$$

Cogley have shown that, in the optically thin limit for a non-gray gas near equilibrium, the radiative heat flux is represented by the following form:

$$\frac{\partial q_r}{\partial y^*} = 4\left(T^* - T_\infty\right)I, \quad where \quad I = \int K_{\lambda w}\frac{\partial e_{b\lambda}}{\partial T^*}d\lambda$$

The initial and boundary conditions are

$$\left.\begin{array}{l} t^* < 0 : u^* = 0, T^* = T_\infty, C^* = C_\infty, \quad \text{forall}\ \ y^* < 0 \\ t^* \geq 0 : u^* = u_0 e^{a^* t^*}, T^* = T_\infty + \left(T_w - T_\infty\right)e^{a^* t^*}, \quad C^* = C_\infty + \left(C_w - C_\infty\right)a^* t^* \quad \text{at}\ \ y^* = 0 \\ u^* \to 0, T^* \to T_\infty, C^* \to C_\infty \quad \text{as}\ \ y^* \to \infty \end{array}\right\} \tag{4}$$

On introducing the following non-dimensional quantities

$$u = \frac{u^*}{u_0},\ t = \frac{t^* u_0^2}{\nu},\ y = \frac{y^* u_0}{\nu},\ \theta = \frac{T^* - T_\infty}{T_w - T_\infty},\ C = \frac{C^* - C_\infty}{C_w - C_\infty}$$

Where

$$a = \frac{a^* \nu}{U_0^2}$$

$$Gr = \frac{\nu g \beta \left(T_w - T_\infty\right)}{u_0^3}, \qquad \text{(Grashof number)}$$

$$Gm = \frac{\nu g \beta^* \left(C_w - C_\infty\right)}{u_0^3}, \qquad \text{(Modified Grashof number)}$$

$$K = \frac{k_1 u_0^2}{\phi \nu^2}, \qquad \text{(Permeability parameter)}$$

$$M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \qquad \text{(Magnetic parameter)}$$

$$\Pr = \frac{\nu \rho C_p}{k}, \qquad \text{(Prandtl number)}$$

$$Ec = \frac{u_0^2}{C_p \left(T_w - T_\infty\right)}, \qquad \text{(Eckert number)}$$

$$Sc = \frac{\nu}{D}, \qquad \text{(Schmidt number)}$$

$$Q = \frac{Q^* \nu}{\rho C_p u_0^2}, \qquad \text{(Heat absorption parameter)}$$

$$R = \frac{4 \nu I}{\rho C_p u_0^2}, \qquad \text{(Radiation parameter)}$$

$$Sr = \frac{D_1 \left(T_w - T_\infty\right)}{\nu \left(C_w - C_\infty\right)}, \qquad \text{(Soret number)}$$

$$Kr = \frac{K_r^* \nu}{u_0^2}$$

$$Df = \frac{D_m k \left(C_w - C_\infty\right)}{\nu C_s C_p \left(T_w - T_\infty\right)}$$

$$\gamma \qquad \text{(Casson, parameter)}$$

In terms of the above non-dimension quantities, Equations (1)-(3) reduces to

$$\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 u}{\partial y^2} - M\,u - \frac{1}{K} u + Gr\,\theta + Gm\,C \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} + Q\,\theta - R\,\theta + Df\,\frac{\partial^2 C}{\partial y^2} + M\,Ec\,u^2 \tag{6}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + Sr\,\frac{\partial^2 \theta}{\partial y^2} - Kr\,C \tag{7}$$

The corresponding initial and boundary conditions are:

$$\left.\begin{array}{l} t < 0 : u = 0, T = 0, C = 0 \ \text{ forall } \ y < 0 \\ t \geq 0 : u = e^{at}, \theta = e^{at}, C = at \ \text{ at}\, y = 0 \\ u \to 0, T \to 0, C^* \to 0 \ \text{ as } y \to \infty \end{array}\right\} \tag{8}$$

3. Method of solution

Equations (5)-(7) are coupled non-linear partial differential equations and are to be solved by using the initial and boundary conditions (8). However exact solution is not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference schemes of equations for (5)-(7) are as follows:

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = \left(1 + \frac{1}{\gamma}\right) \left(\frac{u_{i-1,j} - 2\,u_{i,j} + u_{i+1,j}}{\left(\Delta y\right)^2}\right) - M\,u_{i,j} - \frac{1}{K} u_{i,j} + Gr\,\theta_{i,j} + Gc\,C_{i,j} \tag{9}$$

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}=\frac{1}{\mathrm{Pr}}\left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{(\Delta y)^2}\right)+Q\,\theta_{i,j}-R\,\theta_{i,j}+Df\left(\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}}{(\Delta y)^2}\right)+M\,Ec\,(u_{i,j})^2 \tag{10}$$

$$\frac{C_{i,j+1}-C_{i,j}}{\Delta t}=\frac{1}{Sc}\left(\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}}{(\Delta y)^2}\right)+Sr\left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{(\Delta y)^2}\right)-Kr\,C_{i,j} \tag{11}$$

Here, index i refer to y and j to time. The mesh system is divided by taking $\Delta y = 0.04$. From the initial condition in (8), we have the following equivalent:

$$u(i,0)=0,\ \theta(i,0)=0,\ C(i,0)=0\ for\,all\,i \tag{12}$$

The boundary conditions from (8) are expressed in finite-difference form as follows

$$\begin{aligned}&u(0,j)=\exp\left(a*(j-1)*\Delta t\right),\ \theta(0,j)=\exp\left(a*(j-1)*\Delta t\right),\ C(0,j)=a*(j-1)*\Delta t\ \ for\,all\,j\\&u(i_{\max},j)=0,\ \theta(i_{\max},j)=0,\ C(i_{\max},j)=0\ \ for\,all\,j\end{aligned} \tag{13}$$

(Here i_{max} was taken as 201). First the velocity at the end of time step viz, $u(i, j+1)$, $(i = 1, 201)$ is computed from (9) in terms of velocity, temperature and concentration at points on the earlier time-step. Then $\theta\ (i, j+1)$ is computed from (10) and C $(i, j+1)$ is computed from (11). The procedure is repeated until $t = 0.05$ (i.e. $j = 500$). During computation Δt was chosen as 0.0001.

Skin-friction: The skin-friction in non-dimensional form is given by

$$\tau=-\left(1+\frac{1}{\gamma}\right)\left(\frac{du}{dy}\right)_{y=0},\ where\ \tau^*=\frac{\tau}{\rho u_0^2}$$

Rate of heat transfer: The dimensionless rate of heat transfer is given by

$$Nu=-\left(\frac{d\theta}{dy}\right)_{y=0}$$

Rate of mass transfer: The dimensionless rate of mass transfer is given by

$$Sh=-\left(\frac{dC}{dy}\right)_{y=0}$$

4. Result and Discussion

A Numerical study is carried out on the MHD flow of a Casson fluid. The effects of various physical parameters such as Grashof number, modified Grashof number, Casson parameter, magnetic parameter, permeability parameter, Prandtl number, heat source, radiation parameter, Schmidt number, chemical reaction parameter and Sorret number on velocity, temperature and concentration are discussed with help of graphs whereas Skin friction, Nusselt number and Sherwood are also discussed with the help of tables. In Figure 1, the effect of Grashof number on velocity is presented. As Gr increases, velocity also increases. This is due to the buoyancy which is acting on the fluid particles due to gravitational force that enhances the fluid velocity. A similar effect is observed from Figure 2, here in the presence of modified Grashof number, which also increases fluid velocity. In figure 3, velocity profiles are displayed with the variation in magnetic parameter. From this figure it is noticed that velocity gets reduced by the increase of magnetic parameter. Because the magnetic force which is applied perpendicular to the plate, retards the flow, which is known as Lorentz force. Hence the presence of this retarding force reduces the fluid velocity. Figure 4 shows that the velocity increases with an increase in permeability parameter. This is due to the fact that increasing values of K reduces the drag force which assists the fluid considerably to move fast. Figure 5, demonstrates that the velocity decreases with an increase in Casson parameter. Figure 6 indicates that a rise in Pr substantially reduces the temperature in the viscous fluid. It can be found from Figure 6 that the solutal boundary layer thickness of the fluid enhances with the increase of Pr. Figure 7, depicts the effect of heat source on temperature. It is noticed that the temperature is increased by an increase in the heat source by the fluid. The central reason behind this effect is that the heat source causes a increase in the kinetic energy as well as thermal energy of the fluid. The momentum and thermal boundary layers get thinner in case of heat source fluids. Figure 8, demonstrates the effect of radiation parameter on temperature. It is observed that temperature decreases as radiation parameter increases. Figure 9 depicts the variations in temperature profile for different values of radiation Dufour number. It is noticed that temperature increases as Dufour number increases. Influence of Schmidt number on concentration is shown in figure 10, from this figure it is noticed that concentration decreases with an increase in Schmidt number. Because, Schmidt number is a dimensionless number defined as the ratio of momentum diffusivity and mass diffusivity, and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes. Therefore concentration boundary layer decreases with an increase in Schmidt number. Figure 11 indicates that, concentration profile decreases with an increase in Kr. From figure 12, we observe that the concentration increases as Soret number increases.

Table 1 shows that Skin friction increase with an increase in magnetic parameter while it decreases with an increase in Grashof number, modified Grashof number and permeability parameter. From table 2, we observed that the Nusselt number increase with increasing values of Prandtl number and radiation parameter while it decreases with increasing value of heat source and Dufour number. From table 3, we have seen that the Sherwood number increase with increasing values of Schmidt number and chemical reaction parameter and it shows reverse trend in the case of Soret number.

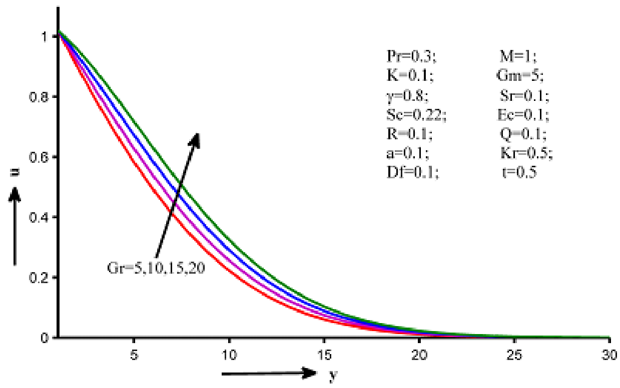

Figure 1. **Effect of Grashof number on velocity**

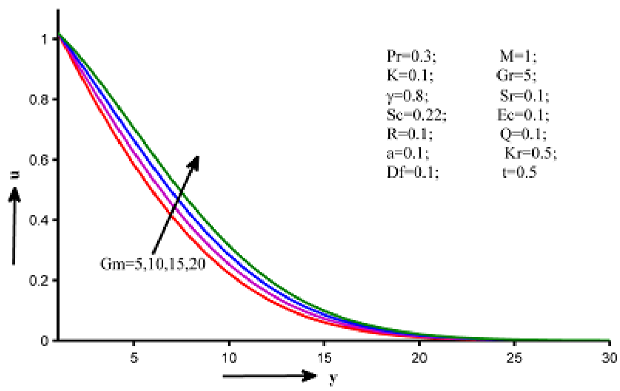

Figure 2. **Effect of modified Grashof number on velocity**

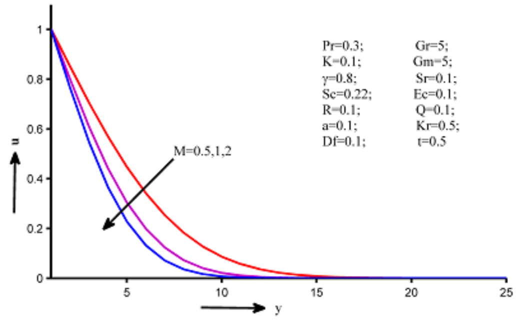

Figure 3. **Effect of magnetic parameter on velocity**

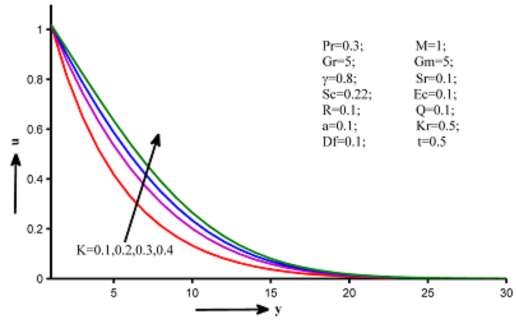

Figure 4. **Effect of permeability parameter on velocity**

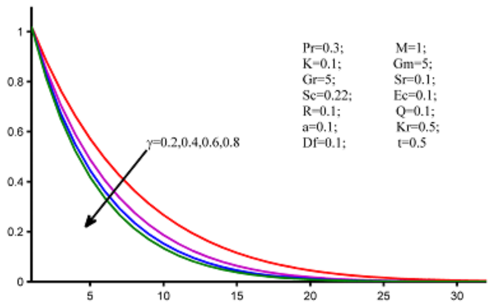

Figure 5. **Effect of Casson parameter on velocity**

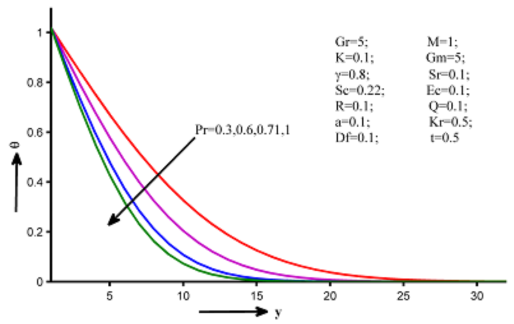

Figure 6. **Effect of Prandtl number on temperature**

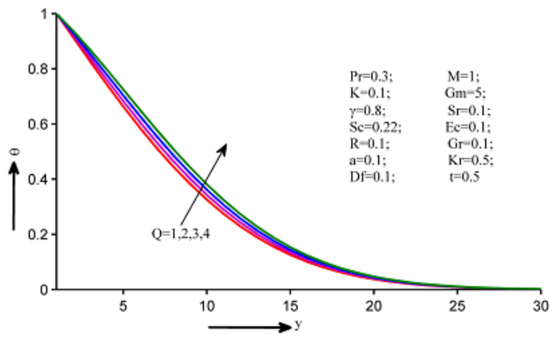

Figure 7. **Effect of heat source parameter on temperature**

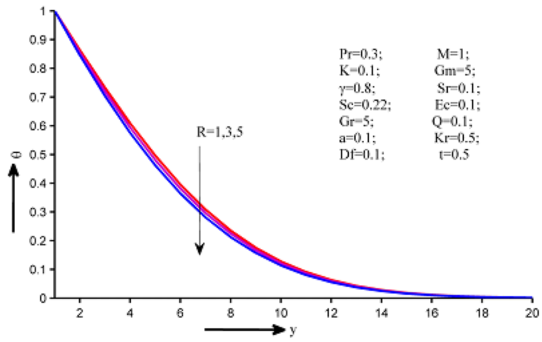

Figure 8. **Effect of radiation parameter on temperature**

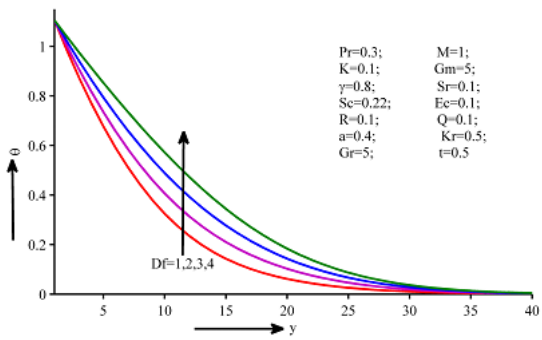

Figure 9. **Effect of Dufour number on temperature**

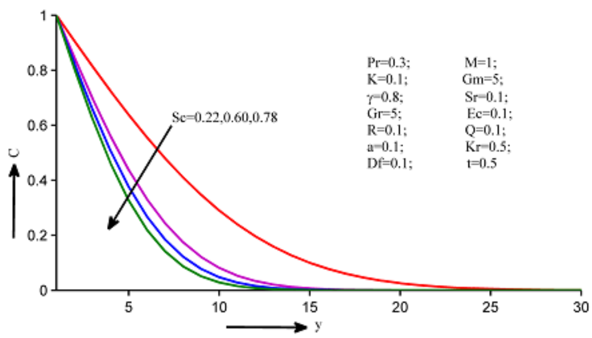

Figure 10. **Effect of Schmidt number on concentration**

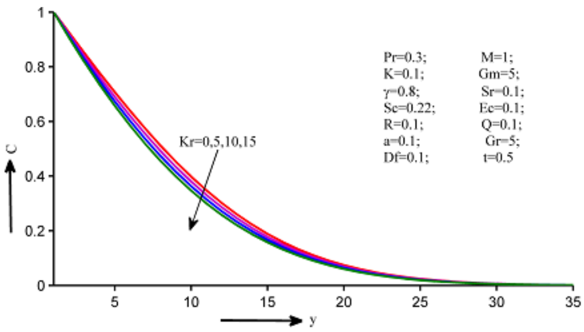

Figure 11. **Effect of chemical reaction parameter on concentration**

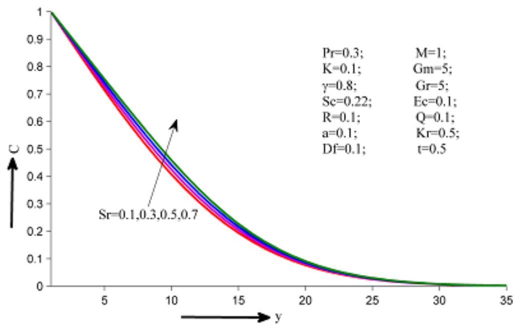

Figure 12. **Effect of soret number on concentration**

Gr	**Gm**	**M**	**K**	τ
5	5	3	0.8	0.0216
10	5	3	0.8	0.0165
15	5	3	0.8	-0.0142
20	5	3	0.8	-0.0524
5	10	3	0.8	0.0584
5	15	3	0.8	0.0326
5	20	3	0.8	-0.0324
5	5	0.5	0.8	0.0984
5	5	1	0.8	0.1428
5	5	2	0.8	0.1598
5	5	3	0.1	0.0624
5	5	3	0.3	0.0425
5	5	3	0.5	0.0261

Table 1. **Variations in skin friction for different values of flow parameters**

Pr	Q	R	χ	Df	**Nu**
0.71	0.8	0.8	0.1	0.5	5.5328
1	0.8	0.8	0.1	0.5	6.4127
3	0.8	0.8	0.1	0.5	9.5368
7.1	0.8	0.8	0.1	0.5	12.4683
0.71	0.1	0.8	0.1	0.5	5.3686
0.71	0.3	0.8	0.1	0.5	5.3491
0.71	0.5	0.8	0.1	0.5	5.3152
0.71	1	0.8	0.1	0.5	5.2984
0.71	0.8	0.5	0.1	0.5	5.4292
0.71	0.8	1	0.1	0.5	5.6134
0.71	0.8	2	0.1	0.5	5.8952

0.71	0.8	0.8	1	0.5	4.4231
0.71	0.8	0.8	2	0.5	3.4861
0.71	0.8	0.8	3	0.5	3.1625
0.71	0.8	0.8	0.1	1	4.8231
0.71	0.8	0.8	0.1	2	3.8963
0.71	0.8	0.8	0.1	3	3.4625
0.71	0.8	0.8	0.1	4	2.6256

Table 2. **Variations in Nusselt number**

Sc	**Kr**	**Sr**	**Sh**
0.22	0.8	0.1	3.8854
0.60	0.8	0.1	4.6416
0.78	0.8	0.1	4.9512
0.96	0.8	0.1	5.5869
0.22	0.1	0.1	3.1542
0.22	0.3	0.1	3.2813
0.22	0.5	0.1	3.3681
0.22	0.9	0.1	3.3895
0.22	0.8	1	2.2165
0.22	0.8	2	1.6871
0.22	0.8	3	0.8562
0.22	0.8	4	0.6586

Table 3. **Variations in Sherwood number**

5. Conclusion

In this manuscript a numerical analysis is carried out to investigate the radiation, Soret number and chemical reaction effects on MHD heat and mass transfer flow of a Casson fluid past an oscillating vertical porous plate with heat source and Dufour number. The non-dimensional governing equations are solved with the help of explicit finite difference method. The effects of various flow parameters on concentration, temperature and velocity profile are demonstrated through graphs. The effects of some flow parameter on Skin-friction, Nusselt number and Sherwood number are also presented in tables. The following are the conclusions of this manuscript.

(1). Velocity of the Casson fluid decreases with increasing values of M, γ whereas it increases with increasing values of Gr, Gm and K.

(2). Temperature of the Casson fluid increases with increasing values of R, Q and Df whereas reverse trend is seen in the case of R and Pr.

(3). Concentration of Casson fluid decreases with an increasing value of Sr and shows revere effect in the case of Sc and Kr.

(4). Skin friction increase with an increase in magnetic parameter while it decreases with an increase in Grashof number, modified Grashof number and permeability parameter.

(5). Nusselt number increase with increasing values of Prandtl number and radiation parameter while it decreases with increasing value of heat source and Dufour number.

(6). Sherwood number increase with increasing values of Schmidt number and chemical reaction parameter and it shows reverse trend in the case of Soret number.

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