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A New Technique in Stability of Infinite Delay Differential Equations With Impulsive Effects

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Abstract: In this work, we consider the stability of impulsive infinite delay differential equations. A new technique is derived to establish the stability criteria for impulsive infinite delay differential equations. By using Lyapunov functions and Razumikhin technique, some results are obtained which are more general than ones existing in literature. Lyapunov functionals are adopted and components of x are divided into several groups, correspondingly, several functions $V_j\left(t, x^{(j)}\right)$, $(j = 1, 2, \ldots, m)$ are employed. It is shown that impulses do contribute to yield stability properties even when the underlying system does not enjoy any stability behaviour. An example is also presented to illustrate the efficiency of the result obtained.

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1. Introduction

It is known that many biological phenomenon involving thresholds, bursting rhythm models in medicine and biology optimal control models in economics and frequency modulate system exhibit the impulse effect. Thus impulsive differential equations, that is, differential equations involving impulse effects, appear as a natural description of observed evolution phenomena for several real world problems. In recent years, qualitative properties of the mathematical theory of impulsive differential equations have been developed by large number of mathematicians ; see [1-10]. Systems with infinite delay deserve study because they describe a kind of system present in the real world. In [4], Lyapunov functionals are adopted and components of x are divided into several groups, correspondingly, several functions $V_j\left(t, x^{(j)}\right)$, $(j = 1, 2, \ldots, m)$ are employed. In that way, to construct the suitable function is rather easy and the imposed conditions ensuring the required stability are less restrictive. There are some results on systems with infinite delay.

In this work, we consider impulsive infinite delay differential equations. By using Lyapunov function and the Razumikhin technique; we get some results that are more general than the ones given in [5]. We extend the new technique developed in [4] to study impulsive systems. We give an example to show that this new technique is rather effective and especially applicable to system of impulsive infinite delay differential equations.

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2. Preliminaries

Consider the following, impulsive infinite delay differential equations

$$\left.\begin{aligned} x'(t) &= f(t, x(t), x(t-\tau(t))), \quad t \geq t_0, \quad t \neq t_k \\ \Delta x(t) &= x(t) - x\left(t^-\right) = I_k\left(x\left(t^-\right)\right), \quad t = t_k; \quad k = 1, 2, \ldots \end{aligned}\right\} \tag{1}$$

Where $t \in R^+$, $f \in C[R^+ \times R^n \times PC\left((-\infty, 0], R^n\right), R^n]$, $PC((-\infty, 0], R^n)$ denotes the space of piecewise right continuous functions $\emptyset : (-\infty, 0] \to R^n$ with the sup norm $\|\emptyset\| = \sup\limits_{-\infty < s \leq 0} |\emptyset(s)|$, $|.|$ is a norm in R^n, $f(t, 0, 0) \equiv 0$, $I_k(0) \equiv 0$, $t \geq \tau(t) \geq 0$, $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_k < \ldots$, $\tau_k \to \infty$ for $k \to \infty$, $x\left(t^+\right) = \lim\limits_{s \to t^+} x(s)$, and $x\left(t^-\right) = \lim\limits_{s \to t^-} x(s)$. The functions $I_k : R^n \to R^n$, $k = 1, 2, \ldots$, and such that if $\|x\| < H$ and $I_k(x) \neq 0$, then $\|x + I_k(x)\| < H$, where $H = \text{const.} > 0$. The initial condition for system (1) is given by

$$x_\sigma = \emptyset \tag{2}$$

Where $\emptyset \in PC((-\infty, 0], R^n)$. We assume that a solution for the initial value problem (1) and (2) does exist and is unique. Since $f(t, 0, 0) = 0$, then $x(t) = 0$ is a solution of (1), which is called zero solution. Let $PC(\rho) = \{\emptyset \in PC\left((-\infty, 0], R^n\right) : \|\emptyset\| < \rho\}$. For $\emptyset \in PC(\rho)$ we define

$$\|\emptyset\| = \|\emptyset\|^{(-\infty, t]} = \sup_{-\infty \leq s \leq t} |\emptyset(s)|$$

For convenience, we define $|x| = \max\limits_{1 \leq i \leq n} |x_i|$, for $x \in R^n$. We introduce some definitions as follows:

Definition 2.1. *The zero solution of (1) and (2) is said to be stable if for any $\sigma \geq t_0$ and $\epsilon > 0$, there is a $\delta = \delta(\sigma, \epsilon)$ such that $[\emptyset \in PC(\delta), \ t \geq \sigma]$ implies that $|x(t, \sigma, \emptyset)| \leq \epsilon$.*

Definition 2.2. *The zero solution of (1) and (2) is said to be uniformly stable if it is stable and δ is independent of σ.*

Definition 2.3. *A continuous function $W : R^+ \to R^+$ is called a wedge function if $W(0) = 0$ and $W(s)$ is strictly increasing.*

The following lemma (c.f [1]) is needed in proving the main result.

Lemma 2.4. *Let u be a continuous and bounded function. Then for any wedge functions W and W^*, any $h > 0$, and for each $\beta > 0$, there is a corresponding $\beta^* > 0$ such that $\int_{t-h}^{t} W(|u(s)|)\, ds \geq \beta$ implies $\int_{t-h}^{t} W^*(|u(s)|) ds \geq \beta^*$.*

In what follows, we will split $\emptyset = (\emptyset_1, \emptyset_2, \emptyset_3, \ldots, \emptyset_n)^T \in PC$ into several vectors, say, $\left({\emptyset_1}^{(1)}, {\emptyset_2}^{(1)}, \ldots, {\emptyset_{n_1}}^{(1)}\right)^T, \left({\emptyset_1}^{(2)}, {\emptyset_2}^{(2)}, \ldots, {\emptyset_{n_2}}^{(2)}\right)^T, \ldots, \left({\emptyset_1}^{(m)}, {\emptyset_2}^{(m)}, \ldots, {\emptyset_{n_m}}^{(m)}\right)^T$ such that $n_1 + n_2 + \cdots + n_m = n$ and

$$\left\{{\emptyset_1}^{(1)}, \ldots, {\emptyset_{n_1}}^{(1)}, {\emptyset_1}^{(2)}, \ldots, {\emptyset_{n_2}}^{(2)}, {\emptyset_1}^{(m)}, \ldots, {\emptyset_{n_m}}^{(m)}\right\} = \{\emptyset_1, \emptyset_2, \ldots\}$$

For convenience, we define

$$\emptyset^{(j)} = \left({\emptyset_1}^{(j)}, {\emptyset_2}^{(j)}, \ldots, {\emptyset_{n_j}}^{(j)}\right), \qquad j = 1, 2, \ldots, m$$

And $\emptyset = \left(\emptyset^{(1)}, \emptyset^{(2)}, \ldots, \emptyset^{(m)}\right)^T$. Note that the order of components in $\emptyset^{(j)}$ is not necessarily same as that in $\emptyset$.
For $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, we adopt the similar notation as for $\emptyset \in PC(\rho)$. Let $\left|\emptyset^{(j)}\right| = \max\limits_{1 \leq k \leq n_j} \left|{x_k}^{(j)}\right|$, $j = 1, 2, \ldots, m$ and thus

$$|x| = \max_{1 \leq j \leq m} \left|x^{(j)}\right|$$

Correspondingly $\left|\emptyset^{(j)}(s)\right| = \max_{1\le k\le n_j}\left|{x_k}^{(j)}(s)\right|$, $j = 1,2,\dots,m$ and

$$|\emptyset(s)| = .\max_{1\le j\le m}\left|\emptyset^{(j)}(s)\right|$$

Let

$$\left\|\emptyset^{(j)}\right\| = \left\|\emptyset^{(j)}\right\|^{(-\infty,t]} = \sup_{-\infty\le s\le t}\left|\emptyset^{(j)}(s)\right|, \quad j = 1,2,\dots,m$$

and denote

$$PC^{(j)}(t) = \left\{\emptyset^{(j)} : (-,t] \to R^{n_j} | \emptyset^{(j)} \text{ is continuous and bounded}\right\},$$

and

$$PC_\rho{}^{(j)}(t) = \left\{\emptyset^{(j)} \in PC^{(j)}(t) \mid \left\|\emptyset^{(j)}\right\| < \rho\right\}$$

3. Main Results

Theorem 3.1. *Let $\Phi_j : R^+ \to R^+$ be continuous, $\Phi_j \in L^1[0,\infty)$, $\Phi_j(t) \le K_j$ for $t \ge 0$ with some constant K_j $(j = 1,2,\dots,m)$ and $W_{ij}(i = 1,2,3,4,\ j = 1,2,\dots,m)$ be wedge functions. Suppose that there exist continuous Lyapunov functionals $V_j : [0,\infty) \times PC_H{}^{(j)}(t) \to R^+$ $(j = 1,2,\dots,m)$ such that*

(i). $W_{1j}(\left|\emptyset^{(j)}(t)\right| \le V_j(t,\emptyset^{(j)}(t)) \le W_{2j}(\left|\emptyset^{(j)}(t)\right| + W_{3j}\left[\int_{-\infty}^t \Phi_j(t-s) W_{4j}\left|\emptyset^{(j)}(s)\right| ds\right]$, $j = 1,2,\dots,m$.

(ii). When $V_j\left(t,x^{(j)}(t)\right) = \max\left\{V_l\left(t,x^{(l)}(t)\right.\middle|\ 1\le l\le m\right\}$, *there holds* $V_j'\left(t,x^{(j)}(t)\right) \le 0$ *if* $V_j(t-\tau(t), x^{(j)}(t-\tau(t))) \le V_j\left(t,x^{(j)}(t)\right)$.

(iii). $V_j\left(\tau_k, x\left({\tau_k}^-\right) + I_k\left(x\left({\tau_k}^-\right)\right)\right) \le (1+b_k)V_j\left({\tau_k}^-, x\left({\tau_k}^-\right)\right)$, $j = 1,2,\dots,m$, $k = 1,2,\dots$ *for which* $b_k \ge 0$ *and* $\sum_{k=1}^{\infty} b_k < \infty$.

Where $x(t) = (x^{(1)}(t), x^{(2)}(t), \dots\dots\dots, x^{(m)}(t))$ *is a solution of* (1) *and* (2) *then the zero solution of* (1) *and* (2) *is uniformly stable.*

Proof. Since $b_k \ge 0$ and $\sum_{k=1}^{\infty} b_k < \infty$, it follows that $\prod_{k=1}^{\infty}(1+b_k) = M$ and $1 \le M \le \infty$. Define a function $V(t)$ as follows: $V(t) = V_k\left(t,x^{(k)}(t)\right)$ if

$$V_k\left(t,x^{(k)}(t)\right) = \max\left\{V_j\left(t,x^{(j)}(t)\right.\middle|\ 1\le j\le m\right\} \tag{3}$$

Obviously $V(t)$ is continuous for all $t \ge \alpha$. In the following, we denote, for the sake of brevity

$$V_j(t) = V_j\left(t,x^{(j)}(t)\right), \quad V_j'(t) = V_j'\left(t,x^{(j)}(t)\right), \quad j = 1,2,\dots,m$$

First we prove that for all $t \in R^+$,

$$\begin{aligned}\frac{\left[\sum_{j=1}^{m} W_{1j}\left(\left|x^{(j)}(t)\right|\right)\right]}{m} &\le V(t)\\ &\le \sum_{j=1}^{m} W_{2j}\left(\left|x^{(j)}(t)\right|\right) + \sum_{j=1}^{m} W_{3j}\left[\int_{-\infty}^{t}\Phi_j(t-s)W_{4j}\left(\left|x^{(j)}(s)\right|\right)ds\right]\end{aligned} \tag{4}$$

Now by (3) and condition (i),

$$V(t) = V_j(t) \ge \frac{\left[W_{11}\left(\left|x^{(1)}(t)\right|\right) + W_{12}\left(\left|x^{(2)}(t)\right|\right) + \cdots + W_{1j}\left(\left|x^{(j)}(t)\right|\right)\right]}{m}$$

On the other hand, the right hand inequality in (4) obviously holds. Now we show that for each $t \geq t_0$, the right hand and the left hand derivatives of $V(t)$, both denoted by $V'(t)$, satisfy $V'(t) \leq 0$ if

$$V(t - \tau(t)) \leq V(t) \tag{5}$$

Let $s_1 > s_0$. By (3), if

$$V_k\left(t, x^{(k)}(t)\right) = \max\left\{V_j\left(t, x^{(j)}(t) \;\middle|\; 1 \leq j \leq m\right\}\right.$$

Then $V(t) = V_j(t)$ for $t \in [s_0, s_1]$. Therefore $V(t - \tau(t)) = V_j(t - \tau(t))$ implies $V(t - \tau(t)) \leq V(t)$. Thus $V'(t) = V_j'(t) \leq 0$ if $V(t - \tau(t)) \leq V(t)$. Now we are in a position to show the U.S of the zero solution of (1) and (2). For any $\epsilon > 0$ ($\epsilon < H$), let

$$M\epsilon^* = \min\{W_{11}(\epsilon), W_{12}(\epsilon), \ldots, W_{1n}(\epsilon)\}$$

We may choose a $\delta(\epsilon) > 0$ such that $\delta < \epsilon$, $W_{2j}(\delta) < \frac{\epsilon^*}{8}$ and $W_{3j}(J_j W_{4j}(\delta)) < \frac{\epsilon^*}{8}$, $j = 1, 2, \ldots, m$, where

$$J_j = \int_0^\infty \Phi_j(s)\, ds \quad (j = 1, 2, \ldots, m)$$

For any $\sigma \geq t_0$, $\emptyset \in PC(\delta)$, $\sigma \in [\tau_{l-1}, \tau_l)$ for some positive integer l, define $x(t) = x(t, \sigma, \emptyset)$ then by (4) we have

$$V(t, x(t)) = V(t, \emptyset(t - \sigma)) \leq \sum_{j=1}^{m} W_{2j}(\delta) + \sum_{j=1}^{m} W_{3j}(J_j W_{4j}(\delta)) < \frac{\epsilon^*}{2} \quad for \quad t \in [0, \sigma]$$

We prove that

$$\sum_{j=1}^{m} \frac{W_{ij}(\left|x^{(j)}(t)\right|}{m} \leq V(t) \leq \frac{\epsilon^*}{2} \quad for \quad \sigma \leq t \leq \tau_l \tag{6}$$

If this does not hold, then there is a $\hat{t} \in (\sigma, \tau_l)$ such that $V(\hat{t}) > \frac{\epsilon^*}{2}$ and $V'(\hat{t}) > 0$, $V(t) \leq V(\hat{t})$ for $t \in [\sigma, \hat{t}]$. Since $t \geq \tau(t) \geq 0$, we have $V(\hat{t} - \tau(\hat{t})) \leq V(\hat{t})$. From (5) we have

$$V'(\hat{t}) \leq 0.$$

This is a contradiction. So (6) holds. From inequality (6) and condition (iii) we have

$$\begin{aligned} V(\tau_l) &= V\left(\tau_l, x\left(\tau_l^-\right) + I_k\left(x\left(\tau_l^-\right)\right)\right) \\ &\leq (1 + b_l)\, V\left(\tau_l^-, x\left(\tau_l^-\right)\right) \leq (1 + b_l)\, \frac{\epsilon^*}{2} \end{aligned}$$

Thus

$$V(\tau_l) \leq (1 + b_l)\, \frac{\epsilon^*}{2}.$$

Next we prove that

$$V(t) \leq (1 + b_l)\, \frac{\epsilon^*}{2} \quad for \quad \tau_l \leq t < \tau_{l+1} \tag{7}$$

If inequality (7) does not hold, then there is a $\hat{s} \in (\tau_l, \tau_{l+1})$ such that $V(\hat{s}) > (1 + b_l)\frac{\epsilon^*}{2}$ and $V'(\hat{s}) > 0$, $V(t) \leq V(\hat{s})$ for $t \in [\tau_l, \hat{s}]$. Since $t \geq \tau(t) \geq 0$, we have $V(\hat{s} - \tau(\hat{s})) \leq V(\hat{s})$. From (5) we have

$$V'(\hat{s}) \leq 0.$$

This is a contradiction. So (7) holds. From inequality (7) and condition (iii) we have

$$\begin{aligned} V(\tau_{l+1}) &= V\left(\tau_{l+1}, x\left(\tau_{l+1}{}^{-}\right) + I_k\left(x\left(\tau_{l+1}{}^{-}\right)\right)\right) \\ &\le (1+b_{l+1})\, V\left(\tau_{l+1}{}^{-}, x\left(\tau_{l+1}{}^{-}\right)\right) \\ &\le (1+b_{l+1})(1+b_l)\frac{\epsilon^*}{2} \end{aligned}$$

Thus

$$V(\tau_{l+1}) \le (1+b_{l+1})(1+b_l)\frac{\epsilon^*}{2}$$

Thus by simple induction, we can prove that, in general $V(t) \le (1+b_{l+i+1})\dots(1+b_l)\frac{\epsilon^*}{2}$ for $\tau_{l+i} \le t \le \tau_{l+i+1}$. Taking this together with (4) and (6) and $\prod_{k=1}^{\infty}(1+b_k) = M$, we have

$$\sum_{j=1}^{m} \frac{W_{ij}(\left|x^{(j)}(t)\right|}{m} \le V(t) \le M\frac{\epsilon^*}{2} \quad for \quad t \ge \sigma \tag{8}$$

Since

$$M\epsilon^* = min\left\{W_{11}(\epsilon), W_{12}(\epsilon), \dots, W_{1m}(\epsilon)\right\},$$

we have

$$W_{11}\left(\left|x^{(1)}(t)\right|\right) \le W_{11}(\epsilon), \quad W_{12}\left(\left|x^{(2)}(t)\right|\right) \le W_{12}(\epsilon), \dots, W_{1m}\left(\left|x^{(m)}(t)\right|\right) \le W_{1m}(\epsilon).$$

Therefore,

$$|x(t)| = \max\{\left(\left|x^{(1)}(t)\right|\right), \left(\left|x^{(2)}(t)\right|, \dots, \left|x^{(m)}(t)\right|\right)\} \le \epsilon$$

Therefore, the zero solution of (1) and (2) is Uniformly stable. □

Corollary 3.2. *Suppose that there exist continuous lyapunov functions* $V_j : (-\infty,\infty) \times B_H{}^{(j)} \to R^+$ *with* $B_H{}^{(j)} = \left\{x^{(j)} \in R^{n(j)} \,\middle|\, \left|x^{(j)}\right| < H\right\}$ $(j = 1,2,\dots,m)$ *and wedge functions* W_{ij} $(i = 1,2,\ j = 1,2,\dots..,m)$ *such that*

(i). $W_{1j}\left|\emptyset^{(j)}(t)\right| \le V_j\left(t, x^{(j)}(t)\right) \le W_{2j}\left|\emptyset^{(j)}(t)\right|$,

(ii). When $V_j\left(t, x^{(j)}(t)\right) = \max\left\{V_l\left(t, x^{(l)}(t)\right) \middle| 1 \le l \le m\right\}$ *there holds* $V_j'\left(t, x^{(j)}(t)\right) \le 0$ *if* $V_j\left(t-\tau(t), x^{(j)}(t-\tau(t))\right) \le V_j\left(t, x^{(j)}(t)\right)$;

(iii). $V_j\left(\tau_k, x\left(\tau_k{}^{-}\right) + I_k\left(x\left(\tau_k{}^{-}\right)\right)\right) \le (1+b_k)V_j\left(\tau_k{}^{-}, x\left(\tau_k{}^{-}\right)\right)$, $j = 1,2,\dots,m$, $k = 1,2,\dots$ *in which* $b_k \ge 0$ *and* $\sum_{k=1}^{\infty} b_k < \infty$,

where $x(t) = (x^{(1)}(t), x^{(2)}(t), \dots, x^{(m)}(t)$ *is a solution of* (1) *and* (2), *then the zero solution of* (1) *and* (2) *is uniformly stable.*

For simplicity, we establish example for $m = 2$.

Example 3.3. *Consider the following impulsive infinite delay differential equations.*

$$\left.\begin{aligned} x_1'(t) &= -a_1x_1(t) + a_2x_2(t) + a_3x_1(t-\tau(t)), \quad t \ge t_0,\ t \ne \tau_k, \quad x_1(\tau_k) = cx_1(\tau_k{}^{-}) \\ x_2'(t) &= b_1x_1(t) - b_2x_2(t) + b_3x_2(t-\tau(t)), \quad t \ge t_0,\ t \ne \tau_k, \quad x_2(\tau_k) = cx_2(\tau_k{}^{-}) \end{aligned}\right\} \tag{9}$$

Where $k = 1,2,\dots,$ $t \ge \tau(t) \ge 0$, $a_1, a_2, a_3, b_1, b_2, b_3 > 0$, $0 \le c < 1$, $a_2 + a_3 \le a_1$, $b_1 + b_3 \le b_2$ *and* $x_j(0) = 0$, $j = 1,2$. *Let* $V_j(t, x_j(t)) = \frac{1}{2}[x_j(t)]^2$ $(j = 1,2)$. *Obviously condition (i) of Theorem 3.1 holds, and moreover when*

$V_1(t, x_1(t)) \geq V_2(t, x_2(t))$ i.e. $|x_1(t)| \geq |x_2(t)|$. If $V_1((t-\tau(t)), x_1(t-\tau(t))) \leq V_1(t, x_1(t)$ i.e. $|x_1(t-\tau(t)) \leq |x_1(t)||$, we have

$$\begin{aligned} V_1'(t, x_1(t)) &= x_1(t) x_1'(t) \\ &= -a_1 {x_1}^2(t) + a_2 x_1(t) x_2(t) + a_3 x_1(t) x_1(t-\tau(t)) \\ &\leq (-a_1 + a_2 + a_3) {x_1}^2(t) \\ &\leq 0 \end{aligned}$$

When $V_1(t, x_1(t)) \leq V_2(t, x_2(t))$ i.e., $|x_1(t)| \leq |x_2(t)|$. If $V_1((t-\tau(t)), x_2(t-\tau(t))) \leq V_2(t, x_2(t)$ i.e. $|x_2(t-\tau(t)) \leq |x_2(t)||$, we have

$$\begin{aligned} V_2'(t, x_2(t)) &= x_2(t) x_2'(t) \\ &= b_1 x_1(t) x_2(t) - b_2 {x_2}^2(t) + b_3 x_2(t) x_2(t-\tau(t)) \\ &\leq (b_1 - b_2 + b_3) {x_2}^2(t) \\ &\leq 0 \end{aligned}$$

and

$$\begin{aligned} V_j\left(x_j\left({\tau_k}^-\right) + I_k(x_j\left({\tau_k}^-\right))\right) &= V_j\left(c x_j\left({\tau_k}^-\right)\right) \\ &= \frac{1}{2} c^2 {x_j}^2\left({\tau_k}^-\right) < \frac{1}{2} {x_j}^2\left({\tau_k}^-\right) \\ &= V_j\left({x_j}^2\left({\tau_k}^-\right)\right), \quad j = 1, 2 \end{aligned}$$

Let $b_k = 0$, $k = 1, 2, \ldots$. Then conditions (ii) and (iii) of Theorem 3.1 hold. Therefore the zero solution of (9) is uniformly stable. Since in this example $\tau(t)$ may be ∞, by the previous theory we cannot obtain this stability result.

Remark 3.4. *It is easy to see that if we employ the usual Razumikhin techniques, that is, to put two variables x_1, x_2 in one Lyapunov function, then the arguments to get the desired stability conclusions (if no possible) would be much more complicated and the imposed conditions would be more restrictive.*

Remark 3.5. *Trivially, the arguments used in the above example are also applicable to systems involving more equations as well as we present in the theorem. Hence, the obtained results are quite flexible and effective especially for systems of infinite delay equations.*

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