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Some Multiplicative Neighborhood Dakhayani Indices of Certain Nanostructures

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Abstract: A topological index is a numerical parameter mathematically derived from the graph structure. Connectivity indices are applied to measure the chemical characteristics of chemical compounds in Chemistry. In this paper, we introduce the multiplicative neighborhood Dakshayani indices, multiplicative hyper neighborhood Dakshayani indices, multiplicative sum connectivity neighborhood Dakshayani index, multiplicative product connectivity neighborhood Dakshayani index, general first and second multiplicative neighborhood Dakshayani indices of a molecular graph and determine exact formulas for these indices of line graphs of subdivision graphs of 2-D lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$.

MSC: 05C05, 05C07, 05C12, 05C35.

Keywords: Multiplicative neighborhood Dakshayani indices, multiplicative sum connectivity neighborhood Dakshayani index, multiplicative product connectivity neighborhood Dakshayani index, nanostructure.

1. Introduction

A molecular graph is a graph such that its vertices correspond to the atoms and edges to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of chemical Sciences, Medical Sciences. In Mathematical Chemistry, several topological indices or graph indices have found some applications especially in chemical documentation, isomer discrimination, QSAR/QSPR study [1, 2]. In this paper, we consider only finite, simple, connected graphs. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The set of all vertices which adjacent to v is called open neighborhood of v and denoted by $N_G(v)$. The closed neighborhood of v is the set $N_G[v] = N_G(v) \cup \{v\}$. The set $N_G[v]$ is the set of closed neighborhood vertices of v. Let $D_G(v) = d_G(v) + \sum_{u \in N_G(v)} d_G(v)$ be the degree sum of closed neighborhood vertices of v. We refer [3] for undefined term and notation. In [4], Kulli introduced the first and second neighborhood Dakshayani indices, defined as

$$ND_1(G) = \sum_{uv \in E(G)} [D_G(u) + D_G(v)], \qquad ND_2(G) = \sum_{uv \in E(G)} D_G(u) D_G(v).$$

The first and second hyper neighborhood Dakshayani indices were proposed by Kulli in [4], defined as

$$HND_1(G) = \sum_{uv \in E(G)} [D_G(u) + D_G(v)]^2, \qquad HND_2(G) = \sum_{uv \in E(G)} [D_G(u) D_G(v)]^2.$$

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Recently, some variants of neighborhood Dakshayani indices were introduced and studied such as F-neighborhood Dakshayani index [5], square neighborhood Dakshayani index [6], sum and product connectivity neighborhood Dakshayani indices [7]. We introduce the first and second multiplicative neighborhood Dakshayani indices, defined as

$$ND_1II(G) = \prod_{uv \in E(G)} \left[D_G(u) + D_G(v)\right], \qquad ND_2II(G) = \prod_{uv \in E(G)} \left[D_G(u) D_G(v)\right].$$

We now propose the first and second multiplicative hyper neighborhood Dakshayani indices of a graph, defined as

$$HND_1II(G) = \prod_{uv \in E(G)} \left[D_G(u) + D_G(v)\right]^2, \qquad HND_2II(G) = \prod_{uv \in E(G)} \left[D_G(u) D_G(v)\right]^2.$$

We introduce the multiplicative sum connectivity neighborhood Dakshayani index of a graph G, defined as

$$SNDII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{D_G(u) + D_G(v)}}.$$

We propose the multiplicative product connectivity neighborhood Dakshayani index of a graph G, defined as

$$PNDII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{D_G(u) D_G(v)}}.$$

We define the reciprocal multiplicative neighborhood Dakshayani index of a graph G as

$$RNDII(G) = \prod_{uv \in E(G)} \sqrt{D_G(u) D_G(v)}.$$

We continue this generalization and define the general first and second multiplicative neighborhood Dakshayani indices of a graph G as

$$ND_1^aII(G) = \prod_{uv \in E(G)} \left[D_G(u) + D_G(v)\right]^a, \tag{1}$$

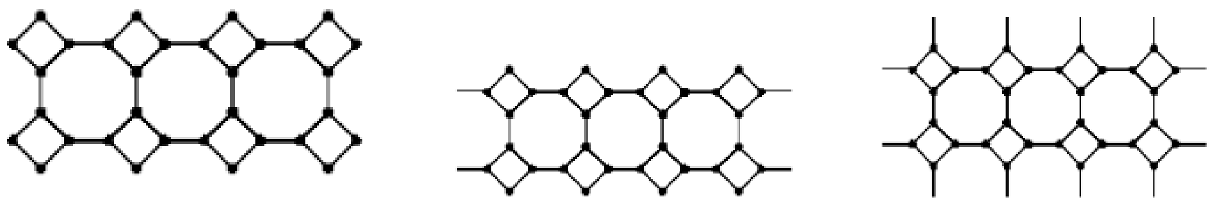
$$ND_2^aII(G) = \prod_{uv \in E(G)} \left[D_G(u) D_G(v)\right]^a. \tag{2}$$

where a is a real number. Recently, some now multiplicative indices were studied in [8–17]. In this paper, some multiplicative neighborhood Dakshayani indices of certain nanostructures were computed. We need the following definitions and results. The subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2. The line graph $L(G)$ of G is the graph whose vertex set is $E(G)$ and two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent.

Lemma 1.1. *Let G be a graph with p vertices of q edges. Then $S(G)$ has $p+q$ vertices and $2q$ edges.*

Lemma 1.2. *Let G be a (p,q) graph. Then $L(G)$ has q vertices and $\frac{1}{2}\sum_{i=1}^{p} d_G(u_i)^2 - q$ edges.*

We consider the graph of 2-D lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$, where p is the number of squares in a row and q is the number of rows of squares. These graphs are shown in Figure 1.

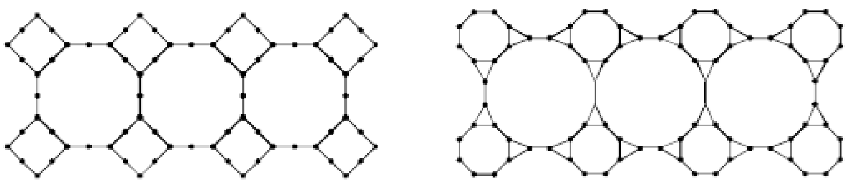


(a) 2D-lattice of $TUC_4C_8[4,2]$ (b) $TUC_4C_8[4,2]$ nanotube (c) $TUC_4C_8[4,2]$ nanotorus

Figure 1:

2. Results for 2D-Lattice of $TUC_4C_8[p,q]$

The line graph of the subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$ is depicted in Figure 2(b).



(a) Subdivision graph of 2D-lattice of $TUC_4C_8[4,2]$ (b) Line graph of the subdivision graph of $TUC_4C_8[4,2]$

Figure 2:

The 2D-lattice of $TUC_4C_8[p,q]$ is a graph with $4pq$ vertices and $6pq-p-q$ edges. By Lemma 2.1, the subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$ is a graph with $10pq-p-q$ vertices and $2(6pq-p-q)$ edges. Thus by Lemma 2.2, G has $2(6pq-p-q)$ vertices and $18pq-5p-5q$ edges, where G is the line graph of subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$. Clearly the vertices of G are either of degree 2 or 3, see Figure 2(b). Thus the edge partition of G based on the degree sum of closed neighborhood vertices of each vertex is obtained as given in Table 1 and Table 2.

$D_G(u), D_G(v) \setminus uv \in E(G)$	(6, 6)	(6,7)	(7, 7)	(7, 11)	(11, 12)	(12, 12)
Number of edges	4	8	$2(p+q-4)$	$4(p+q-2)$	$8(p+q-2)$	$2(9pq+10)-19(p+q)$

Table 1: Edge partition of G with $p>1$, $q>1$

$D_G(u), D_G(v) \setminus uv \in E(G)$	(6, 6)	(6,7)	(7, 7)	(7, 11)	(11, 11)	(11, 12)	(12, 12)
Number of edges	6	4	$2(p-2)$	$4(p-1)$	$2(p-1)$	$4(p-1)$	$(p-1)$

Table 2: Edge partition of G with $p>1$ and $q=1$

Theorem 2.1. *Let G be the line graph of subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$. Then*

$$ND_1^aII(G) = 12^{4a} \times 13^{8a} \times 14^{2a(p+q-4)} \times 18^{4a(p+q-2)} \times 23^{8a(p+q-2)} \times 24^{a[2(9pq+10)-19(p+q)]}, \quad if \quad p>1, q>1 \qquad (3)$$

$$= 12^{6a} \times 13^{4a} \times 14^{2a(p-2)} \times 18^{4a(p-1)} \times 22^{2a(p-1)} \times 23^{4a(p-1)} \times 24^{a(p-1)}, \qquad if \quad p>1, q=1 \qquad (4)$$

Proof. **Case 1:** Suppose $p>1$, $q>1$. From equation (1) and by using Table 1, we deduce

$$ND_1^a II(G) = \prod_{uv\in E(G)} [D_G(u)+D_G(v)]^a$$
$$= (6+6)^{4a} \times (6+7)^{8a} \times (7+7)^{2(p+q-4)a} \times (7+11)^{4(p+q-2)a} \times (11+12)^{8(p+q-2)a} \times (12+12)^{[2(9pq+10)-19(p+q)]a}$$
$$= 12^{4a} \times 13^{8a} \times 14^{2a(p+q-4)} \times 18^{4a(p+q-2)} \times 23^{8a(p+q-2)} \times 24^{a[2(9pq+10)-19(p+q)]}$$

Case 2: Suppose $p>1$, $q=1$. From equation (1) and by using Table 2, we derive

$$ND_1^a II(G) = \prod_{uv\in E(G)} [D_G(u)+D_G(v)]^a$$
$$= (6+6)^{6a} \times (6+7)^{4a} \times (7+7)^{2(p-2)a} \times (7+11)^{4(p-1)a} \times (11+11)^{2(p-1)a} \times (11+12)^{4(p-1)a} \times (12+12)^{(p-1)a}$$
$$= 12^{6a} \times 13^{4a} \times 14^{2a(p-2)} \times 18^{4a(p-1)} \times 22^{2a(p-1)} \times 23^{4a(p-1)} \times 24^{a(p-1)}.$$

□

We obtain the following results by using Theorem 2.1.

Corollary 2.2. *Let G be the line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$. Suppose $p>1$, $q>1$. Then*

(1). $ND_1II(G) = 12^4 \times 13^8 \times 14^{2(p+q-4)} \times 18^{4(p+q-2)} \times 23^{8(p+q-2)} \times 24^{2(9pq+10)-19(p+q)}$.

(2). $HND_1II(G) = 12^8 \times 13^{16} \times 14^{4(p+q-4)} \times 18^{8(p+q-2)} \times 23^{16(p+q-2)} \times 24^{2[2(9pq+10)-19(p+q)]}$.

(3). $SNDII(G) = \left(\frac{1}{\sqrt{12}}\right)^4 \times \left(\frac{1}{\sqrt{13}}\right)^8 \times \left(\frac{1}{\sqrt{14}}\right)^{2(p+q-4)} \times \left(\frac{1}{\sqrt{18}}\right)^{4(p+q-2)} \times \left(\frac{1}{\sqrt{23}}\right)^{8(p+q-2)} \times \left(\frac{1}{\sqrt{24}}\right)^{2(9pq+10)-19(p+q)}$.

Proof. Put $a = 1, 2, -\frac{1}{2}$ in equation (3), we get the desired results. □

Corollary 2.3. *Let G be the line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$. Suppose $p>1$, $q=1$. Then*

(1). $ND_1II(G) = 12^6 \times 13^4 \times 14^{2(p-2)} \times 18^{4(p-1)} \times 22^{2(p-1)} \times 23^{4(p-1)} \times 24^{(p-1)}$.

(2). $HND_1II(G) = 12^{12} \times 13^8 \times 14^{4(p-2)} \times 18^{8(p-1)} \times 22^{4(p-1)} \times 23^{8(p-1)} \times 24^{2(p-1)}$.

(3). $SNDII(G) = \left(\frac{1}{\sqrt{12}}\right)^6 \times \left(\frac{1}{\sqrt{13}}\right)^4 \times \left(\frac{1}{\sqrt{14}}\right)^{2(p-2)} \times \left(\frac{1}{\sqrt{18}}\right)^{4(p-1)} \times \left(\frac{1}{\sqrt{22}}\right)^{2(p-1)} \times \left(\frac{1}{\sqrt{23}}\right)^{4(p-1)} \times \left(\frac{1}{\sqrt{24}}\right)^{p-1}$.

Proof. Put $a = 1, 2, -\frac{1}{2}$ in equation (4), we obtain the desired results. □

Theorem 2.4. *Let G be the line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$. Then*

$$ND_2^a II(G) = 36^{4a} \times 42^{8a} \times 49^{2(p+q-4)a} \times 77^{4(p+q-2)a} \times 132^{8(p+q-2)a} \times 144^{[2(9pq+10)-19(p+q)]a} \quad if \quad p>1, q>1 \qquad (5)$$
$$= 36^{6a} \times 42^{4a} \times 49^{2(p-2)a} \times 77^{4(p-1)a} \times 121^{2(p-1)a} \times 132^{4(p-1)a} \times 144^{(p-1)a}, \qquad if \quad p>1, q=1 \qquad (6)$$

Proof. **Case 1:** Suppose $p>1$, $q>1$. By using equation (2) and Table 1, we obtain

$$ND_2^a II(G) = \prod_{uv\in E(G)} [D_G(u)\,D_G(v)]^a$$
$$= (6\times 6)^{4a} \times (6\times 7)^{8a} \times (7\times 7)^{2(p+q-4)a} \times (7\times 11)^{4(p+q-2)a} \times (11\times 12)^{8(p+q-2)a} \times (12\times 12)^{[2(9pq+10)-19(p+q)]a}$$

$$= 36^{4a} \times 42^{8a} \times 49^{2(p+q-4)} \times 77^{4(p+q-2)a} \times 132^{8(p+q-2)a} \times 144^{[2(9pq+10)-19(p+q)]a}$$

Case 2: Suppose $p > 1$, $q = 1$. From equation (2) and by using Table 2, we deduce

$$ND_2^a II(G) = \prod_{uv \in E(G)} [D_G(u) D_G(v)]^a$$

$$= (6 \times 6)^{6a} \times (6 \times 7)^{4a} \times (7 \times 7)^{2(p-2)a} \times (7 \times 11)^{4(p-1)a} \times (11 \times 11)^{2(p-1)a} \times (11 \times 12)^{4(p-1)a} \times (12 \times 12)^{(p-1)a}$$

$$= 36^{6a} \times 42^{4a} \times 49^{2(p-2)a} \times 77^{4(p-1)a} \times 121^{2(p-1)a} \times 132^{4(p-1)a} \times 144^{(p-1)a}.$$

□

We establish the following results by Theorem 2.4.

Corollary 2.5. *Let G be the line graph of subdivision graph of 2D-lattice of $TUC_4C_8[p, q]$. Suppose $p > 1$, $q > 1$. Then*

(1). $ND_2II(G) = 36^4 \times 42^8 \times 49^{2(p+q-4)} \times 77^{4(p+q-2)} \times 132^{8(p+q-2)} \times 144^{2(9pq+10)-19(p+q)}$.

(2). $HND_2II(G) = 36^8 \times 42^{16} \times 49^{4(p+q-4)} \times 77^{8(p+q-2)} \times 132^{16(p+q-2)} \times 144^{2[2(9pq+10)-19(p+q)]}$.

(3). $PNDII(G) = \left(\frac{1}{6}\right)^4 \times \left(\frac{1}{\sqrt{42}}\right)^8 \times \left(\frac{1}{\sqrt{7}}\right)^{2(p+q-4)} \times \left(\frac{1}{\sqrt{77}}\right)^{4(p+q-2)} \times \left(\frac{1}{\sqrt{132}}\right)^{8(p+q-2)} \times \left(\frac{1}{12}\right)^{2(9pq+10)-19(p+q)}$.

(4). $RNDII(G) = 6^4 \times 42^4 \times 7^{2(p+q-4)} \times 77^{2(p+q-2)} \times 132^{4(p+q-2)} \times 12^{2(9pq+10)-19(p+q)}$.

Proof. Put $a = 1, 2, -\frac{1}{2}, \frac{1}{2}$ in equation (5), we get the desired results. □

Corollary 2.6. *Let G be the line graph of subdivision graph of 2D-lattice of $TUC_4C_8[p, q]$. Suppose $p > 1$, $q = 1$. Then*

(1). $ND_2II(G) = 36^6 \times 42^4 \times 49^{2(p-2)} \times 77^{4(p-1)} \times 121^{2(p-1)} \times 132^{4(p-1)} \times 144^{p-1}$.

(2). $HND_2II(G) = 36^{12} \times 42^8 \times 49^{4(p-2)} \times 77^{8(p-1)} \times 121^{4(p-1)} \times 132^{8(p-1)} \times 144^{2(p-1)}$.

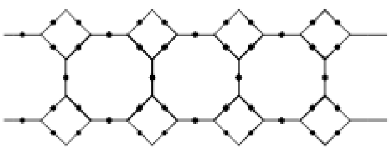
(3). $PNDII(G) = \left(\frac{1}{6}\right)^6 \times \left(\frac{1}{42}\right)^2 \times \left(\frac{1}{7}\right)^{2(p-2)} \times \left(\frac{1}{77}\right)^{2(p-1)} \times \left(\frac{1}{121}\right)^{p-1} \times \left(\frac{1}{132}\right)^{2(p-1)} \times \left(\frac{1}{12}\right)^{p-1}$.

(4). $RNDII(G) = 6^6 \times 42^2 \times 7^{2(p-2)} \times 77^{2(p-1)} \times 121^{p-1} \times 132^{2(p-1)} \times 12^{p-1}$.

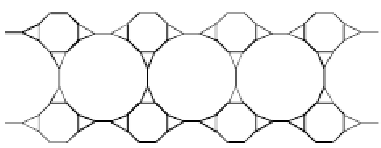
Proof. Put $a = 1, 2, -\frac{1}{2}, \frac{1}{2}$ in equation (6), we obtain the desired results. □

3. Results for $TUC_4C_8[p, q]$ Nanotube

The line graph of subdivision graph of $TUC_4C_8[4, 2]$ nanotube is shown in Figure 3(b).



(a) Subdivision graph of $TUC_4C_8[4, 2]$ nanotube



(b) Line graph of subdivision graph of $TUC_4C_8[4, 2]$ nanotube

Figure 3:

Let H be the line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotube. A graph of $TUC_4C_8[p, q]$ nanotube is a graph with $4pq$ vertices and $6pq - p$ edges. By Lemma 1.1, the subdivision graph of $TUC_4C_8[p, q]$ nanotube is a graph with $10pq - p$

vertices and $12pq - 2p$ edges. Thus by Lemma 1.2, H has $12pq - 2p$ vertices and $18pq - 5p$ edges. The vertices of H are either of degree 1 or 3, see Figure 3(b). Hence the edge partition of H based on the degree sum of closed neighborhood vertices of each vertex is obtained as given in Table 3 and Table 4.

$D_H(u), D_H(v) \setminus uv \in E(H)$	(7,7)	(7, 11)	(11, 12)	(12, 12)
Number of edges	$2p$	$4p$	$8p$	$18pq - 19p$

Table 3: Edge partition of H when $p > 1$, $q > 1$

$D_H(u), D_H(v) \setminus uv \in E(H)$	(7,7)	(7, 11)	(11, 11)	(11, 12)	(12, 12)
Number of edges	$2p$	$4p$	$2p$	$4p$	p

Table 4: Edge partition of H when $p > 1$, $q = 1$

Theorem 3.1. *Let H be the line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotube. Then*

$$ND_1^a II\,(H) = 14^{2ap} \times 18^{4ap} \times 23^{8ap} \times 24^{a(18pq-19p)}, \quad if \quad p > 1, q > 1 \tag{7}$$

$$= 14^{2ap} \times 18^{4ap} \times 22^{2ap} \times 23^{4ap} \times 24^{ap}, \quad if \quad p > 1, q = 1 \tag{8}$$

Proof. **Case 1:** Suppose $p > 1$, $q > 1$. By using equation (1) and Table 3, we obtain

$$\begin{aligned} ND_1^a II\,(H) &= \prod_{uv \in E(H)} [D_H(u) + D_H(v)]^a \\ &= (7+7)^{2pa} \times (7+11)^{4pa} \times (11+12)^{8pa} \times (12+12)^{(18pq-19p)a} \\ &= 14^{2ap} \times 18^{4ap} \times 23^{8ap} \times 24^{a(18pq-19p)}. \end{aligned}$$

Case 2: Suppose $p > 1$, $q = 1$. From equation (1) and Table 4, we deduce

$$\begin{aligned} ND_1^a II\,(H) &= \prod_{uv \in E(H)} [D_H(u) + D_H(v)]^a \\ &= (7+7)^{2pa} \times (7+11)^{4pa} \times (11+11)^{2pa} \times (11+12)^{4pa} \times (12+12)^{pa} \\ &= 14^{2ap} \times 18^{4ap} \times 22^{2ap} \times 23^{4ap} \times 24^{ap}. \end{aligned}$$

□

We obtain the following results by Theorem 3.1.

Corollary 3.2. *Let H be the line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotube. Suppose $p > 1$, $q > 1$. Then*

(1). $ND_1II\,(H) = 14^{2p} \times 18^{4p} \times 23^{8p} \times 24^{18pq-19p}$.

(2). $HND_1II\,(H) = 14^{4p} \times 18^{8p} \times 23^{16p} \times 24^{2(18pq-19p)}$.

(3). $SNDII\,(H) = \left(\frac{1}{\sqrt{14}}\right)^{2p} \times \left(\frac{1}{\sqrt{18}}\right)^{4p} \times \left(\frac{1}{\sqrt{23}}\right)^{8p} \times \left(\frac{1}{\sqrt{24}}\right)^{18pq-19p}$.

Proof. Put $a = 1, 2, -\frac{1}{2}$, in equation (7), we obtain the desired results. □

Corollary 3.3. *Let H be the line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Suppose $p>1$, $q=1$. Then*

(1). $ND_1II(H) = 14^{2p} \times 18^{4p} \times 22^{2p} \times 23^{4p} \times 24^{p}$.

(2). $HND_1II(H) = 14^{4p} \times 18^{8p} \times 22^{4p} \times 23^{8p} \times 24^{2p}$.

(3). $SNDII(H) = \left(\frac{1}{\sqrt{14}}\right)^{2p} \times \left(\frac{1}{\sqrt{18}}\right)^{4p} \times \left(\frac{1}{\sqrt{22}}\right)^{2p} \times \left(\frac{1}{\sqrt{23}}\right)^{4p} \times \left(\frac{1}{\sqrt{24}}\right)^{p}$.

Proof. Put $a = 1, 2, -\frac{1}{2}$, in equation (8), we obtain the desired results. □

Theorem 3.4. *Let H be the line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then*

$$ND_2^aII(H) = 49^{2pa} \times 77^{4pa} \times 132^{8pa} \times 144^{(18pq-19p)a}, \quad if \quad p>1, q>1 \tag{9}$$

$$= 49^{2ap} \times 77^{4ap} \times 121^{2pa} \times 132^{4pa} \times 144^{pa}, \quad if \quad p>1, q=1 \tag{10}$$

Proof. **Case 1:** Suppose $p=1$, $q>1$. From equation (2) and by using Table 3, we deduce

$$\begin{aligned} ND_2^aII(H) &= \prod_{uv\in E(H)} [D_H(u)\, D_H(v)]^a \\ &= (7\times 7)^{2pa} \times (7\times 11)^{4pa} \times (11\times 12)^{8pa} \times (12\times 12)^{(18pq-19p)a} \\ &= 49^{2pa} \times 77^{4pa} \times 132^{8pa} \times 144^{(18pq-19p)a}. \end{aligned}$$

Case 2: Suppose $p>1$, $q=1$. By using equation (2) and Table 4, we derive

$$\begin{aligned} ND_2^aII(H) &= \prod_{uv\in E(H)} [D_H(u)\, D_H(v)]^a \\ &= (7\times 7)^{2pa} \times (7\times 11)^{4pa} \times (11\times 11)^{2pa} \times (11\times 12)^{4pa} \times (12\times 12)^{pa} \\ &= 49^{2pa} \times 77^{4pa} \times 121^{2pa} \times 132^{4pa} \times 144^{pa}. \end{aligned}$$

□

We establish the following results by Theorem 3.4,

Corollary 3.5. *Let H be the line graph of sub-division graph of $TUC_4C_8[p,q]$ nanotube. Suppose $p>1$, $q>1$. Then*

(1). $ND_2II(G) = 49^{2p} \times 77^{4p} \times 132^{8p} \times 144^{18pq-19p}$.

(2). $HND_2II(H) = 49^{4p} \times 77^{8p} \times 132^{16p} \times 144^{2(18pq-19p)}$.

(3). $PNDII(H) = \left(\frac{1}{7}\right)^{2p} \times \left(\frac{1}{\sqrt{77}}\right)^{4p} \times \left(\frac{1}{\sqrt{132}}\right)^{8p} \times \left(\frac{1}{12}\right)^{18pq-19p}$.

(4). $RNDII(H) = 7^{2p} \times \left(\sqrt{77}\right)^{4p} \times \left(\sqrt{132}\right)^{8p} \times 12^{18pq-19p}$.

Proof. Put $a = 1, 2, -\frac{1}{2}, \frac{1}{2}$ in equation (9), we get the desired results. □

Corollary 3.6. *Let H be the line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Suppose $p>1$, $q=1$. Then*

(1). $ND_2II(G) = 49^{2p} \times 77^{4p} \times 121^{2p} \times 132^{4p} \times 144^{p}$.

(2). $HND_2II(H) = 49^{4p} \times 77^{8p} \times 121^{4p} \times 132^{8p} \times 144^{2p}$.

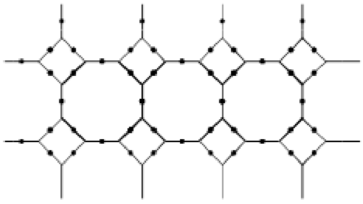
(3). $PNDII(H) = \left(\frac{1}{7}\right)^{4p} \times \left(\frac{1}{\sqrt{77}}\right)^{4p} \times \left(\frac{1}{\sqrt{121}}\right)^{2p} \times \left(\frac{1}{\sqrt{132}}\right)^{4p} \times \left(\frac{1}{12}\right)^{p}$.

(4). $RNDII(H) = 7^{4p} \times \left(\sqrt{77}\right)^{4p} \times \left(\sqrt{121}\right)^{2p} \times \left(\sqrt{132}\right)^{4p} \times 12^{p}$.

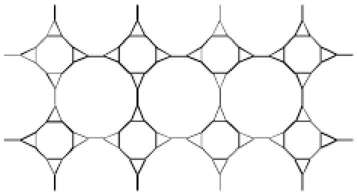
Proof. Put $a = 1, 2, -\frac{1}{2}, \frac{1}{2}$ in equation (10), we obtain the desired results. □

4. Results for $TUC_4C_8[p, q]$ Nanotorus

The line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotorus is depicted in Figure 4(b).



(a) Subdivision graph of $TUC_4C_8[p, q]$ nanotorus



(b) Line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotorus

Figure 4:

Let K be the line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotorus. A graph of $TUC_4C_8[p, q]$ nanotorus has $4pq$ vartices and $6pq$ edges. By Lemma, the subdivision graph of $TUC_4C_8[p, q]$ nanotorus has $10pq$ vertices and $12pq$ edges. Thus by Lemma 2, K has $12pq$ vertices and $18pq$ edges. Clearly the degree of each vertex is 3. The edge partition based on the degree sum of closed neighborhood vertices of each vertex is given in Table 5.

$D_K(u), D_K(v) \setminus uv \in E(K)$	(12, 12)
Number of edges	$18pq$

Table 5: Edge partition of K

Theorem 4.1. *Let K be the line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotorus. Then by using definitions and Table 5, we establish*

(1). $ND_1^aII\,(K) = \prod_{uv \in E(K)} [D_K\,(u) + D_K\,(v)]^a = 24^{18apq}$.

(2). $ND_1II\,(K) = 24^{18pq}$.

(3). $HND_1II\,(K) = 24^{36pq}$.

(4). $SNDII\,(K) = \left(\frac{1}{24}\right)^{9pq}$.

(5). $ND_2^aII\,(K) = \prod_{uv \in E(K)} [D_K\,(u)\,D_K\,(v)]^a = 144^{18apq}$.

(6). $ND_2II\,(K) = 144^{18pq}$.

(7). $HND_2II\,(K) = 144^{36pq}$.

(8). $PNDII\,(K) = \left(\frac{1}{12}\right)^{18pq}$.

(9). $RNDII\,(K) = 12^{18pq}$.

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