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Application of Soft Matrix Theory in Decision Making

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Abstract: In this paper, the traditional soft set theory is expanded to be a fuzzy one the fuzzy membership is used to describe parameter-approximate elements of fuzzy soft set. We then define products of soft matrices and their properties. We finally construct a soft max-min decision making method which can be successfully applied to the problem that contains uncertainties.

Keywords: Soft sets Soft matrix, Products of soft matrices, Soft max-min decision making.

1. Introduction

Most of our traditional tools for formal modelling, computing, reasoning, and data mining are crisp, deterministic. To solve complicated problems in economics and environment, medical science, we can't successfully use classical methods because of various types of uncertainties present in this problem. The theory of probability, the theory of fuzzy sets and the internal mathematics are three theories which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Molodtsov [1-4] introduce the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty which is free from the above difficulties .The soft sets that can be seen as a new mathematical theory for dealing with uncertainty which is free from the above difficulties. The soft set introduced by Pawlak [5-7], etc. is a set associated with a set of parameters and has been applied in several directions. Soft set theory has a rich potential for application in several directions, few of which had been shown by [8].

2. Theory of Soft Fuzzy Soft Set

Definition 2.1. *Let u be an initial universe, $P(U)$ be the power set of U, E be the set of all parameters and $A \subseteq E$. A pain (F, A) is called a soft set over U, where F is a mapping denoted by $F : E \to P(U)$. A soft set (F, E) will be denoted by (F_A, E) such that $(F_A, E) = \{[x, F(x)] : x \in E, F(x) \in P(U)\}$, where, $F(x) = \phi$ if $x \notin A$.*

Definition 2.2. *Let $U = \{u_1, u_2, u_3, \ldots, u_r\}$; $E = \{x_1, x_2, x_3, \ldots, x_S\}$ and $A \subseteq E$. Let, (F_A, E) be a soft set over U_{e_q}. If*

$$a_{jq} = \begin{cases} 1 & \text{if } U_j \in F(e_q) \\ 0 & \text{if } U_j \notin F(e_q) \end{cases}$$

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then the matrix

$$[a_{jq}]_{r\times s} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1s} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2s} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{r1} & a_{r2} & a_{r3} & \cdots & a_{rs} \end{bmatrix}$$

is called $r \times s$ *soft matrix of the soft set* (F_A, E) *over* U. *The set of all* $r \times s$ *soft matrices over* u *be denoted by* $SM_{r\times s}$. *Therefore* $[a_{jq}]$ *is a* $r \times s$ *soft matrix i.e.,* $[a_{jq}] \in SM_{r\times s}$.

Example 2.3. *Let* $U = \{u_1, u_2, u_3, u_4\}$ *is a universal set and* $E = \{x_1, x_2, x_3, x_4\}$ *is a set of parameters. Now if* $A = \{x_1, x_2, x_3\}$ *and* $F_A(x_1) = \{u_1, u_3, u_4\}$, $F_A(x_2) = \{u_2, u_3, u_4\}$, $F_A(x_4) = \phi$, $F_A(x_3) = \{u_4\}$,

$$(F_A, E) = \{(x_1, \{u_1, u_3, u_4\}), (x_2, \{u_2, u_3, u_4\}), (x, \{u_4\})\}$$

$$= \begin{bmatrix} (u_1, x_1) & (u_1, x_2) & (u_1, x_3) & (u_1, x_4) \\ (u_2, x_1) & (u_2, x_2) & (u_2, x_3) & (u_2, x_4) \\ (u_3, x_1) & (u_3, x_2) & (u_3, x_3) & (u_3, x_4) \\ (u_4, x_1) & (u_4, x_2) & (u_4, x_3) & (u_4, x_4) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Definition 2.4. *Assume,* $[a_{jq}]$ *and* $[b_{jq}] \in SM_{r\times s}$. *Union of* $[a_{jq}]$ *and* $[b_{jq}]$, *denoted* $[a_{jq}]\widetilde{\cup}[b_{jq}]$, *if* $c_{jq} = \max\{a_{jq}, b_{jq}\}$ *for all* j *and* q. *Intersection of* $[a_{jq}]$ *and* $[b_{jq}]$ *is* $[a_{jq}]\widetilde{\cap}[b_{jq}]$, *if* $c_{jq} = \min\{a_{jq}, b_{jq}\}$ *for all* j *and* q. *Complement of* $[a_{jq}]$ *is* $[a_{jq}]^c = [c_{jq}]$ *if* $c_{jq} = 1 - a_{jq}$ *for all* j *and* q.

Proposition 2.5. *Let* $[a_{jq}] \in SMr \times s$ *then* $[[a_{jq}]^c]^c = [a_{jq}]$; $[0]^c = 1$, *where 'c' denotes the complement matrix.*

Proof. For all j and q. Let

$$[a_{jq}] = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$[a_{jq}]^c = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$[[a_{jq}]^c]^c = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}^c$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [a_{jq}]$$

Hence proved.

For all j and q

$$[0] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[0]^c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^c$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= [1]$$

□

Proposition 2.6. *Let $[a_{jq}], [b_{jq}] \in SM_{m\times n}$. Then By De-Morgan's laws are valid*

$$\left([a_{jq}] \,\widetilde{\cup}\, [b_{jq}]\right)^c = [a_{jq}]^c \,\widetilde{\cap}\, [b_{jq}]^c$$

$$\left([a_{jq}] \,\widetilde{\cap}\, [b_{jq}]\right)^c = [a_{jq}]^c \,\widetilde{\cup}\, [b_{jq}]^c$$

Proof. Let $[a_{jq}] = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \Rightarrow [a_{jq}]^c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ and $[b_{jq}] = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \Rightarrow [b_{jq}]^c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

$$([a_{jq}]\widetilde{\cup}[b_{jq}])^c = \left(\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \widetilde{\cup} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \right)^c$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}^c$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[a_{jq}]^c \widetilde{\cap} [b_{jq}]^c = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \widetilde{\cap} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

□

3. Product of Soft Matrices

Definition 3.1. *Let $[a_{jq}], [b_{jk}] \in SM_{m\times n}$. Then And-product of $[a_{jq}]$ and $[b_{jk}]$ is given by $\wedge : SM_{r\times s_1} \times SM_{r\times s_2} \rightarrow SM_{r\times s_1.s_2}$; $[a_{jq}] \wedge [b_{jk}] = [c_{il}]$, where, $c_{jl} = \min\{a_{jq}, b_{jk}\}$ such that $l = s_2(\beta - 1) + k$, where, $q = \beta$ such that $l \leq \beta s_2$.*

Example 3.2. *Assume that $[a_{jq}] \in SM_{4\times 5}$ and $[b_{jk}] \in SM_{4\times 3}$ are given as follows*

$$[a_{jq}] = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}_{4\times 5} \quad \textit{and} \quad [b_{jk}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{4\times 3}$$

Then

$$[a_{jq}] \wedge [b_{jk}] = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}_{4\times 15}$$

$$= [c_{jl}]$$

Definition 3.3. *Let $[a_{jq}]$, $[b_{jk}] \in SM_{m\times n}$. Then And-product of $[a_{jq}]$ and $[b_{jk}]$ is defined by $\vee : SM_{r\times s_1} \times SM_{r\times s_2} \rightarrow SM_{r\times s_1.s_2}$; $[a_{jq}] \vee [b_{jk}] = [c_{jl}]$, where, $c_{jl} = \max\{a_{jq}, b_{jk}\}$ such that $l = s_2(\beta - 1) + k$, where, $q = \beta$ such that $l \leq \beta s_2$.*

Example 3.4. *Assume that $[a_{jq}] \in SM_{5\times 4}$ and $[b_{jk}] \in SM_{5\times 6}$ are given as follows*

$$[a_{jq}] = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}_{5\times 4}$$

$$[b_{jk}] = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}_{5\times 6}$$

$$[a_{jq}] \vee [b_{jk}] = \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&1&1&1&0&1 \\ 1&1&1&1&1&1&0&1&0&0&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&1&0&0&1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0 \\ 0&1&1&1&0&1&1&1&1&1&1&1&0&1&1&1&0&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&0&0&1&0&1&1&0&0&1&0&1&1 \end{bmatrix}_{5\times 24}$$

Definition 3.5. *Let* $[a_{jq}], [b_{jk}] \in SM_{m\times n}$. *Then And-product of* $[a_{jq}]$ *and* $[b_{jk}]$ *is defined by* $\overline{\wedge} : SM_{r\times s_1} \times SM_{r\times s_2} \to SM_{r\times s_1.s_2}$; $[a_{jq}]\overline{\wedge}[b_{jk}] = [c_{il}]$, *where,* $c_{jl} = \min\{a_{jq}, 1 - b_{jk}\}$ *such that* $l = s_2(\beta - 1) + k$, *where,* $q = \beta$ *such that* $l \leq \beta s_2$.

Example 3.6. *Assume that* $[a_{jq}] \in SM_{5\times 4}$ *and* $[b_{jk}] \in SM_{5\times 4}$ *are given as follows.*

$$[a_{jq}] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5\times 4}$$

$$[b_{jk}] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}_{5\times 4}$$

$$[a_{jq}]\overline{\wedge}[b_{jk}] = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&1&1&0&0&1&1&0 \\ 0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0 \\ 1&0&0&1&1&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&1&0&0&0&0&0&1&1&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1 \end{bmatrix}_{5\times 16}$$

$$= [c_{jl}]$$

Definition 3.7. *Let* $[a_{jq}], [b_{jk}] \in SM_{r\times s}$. *Then OR-NOT product of* $[a_{jq}]$ *and* $[b_{jk}]$ *is defined by* $\overline{\vee} : SM_{r\times s_1} \times SM_{r\times s_2} \to SM_{r\times s_1.s_2}$; $[a_{jq}]\overline{\vee}[b_{jk}] = [c_{il}]$, *where,* $c_{jl} = \max\{a_{jq}, 1 - b_{jk}\}$ *such that* $l = s_2(\beta - 1) + k$, *where,* $q = \beta$ *such that* $l \leq \beta s$.

Example 3.8. *Assume that* $[a_{jq}] \in SM_{4\times 3}$ *and* $[b_{jk}] \in SM_{4\times 5}$ *are given as follows.*

$$[a_{jq}] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}_{4\times 3}$$

$$[b_{jk}] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}_{4\times 5}$$

$$[a_{jq}]\nabla[b_{jk}] = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}_{4\times 15}$$

$$= [c_{jl}]$$

4. Soft Max-Min Decision Making

In this section, we firstly define soft max-min function, and then we construct a soft max- min decision making (SMmDM) method. It can be select optimum alternatives from the set of alternatives.

Definition 4.1. *Let,* $[c_{jl}]$ *a soft matrix,* $H_k = \{l : j, c_{jl} \neq 0,\ (k-1)\,s < l \leq ks\}$ *for all* $k \in H = \{1, 2, 3, \ldots, s\}$*. Then the soft max-min decision function denoted by Mm, is defined as follows:* $Mm : SM_{r\times s^2} \to SM_{r\times 1}$*;* $Mm[c_{jl}] = \left[\max_{k\in H}\{d_k\}\right]$*, where,* $d_k = \begin{cases} 0, & \text{if } H_k = \phi \\ \min\{c_{jl}\}, & \text{if } H_k \neq \phi \end{cases}$*. The single column soft matrix* $Mm[c_{jl}]$ *is called max-min decision soft matrix.*

Definition 4.2. *Let* $U = \{u_1, u_2, u_3, \ldots, u_r\}$ *be an initial universe and* $Mm[c_{jl}] = [e_{j1}]$*, then a subset of* U *can be obtained by using* $[e_{j1}]$ *as in the following way* $Opt[e_{j1}](U) = \{u_i : u_j \in U;\ e_{j1} = 1\}$*. Which is called an optimum set of* U*.*

Formulation of Problem: Assume that a real estate agent has a set of different types of houses $U = \{u_1, u_2, u_3, u_4\}$ which may be characterized by a set of parameters $E = \{x_1, x_2, x_3, x_4\}$. For $q = 1, 2, 3, 4$ the parameters x_q Stand for "cheap", 'in good location", "modern' 'large", respectively. These four attributes are characterized by the value sets {best, good, fair, poor}, {expensive, middle, cheap}, and {village, city} respectively.

Example 4.3. *Suppose that two men, Mr. Ram and Mr. Rahim, come to the real estate agent to buy a house. If each partner has to consider their own set of parameters, then we select a house on the basis of the sets of partners' parameters by using the SMmDM as follows*

Assume that $U = \{u_1, u_2, u_3, u_4\}$ *is a universal set and* $E = \{x_1, x_2, x_3, x_4\}$ *is a set of all parameters.*

Step 1: First, Mr. Ram and Mr. Rahim have to choose the sets of their parameters, $A = \{x_2, x_3, x_4\}$ *and* $B = \{x_1, x_2, x_4\}$ *respectively.*

Step 2: Then we can write the following soft matrices which are constructed according to their parameters

$$[a_{jq}] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}; \quad [b_{jk}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Step 3: Now, we can find a product of the soft matrices $[a_{jq}]$ and $[b_{jk}]$ by using and-product as follows.

$$[a_{jq}] \wedge [b_{jk}] = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1 \\ 0&0&0&0&1&1&0&0&1&1&0&0&1&1&0&0 \\ 0&0&0&0&0&1&0&1&0&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \end{bmatrix}_{4\times 16}$$

$$= [c_{jl}]$$

Here, we use And-product since both Mr. Ram and Mr. Rahim's choices have to be considered.

Step 4: We can find a max-min decision soft matrix as

$$Mm([a_{jq}] \wedge [b_{jk}]) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Step 5: Finally, we can find an optimum set of U according to $Mm([a_{jq}] \wedge [b_{jk}])$

$$Opt\ Mm([a_{jq}] \wedge [b_{jk}])(U) = \{u_2\}$$

Where u_2 is an optimum house to buy for Mr. Ram and Mr. Rahim. It is noted that the optimal set of U may contain more than one element.

5. Conclusion

In this paper, we define soft matrices which are a matrix representation of the soft sets. Then we introduced some operations of the soft matrix. Such as, And product, And-Not product, Or product, Or-Not product and then presented a decision making method using these products. We give an application for a real estate agent to choose an optimal house. We think these methods will present a new perspective to handle the decision making problems.

References

[1] D. A. Molodtsov, *Soft set theory-first results*, Computers and Mathematics with Applications, 37(1999), 19-31.

[2] D. A. Molodtsov, *The description of a dependence with the help of soft sets*, Journal of Computer and Systems Sciences International, 40(6)(2001), 977-984.

[3] D. A. Molodtsov, *The Theory of Soft Sets*, URSS Publishers, Moscow, (2004), (in Russian).

[4] D. A. Molodtsov, V. Yu. Leonov and D. V. Kovkov, *Soft sets technique and its application*, Nechetkie Sistemy i Myagkie Vychisleniya, 1(1)(2006), 8-39.

[5] Z. Pawlak, *Rough sets*, International Journal of Information and Computer Sciences, 11(1982), 341-356.

[6] Z. Pawlak, *Hard set and soft sets*, ICS Research Report, Institute of computer Science, Poland, (1994).

[7] D. V. Kovkov, V. M. Kolbanov and D. A. Molodtsov, *Soft sets theory-based optimization*, Journal of Computer and Systems Sciences International, 46(6).

[8] A. R. Roy and P. K. Maji, *A fuzzy soft set theoretic approach to decision making problems*, Journal of Computational and Applied Mathematics, 203(2007), 412-418.