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Non-Neighbor Reduced Randic and Sum-Connectivity Index

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Abstract: In this article, we have computed the non-neighbor reduced-Randic, sum-connectivity index and multiplicative non-neighbor reduced-Randic, sum-connectivity index for some standard graphs and for corona product of some graphs.

Keywords: Topological index, non-neighbors vertices, reduced topological indices.

1. Introduction

We consider finite, undirected simple graph G, having n vertices with m edges. Let $V(G)$ be the vertex set and $E(G)$ be the edge set of G, $d_G(u)$ denotes the degree of vertex u, $d(u,v)$ is the distance between the vertices u and v. Also, uv represent an edge between the two vertices u and v. For undefined terminologies we refer to [1]. A topological index is a numeric value mathematically derived from the graph representing a molecule. The mathematical and computational chemistry involving the computation of topological indices is a trending research topic. Topological indices are of two main categories, one depends on vertex distance and the other depends on vertex degree.

Among the topological indices, Zagreb indices is the oldest given by Gutman and Trinajstic [2] defined as $M_1(G) = \sum\limits_{uv\in E(G)} [d_G(u) + d_G(v)]$ and $M_2(G) = \sum\limits_{uv\in E(G)} [d_G(u) \times d_G(v)]$. As the years passed many degree based topological indices were introduced, among which Randic and sum-connectivity index are two such topological indices. Randic index was introduced in 1975 [3] which is defined as $R(G) = \sum\limits_{uv\in E(G)} \frac{1}{[d_G(u)\times d_G(v)]}$. Sum-connectivity index was introduced in 2009 [5] which is defined as $SCI(G) = \sum\limits_{uv\in E(G)} \frac{1}{[d_G(u)+d_G(v)]}$. The first multiplicative topological index was introduced in 1984 by Narumi and Katayama [4] which is defined as $NK(G) = \prod\limits_{u\in V(G)} d_G(u)$. The first reduced Zagreb index was studied in [6] which is $RM_2(G) = \sum\limits_{uv\in E(G)} [(d_G(u)-1)\times(d_G(v)-1)]$ and $RM_1(G) = \sum\limits_{uv\in E(G)} [(d_G(u)-1)+(d_G(v)-1)]$. Some of the non-neighbor topological indices are studied in [7]. Also, some work on Randic and multiplicative topological indices can be referred in [9, 10].

Motivated by these works, we define non-neighbor reduced Randic, sum-connectivity index and multiplicative non-neighbor reduced Randic, sum-connectivity index. The non-neighbors of vertex $u = \overline{N(G)} = \{v \in V(G) : d(u,v) \neq 1\}$, $|N(G)|$ is denoted by $\overline{d_G(u)} = n - 1 - d_G(u)$.

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Definition 1.1. *Non-neighbor reduced Randic index:*

$$\overline{RR}(G) = \sum_{uv \in E(G)} \frac{1}{\left[\left(\overline{d_G(u)} - 1\right) \times \left(\overline{d_G(v)} - 1\right)\right]}$$

Definition 1.2. *Non-neighbor reduced sum-connectivity index:*

$$\overline{RSCI}(G) = \sum_{uv \in E(G)} \frac{1}{\left[\left(\overline{d_G(u)} - 1\right) + \left(\overline{d_G(v)} - 1\right)\right]}$$

Definition 1.3. *Multiplicative non-neighbor reduced Randic index:*

$$\Pi\overline{RR(G)} = \prod_{uv \in E(G)} \frac{1}{\left[\left(\overline{d_G(u)} - 1\right) \times \left(\overline{d_G(v)} - 1\right)\right]}$$

Definition 1.4. *Multiplicative non-neighbor reduced sum connectivity index:*

$$\Pi\overline{RSCI(G)} = \prod_{uv \in E(G)} \frac{1}{\left[\left(\overline{d_G(u)} - 1\right) + \left(\overline{d_G(v)} - 1\right)\right]}$$

In this article, non-neighbor reduced - Randic, sum-connectivity index and multiplicative non-neighbor reduced Randic, sum-connectivity index are introduced. In section II, these new indices are obtained for some class of graphs. In section III, these new indices are computed for some corona product of graphs.

2. Non-Neighbor Reduced-Randic, Sum-Connectivity Index And Multiplicative Non-Neighbor Reduced-Randic, Sum-Connectivity Index For Class of Graphs

Here expression for non-neighbor reduced-Randic, sum-connectivity index and Multiplicative non-neighbor reduced-Randic, sum-connectivity Index of k-regular graph, cycle, path, complete bipartite graph, star graph and wheel graph are computed.

Theorem 2.1. *For the k-regular graph G $(k \geq 2)$ of order $n \geq 3$, $\overline{RR(G)} = \frac{nk}{2(n-2-k)}$; $\overline{RSCI(G)} = \frac{nk}{2\sqrt{2(n-2-k)}}$.*

Proof. A k-regular graph of order $n \geq 3$ has $\frac{nk}{2}$ number of edges. In these graphs the non-neighbors of each vertex is $(n-1-k)$. Hence for a k-regular graph G,

$$\overline{RR(G)} = \frac{nk}{2}\left[\frac{1}{\sqrt{(n-2-k)^2}}\right] = \frac{nk}{2(n-2-k)} \quad \textit{and}$$

$$\overline{RSCI(G)} = \frac{nk}{2}\left[\frac{1}{\sqrt{2(n-2-k)}}\right] = \frac{nk}{2\sqrt{2(n-2-k)}}.$$

□

Corollary 2.2. *For the k-regular graph G $(k \geq 2)$ of order $n \geq 3$, $\Pi\overline{RR(G)} = [n-2-k]^{-\frac{nk}{2}}$; $\Pi\overline{RSCI(G)} = [2(n-2-k)]^{-\frac{nk}{4}}$.*

Proof. $\Pi\overline{RR(G)} = \left[\frac{1}{\sqrt{(n-2-k)^2}}\right]^{\frac{nk}{2}} = [n-2-k]^{-\frac{nk}{2}}$ and $\Pi\overline{RSCI(G)} = \left[\frac{1}{\sqrt{2(n-2-k)}}\right]^{\frac{nk}{2}} = [2(n-2-k)]^{-\frac{nk}{4}}$. □

Corollary 2.3. *For a cycle C_n $(n \geq 4)$, $\overline{RR(G)} = \frac{n}{(n-4)}$; $\overline{RSCI(G)} = \frac{n}{\sqrt{2(n-4)}}$; $\Pi\overline{RR(G)} = [n-4]^{-n}$; $\Pi\overline{RSCI(G)} = [2(n-4)]^{-\frac{n}{2}}$.*

Proof. Putting $k = 2$, we get the results. □

Remark 2.4. *In the complete graph K_n, diameter is 1. Hence non-neighbor topological indices cannot be defined for K_n.*

Theorem 2.5. *For a path P_n $(n \geq 3)$, $\overline{RR(P_n)} = \frac{2}{\sqrt{(n-3)(n-4)}} + \frac{(n-3)}{(n-4)}$; $\overline{RSCI(P_n)} = \frac{2}{\sqrt{2n-7}} + \frac{(n-3)}{\sqrt{2(n-4)}}$.*

Proof. Let $u \in V(P_n)$, then $\overline{d_G(u)} = \begin{cases} (n-2) & \text{if } d_G(u) = 1 \\ (n-3) & \text{if } d_G(u) = 2 \end{cases}$ and $|E(P_n)| = (n-1)$.

$$\overline{RR(P_n)} = \frac{2}{\sqrt{(n-3)(n-4)}} + \frac{(n-3)}{\sqrt{(n-4)^2}} = \frac{2}{\sqrt{(n-3)(n-4)}} + \frac{(n-3)}{(n-4)}$$

$$\overline{RSCI(P_n)} = \frac{2}{\sqrt{(n-3)+(n-4)}} + \frac{(n-3)}{\sqrt{2(n-4)}} = \frac{2}{\sqrt{2n-7}} + \frac{(n-3)}{\sqrt{2(n-4)}}.$$

□

Corollary 2.6. *For a path P_n $(n \geq 3)$, $\Pi\overline{RR(P_n)} = \left[(n-3)(n-4)^{(n-2)}\right]^{-1}$; $\Pi\overline{RSCI(P_n)} = \frac{[2(n-4)]^{-\frac{(n-3)}{2}}}{(2n-7)}$.*

Proof.

$$\Pi\overline{RR(P_n)} = \left(\frac{1}{\sqrt{(n-3)(n-4)}}\right)^2 \left(\frac{1}{\sqrt{(n-4)^2}}\right)^{(n-3)} = \left[(n-3)(n-4)^{(n-2)}\right]^{-1} \quad \textit{and}$$

$$\Pi\overline{RSCI(P_n)} = \left(\frac{1}{\sqrt{(n-3)+(n-4)}}\right)^2 \left(\frac{1}{\sqrt{2(n-4)}}\right)^{(n-3)} = \frac{[2(n-4)]^{-\frac{(n-3)}{2}}}{(2n-7)}.$$

□

Theorem 2.7. *For a complete bipartite graph $K_{p,q}$ $(p, q \geq 1)$, $\overline{RR(K_{p,q})} = \frac{pq}{\sqrt{pq+2(2-(p+q))}}$; $\overline{RSCI(K_{p,q})} = \frac{pq}{\sqrt{p+q-4}}$.*

Proof. Let V_1 and V_2 be the partite sets of complete bipartite set with $|V_1| = p$ and $|V_2| = q$. Let $u \in V(K_{p,q})$, then $\overline{d_G(u)} = \begin{cases} (p-1) & \text{if } u \in V_1 \\ (q-1) & \text{if } u \in V_2 \end{cases}$ and $|E(K_{p,q})| = pq$.

$$\overline{RR(K_{p,q})} = pq\left[\frac{1}{\sqrt{(p-2)(q-2)}}\right] = \frac{pq}{\sqrt{pq+2(2-(p+q))}}$$

$$\overline{RSCI(K_{p,q})} = pq\left[\frac{1}{\sqrt{(p-2)+(q-2)}}\right] = \frac{pq}{\sqrt{p+q-4}}.$$

□

Corollary 2.8. *For a complete bipartite graph $K_{p,q}$ $(p, q \geq 1)$, $\Pi\overline{RR(K_{p,q})} = [pq + 2(2-(p+q))]^{-\frac{pq}{2}}$; $\Pi\overline{RSCI(K_{p,q})} = [p+q-4]^{-\frac{pq}{2}}$.*

Proof.

$$\Pi\overline{RR(K_{p,q})} = \left[\frac{1}{\sqrt{(p-2)(q-2)}}\right]^{pq} = [pq + 2(2-(p+q))]^{-\frac{pq}{2}} \quad \textit{and}$$

$$\Pi\overline{RSCI(K_{p,q})} = \left[\frac{1}{\sqrt{(p-2)+(q-2)}}\right]^{pq} = [p+q-4]^{-\frac{pq}{2}}.$$

□

Corollary 2.9. *For a star graph $K_{1,n}$ $(n \geq 2)$, $\overline{RR(K_{1,n})} = \frac{n}{\sqrt{2-n}}$; $\overline{RSCI(K_{1,n})} = \frac{n}{\sqrt{n-3}}$; $\Pi\overline{RR(K_{1,n})} = (2-n)^{-\frac{n}{2}}$; $\Pi\overline{RSCI(K_{1,n})} = (n-3)^{-\frac{n}{2}}$.*

Proof. Putting $p = 1$ and $q = n$, we get the results. □

Theorem 2.10. *For a wheel graph $W_{1,n}$ $(n \geq 4)$, $\overline{RR(W_{1,n})} = -n\left[\frac{i}{\sqrt{n-4}} - \frac{1}{n-4}\right]$; $\overline{RSCI(W_{1,n})} = n\left[\frac{1}{\sqrt{n-5}} - \frac{1}{\sqrt{2(n-4)}}\right]$.*

Proof. Let $u \in V(W_{1,n})$, then $\overline{d_G(u)} = \begin{cases} 0 & \text{if u is a central vertex} \\ (n-3) & \text{otherwise} \end{cases}$ and $|E(W_{1,n})| = 2n$.

$$\overline{RR(W_{1,n})} = \frac{n}{\sqrt{(-1)(n-4)}} + \frac{n}{\sqrt{(n-4)^2}} = -n\left[\frac{i}{\sqrt{n-4}} - \frac{1}{n-4}\right]$$
$$\overline{RSCI(W_{1,n})} = \frac{n}{\sqrt{(-1)+(n-4)}} + \frac{n}{\sqrt{2(n-4)}} = n\left[\frac{1}{\sqrt{n-5}} - \frac{1}{\sqrt{2(n-4)}}\right].$$

□

Corollary 2.11. *For a wheel graph $W_{1,n}$ $(n \geq 4)$, $\Pi\overline{RR(W_{1,n})} = \left[i(n-4)^{\frac{3}{2}}\right]^{-n}$; $\Pi\overline{RSCI(W_{1,n})} = [2(n-4)(n-5)]^{-\frac{n}{2}}$.*

Proof.

$$\Pi\overline{RR(W_{1,n})} = \left[\frac{1}{\sqrt{(-1)(n-4)}}\right]^n \left[\frac{1}{\sqrt{(n-4)^2}}\right]^n = \left[i(n-4)^{\frac{3}{2}}\right]^{-n} \quad \textit{and}$$
$$\Pi\overline{RSCI(W_{1,n})} = \left[\frac{1}{\sqrt{(-1)+(n-4)}}\right]^n \left[\frac{1}{\sqrt{2(n-4)}}\right]^n = [2(n-4)(n-5)]^{-\frac{n}{2}}.$$

□

3. Non-Neighbor Reduced-Randic, Sum-Connectivity Index And Multiplicative Non-Neighbor Reduced-Randic, Sum-Connectivity Index For Corona Product Of Some Graphs

In this section, we give expression for non-neighbor reduced-Randic, sum-connectivity index and Multiplicative non-neighbor reduced-Randic, sum-connectivity index of comb graph, sunlet graph, helm graph, fan graph and friendship graph. The corona product $G \bigodot H$ [8] of two graphs G and H, is the graph obtained by taking one copy of G and $|V(G)|$ copies of H, and by joining each vertex of the $\mathrm{i^{th}}$ copy of H to the $\mathrm{i^{th}}$ vertex of G; where $1 \leq i \leq |V(G)|$.

Theorem 3.1. *For a comb graph $G = P_n \odot K_1$ $(n \geq 3)$,*

$$\overline{RR(G)} = [2(n-2)]^{\frac{1}{2}}\left[(2n-3)^{-\frac{1}{2}} + (2n-5)^{-\frac{1}{2}}\right] + (2n-5)^{-1}\left[(n-3) + (n-2)(2n-3)^{-\frac{1}{2}}(2n-5)^{\frac{1}{2}}\right];$$
$$\overline{RSCI(G)} = 2\left[(4n-7)^{-\frac{1}{2}} + (4n-9)^{-\frac{1}{2}}\right] + 2^{-1}(n-2)^{\frac{1}{2}} + (n-3)[2(2n-5)]^{-\frac{1}{2}}.$$

Proof. Let $u \in V(G)$, then $\overline{d_G(u)} = \begin{cases} (2n-2) & \text{if } d_G(u) = 1 \\ (2n-3) & \text{if } d_G(u) = 2 \\ 2(n-2) & \text{if } d_G(u) = 3 \end{cases}$ and $|E(G)| = 2n-1$.

$$\overline{RR(G)} = 2[(2n-3)(2n-4)]^{-\frac{1}{2}} + 2[(2n-4)(2n-5)]^{-\frac{1}{2}} + (n-2)[(2n-3)(2n-5)]^{-\frac{1}{2}} + (n-3)\left[(2n-5)^2\right]^{-\frac{1}{2}}$$

$$= [2(n-2)]^{\frac{1}{2}}\left[(2n-3)^{-\frac{1}{2}}+(2n-5)^{-\frac{1}{2}}\right]+(2n-5)^{-1}\left[(n-3)+(n-2)(2n-3)^{-\frac{1}{2}}(2n-5)^{\frac{1}{2}}\right]$$

$$\overline{RSCI(G)} = 2[(2n-3)+(2n-4)]^{-\frac{1}{2}}+2[(2n-4)+(2n-5)]^{-\frac{1}{2}}+(n-2)[(2n-3)+(2n-5)]^{-\frac{1}{2}}+(n-3)[2(2n-5)]^{-\frac{1}{2}}$$

$$= 2\left[(4n-7)^{-\frac{1}{2}}+(4n-9)^{-\frac{1}{2}}\right]+2^{-1}(n-2)^{\frac{1}{2}}+(n-3)[2(2n-5)]^{-\frac{1}{2}}.$$

□

Corollary 3.2. *For a comb graph* $= P_n \odot K_1$ $(n \geq 3)$,

$$\Pi\overline{RR(G)} = \left[2^2(2n-3)^{\frac{n}{2}}(n-2)^2(2n-5)^{\frac{3(n-2)}{2}}\right]^{-1};$$

$$\Pi\overline{RSCI(G)} = [(4n-7)(4n-9)]^{-1}\left[2^{(3n-7)}(n-2)^{(n-2)}(2n-5)^{(n-3)}\right]^{-\frac{1}{2}}.$$

Proof.

$$\Pi\overline{RR(G)} = [(2n-3)(2n-4)]^{-1}[(2n-4)(2n-5)]^{-1}[(2n-3)(2n-5)]^{-\frac{(n-2)}{2}}\left[(2n-5)^2\right]^{-(n-3)}$$

$$= \left[2^2(2n-3)^{\frac{n}{2}}(n-2)^2(2n-5)^{\frac{3(n-2)}{2}}\right]^{-1} \quad and$$

$$\Pi\overline{RSCI(G)} = [(2n-3)+(2n-4)]^{-1}[(2n-4)+(2n-5)]^{-1}[(2n-3)+(2n-5)]^{-\frac{(n-2)}{2}}[2(2n-5)]^{-\frac{(n-3)}{2}}$$

$$= [(4n-7)(4n-9)]^{-1}\left[2^{(3n-7)}(n-2)^{(n-2)}(2n-5)^{(n-3)}\right]^{-\frac{1}{2}}.$$

□

Theorem 3.3. *For a sunlet graph* $G = C_n \odot K_1$ $(n \geq 3)$,

$$\overline{RR(G)} = n(2n-5)^{-1}\left\{\left[(2n-3)^{-1}(2n-5)\right]^{\frac{1}{2}}+1\right\};$$

$$\overline{RSCI(G)} = \frac{n}{2}\left\{(n-2)^{-\frac{1}{2}}+\left[2(2n-5)^{-1}\right]^{\frac{1}{2}}\right\}.$$

Proof. Let $u \in V(G)$, then $\overline{d_G(u)} = \begin{cases} (2n-2) & \text{if } d_G(u)=1 \\ 2(n-2) & \text{if } d_G(u)=3 \end{cases}$ and $|E(G)| = 2n$.

$$\overline{RR(G)} = n\left\{[(2n-3)(2n-5)]^{-\frac{1}{2}}+(2n-5)^{-1}\right\}$$

$$= n(2n-5)^{-1}\left\{\left[(2n-3)^{-1}(2n-5)\right]^{\frac{1}{2}}+1\right\}$$

$$\overline{RSCI(G)} = n\left\{[(2n-3)+(2n-5)]^{-\frac{1}{2}}+[2(2n-5)]^{-\frac{1}{2}}\right\}$$

$$= \frac{n}{2}\left\{(n-2)^{-\frac{1}{2}}+\left[2(2n-5)^{-1}\right]^{\frac{1}{2}}\right\}.$$

□

Corollary 3.4. *For a sunlet graph* $G = C_n \odot K_1$ $(n \geq 3)$, $\Pi\overline{RR(G)} = \left[(2n-3)(2n-5)^3\right]^{-\frac{n}{2}}$; $\Pi\overline{RSCI(G)} = \left[2^3(n-2)(2n-5)\right]^{-\frac{n}{2}}$.

Proof.

$$\Pi\overline{RR(G)} = \left\{[(2n-3)(2n-5)]^{-\frac{1}{2}}(2n-5)^{-1}\right\}^n$$

$$= \left[(2n-3)(2n-5)^3\right]^{-\frac{n}{2}} \quad and$$

$$\Pi\overline{RSCI(G)} = \left\{[(2n-3)+(2n-5)]^{-\frac{1}{2}}[2(2n-5)]^{-\frac{1}{2}}\right\}^n$$

$$= \left[2^3(n-2)(2n-5)\right]^{-\frac{n}{2}}.$$

□

Theorem 3.5. *For helm graph* $G = W_{1,n} \odot K_1 \backslash v_o v_o'$ *where* v_o *is the central vertex* $(n \geq 3)$, $\overline{RR(G)} = n[(n-1)(2n-5)]^{-\frac{1}{2}} \left\{2^{-\frac{1}{2}} + (n-1)^{\frac{1}{2}}(2n-5)^{-\frac{1}{2}} + 1\right\}$; $\overline{RSCI(G)} = n\left[(4n-7)^{-\frac{1}{2}} + [2(2n-5)]^{-\frac{1}{2}} + [3(n-2)]^{-\frac{1}{2}}\right]$.

Proof. Let $u \in V(G)$, then $\overline{d_G(u)} = \begin{cases} (2n-1) & \text{if } d_G(u) = 1 \\ (2n-4) & \text{if } d_G(u) = 4 \\ n & \text{if } d_G(u) = n \end{cases}$ and $|E(G)| = 3n$.

$$\begin{aligned}
\overline{RR(G)} &= n\left\{[(2n-2)(2n-5)]^{-\frac{1}{2}} + (2n-5)^{-1} + [(n-1)(2n-5)]^{-\frac{1}{2}}\right\} \\
&= n[(n-1)(2n-5)]^{-\frac{1}{2}}\left\{2^{-\frac{1}{2}} + (n-1)^{\frac{1}{2}}(2n-5)^{-\frac{1}{2}} + 1\right\} \\
\overline{RSCI(G)} &= n\left\{[(2n-2)+(2n-5)]^{-\frac{1}{2}} + [2(2n-5)]^{-\frac{1}{2}} + [(n-1)+(2n-5)]^{-\frac{1}{2}}\right\} \\
&= n\left[(4n-7)^{-\frac{1}{2}} + [2(2n-5)]^{-\frac{1}{2}} + [3(n-2)]^{-\frac{1}{2}}\right]
\end{aligned}$$

□

Corollary 3.6. *For helm graph* $(n \geq 3)$, $\Pi\overline{RR(G)} = \left[\sqrt{2}(n-1)(2n-5)^2\right]^{-n}$; $\Pi\overline{RSCI(G)} = [6(n-2)(2n-5)(4n-7)]^{-\frac{n}{2}}$.

Proof.

$$\begin{aligned}
\Pi\overline{RR(G)} &= \left\{[(2n-2)(2n-5)]^{-\frac{1}{2}}(2n-5)^{-1}[(n-1)(2n-5)]^{-\frac{1}{2}}\right\}^n = \left[\sqrt{2}(n-1)(2n-5)^2\right]^{-n} \quad \textit{and} \\
\Pi\overline{RSCI(G)} &= \left\{[(2n-2)+(2n-5)]^{-\frac{1}{2}}[2(2n-5)]^{-\frac{1}{2}}[(n-1)+(2n-5)]^{-\frac{1}{2}}\right\}^n \\
&= [6(n-2)(2n-5)(4n-7)]^{-\frac{n}{2}}.
\end{aligned}$$

□

Theorem 3.7. *For fan graph* $f_n = K_1 \odot P_n$ $(n \geq 4)$,

$$\begin{aligned}
\overline{RR(f_n)} &= [(n-3)(n-4)]^{-\frac{1}{2}}\left[-2i(n-4)^{\frac{1}{2}} - i(n-2)(n-3)^{\frac{1}{2}} + 2 + \left[(n-3)^{\frac{3}{2}}(n-4)^{\frac{1}{2}}\right]\right]; \\
\overline{RSCI(f_n)} &= [2(n-4)]^{-\frac{1}{2}}\left[2^{\frac{3}{2}} + (n-3)\right] + 2(2n-7)^{-\frac{1}{2}} + (n-2)(n-5)^{-\frac{1}{2}}.
\end{aligned}$$

Proof. Let $u \in V(f_n)$, then $\overline{d_G(u)} = \begin{cases} (n-2) & \text{if } d_G(u) = 2 \\ (n-3) & \text{if } d_G(u) = 3 \\ 0 & \text{if } d_G(u) = n \end{cases}$ and $|E(f_n)| = 2n-1$.

$$\begin{aligned}
\overline{RR(f_n)} &= 2[-(n-3)]^{-\frac{1}{2}} + (n-2)[-(n-4)]^{-\frac{1}{2}} + 2[(n-3)(n-4)]^{-\frac{1}{2}} + (n-3)(n-4)^{-1} \\
&= [(n-3)(n-4)]^{-\frac{1}{2}}\left[-2i(n-4)^{\frac{1}{2}} - i(n-2)(n-3)^{\frac{1}{2}} + 2 + \left[(n-3)^{\frac{3}{2}}(n-4)^{\frac{1}{2}}\right]\right] \\
\overline{RSCI(f_n)} &= 2[n-4]^{-\frac{1}{2}} + (n-2)[n-5]^{-\frac{1}{2}} + 2[(n-3)+(n-4)]^{-\frac{1}{2}} + (n-3)[2(n-4)]^{-\frac{1}{2}} \\
&= [2(n-4)]^{-\frac{1}{2}}\left[2^{\frac{3}{2}} + (n-3)\right] + 2(2n-7)^{-\frac{1}{2}} + (n-2)(n-5)^{-\frac{1}{2}}.
\end{aligned}$$

□

Corollary 3.8. *For fan graph* $f_n\,(n \geq 4)$, $\Pi\overline{RR(f_n)} = (-1)(n-3)^{-2}(4-n)^{-\frac{(n-2)}{2}}(n-4)^{-(n-2)}$; $\Pi\overline{RSCI(f_n)} = 2^{-\frac{(n-3)}{2}}(n-4)^{-\frac{(n-1)}{2}}(n-5)^{-\frac{(n-2)}{2}}(2n-7)^{-1}$.

Proof.

$$\begin{aligned}
\Pi\overline{RR(f_n)} &= (-1)(n-3)^{-1}(4-n)^{-\frac{(n-2)}{2}}[(n-3)(n-4)]^{-1}(n-4)^{-(n-3)} \\
&= (-1)(n-3)^{-2}(4-n)^{-\frac{(n-2)}{2}}(n-4)^{-(n-2)} \quad \textit{and} \\
\Pi\overline{RSCI(f_n)} &= [n-4]^{-1}[n-5]^{-\frac{(n-2)}{2}}[(n-3)+(n-4)]^{-1}[2(n-4)]^{-\frac{(n-3)}{2}} \\
&= 2^{-\frac{(n-3)}{2}}(n-4)^{-\frac{(n-1)}{2}}(n-5)^{-\frac{(n-2)}{2}}(2n-7)^{-1}.
\end{aligned}$$

□

Theorem 3.9. *For friendship graph* $F_n = K_1 \odot nK_2$ $(n \geq 2)$, $\overline{RR(F_n)} = n\left[2(3-2n)^{-\frac{1}{2}} + (2n-3)^{-1}\right]$; $\overline{RSCI(F_n)} = n\left[\sqrt{\frac{2}{n-2}} + \frac{1}{\sqrt{2(2n-3)}}\right]$.

Proof. Let $u \in V(F_n)$, then $\overline{d_G(u)} = \begin{cases} (2n-2) & \text{if } d_G(u) = 2 \\ 0 & \text{if } d_G(u) = 2n \end{cases}$ and $|E(F_n)| = 3n$.

$$\overline{RR(F_n)} = 2n(3-2n)^{-\frac{1}{2}} + n(2n-3)^{-1} = n\left[2(3-2n)^{-\frac{1}{2}} + (2n-3)^{-1}\right]$$

$$\overline{RSCI(F_n)} = 2n(2n-4)^{-\frac{1}{2}} + n[2(2n-3)]^{-\frac{1}{2}} = n\left[\sqrt{\frac{2}{n-2}} + \frac{1}{\sqrt{2(2n-3)}}\right].$$

□

Corollary 3.10. *For friendship graph* F_n $(n \geq 2)$, $\Pi\overline{RR(F_n)} = \left[(-1)(2n-3)^2\right]^{-n}$; $\Pi\overline{RSCI(F_n)} = \left[2^{\frac{3}{2}}(n-2)(2n-3)^{\frac{1}{2}}\right]^{-n}$.

Proof.

$$\Pi\overline{RR(F_n)} = [(-1)(2n-3)]^{-n}(2n-3)^{-n} = \left[(-1)(2n-3)^2\right]^{-n} \quad \textit{and}$$

$$\Pi\overline{RSCI(F_n)} = [2(n-2)]^{-n}[2(2n-3)]^{-\frac{n}{2}} = \left[2^{\frac{3}{2}}(n-2)(2n-3)^{\frac{1}{2}}\right]^{-n}.$$

□

4. Conclusion

In this paper we have introduced new topological index which are non-neighbor reduced-Randic, sum-connectivity index and multiplicative reduced-Randic, sum-connectivity index. These topological indices have been computed for some standard graphs and for corona product of some graphs.

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